

Chapter six The goodness of fit test

9)MLR test 2:

9.1)The multi-nominal distribution is the sampling distribution, each category has a probability and the summation of probability is 1.

category	1	2	k
probability	p_1	p_2	p_k

$$p_1 + \dots + p_k = 1,$$

$$f(x_1, \dots, x_k) = \frac{n!}{x_1! \times \dots \times x_k!} \times p_1^{x_1} \times \dots \times p_k^{x_k}, x_1 + \dots + x_k = n \quad \text{that is trial number.}$$

9.2)

$$X_i = np_i + \varepsilon_i, E(\varepsilon_i) = 0, E[(\varepsilon_i)^2] = np_i(1 - p_i), i = 1, 2, \dots, k$$

$$X_i - np_i = \varepsilon_i, E\left[\frac{(\varepsilon_i)^2}{np_i}\right] = E\left[\frac{(X_i - np_i)^2}{np_i}\right] = 1 - p_i,$$

$$\sum_{i=1}^k E\left[\frac{(X_i - np_i)^2}{np_i}\right] = E\left[\sum_{i=1}^k \left(\frac{(X_i - np_i)^2}{np_i}\right)\right] = k - \sum_{i=1}^k p_i = k - 1,$$

$$\sum_{i=1}^k \left(\frac{(X_i - np_i)^2}{np_i}\right) \rightarrow \chi_{k-1}^2$$

$$H_0 : p_1 = p_{01}, p_2 = p_{02}, \dots, p_k = p_{0k} \quad H_1 : \text{against } H_0$$

$p_{01}, p_{02}, \dots, p_{0k}$ are known value and $p_{01} + p_{02} + \dots + p_{0k} = 1,$

$$E_i = np_{i0}, i = 1, 2, \dots, k, O_i = x_i,$$

O_i : The observed sample number of cell i,

E_i : The expected sample number of cell i,

$$\chi_v^2 = \sum_{i=1}^k \frac{(O_i - E_i)^2}{O_i}, \quad v = k - 1 - (\text{number of point estimator}) \text{ is degree of freedom of}$$

pearsson chi square test.

$$\chi_v^2 > \chi_{\alpha, v}^2 \Rightarrow \text{reject } H_0.$$

9.3)The process of test

H_0 : Population distribution is a continuous probability distribution,

H_1 : against H_0

The sample size is n.

The process: i)The class number of frequency distribution: $k = \log_2(n) + 1.$

ii)The class limit of frequency distribution:

The class limit will be found by the a general frequency distribution table.

iii)The class sample number of frequency distribution(O_i):

The frequency distribution is done and getting the O_i of each class.

iv) The class expected number of frequency distribution(E_i):

$E_i = n \times$ the probability of each class.

Note: There are 20 kinds of continuous probability distribution that is can be assigned to null hypothesis.

9.4) Example (The simulated sample data and computing the result by the P_S_CCC)

The Likelihood ratio chi square test (goodness of fit) ,the traditional frequency distribution, please select the population distribution

1.H0:Uniform distribution	13.H0:Gumbel distribution
2.H0:Normal distribution	14.H0:Triangular 1 distribution
3.H0:Shifted exponential distribution	15.H0:Trapezoid distribution
4.H0:Pareto 1 distribution	16.H0:U-quadractic distribution
5.H0:Pareto 2 distribution	17.H0:Semi-circle distribution
6.H0:Rayleigh distribution	18.H0:Logistic distribution
7.H0:Double expoenoential distribution	19.H0:Weibull distribution
8.H0:Log normal distribution	20.H0:Pareto 3 distribution
9.H0:Gamma distribution	** Above Ho population all do once
10.H0:Beta distribution	
11.H0:Cauchy distribution	
12.H0:Arcsin distribution	

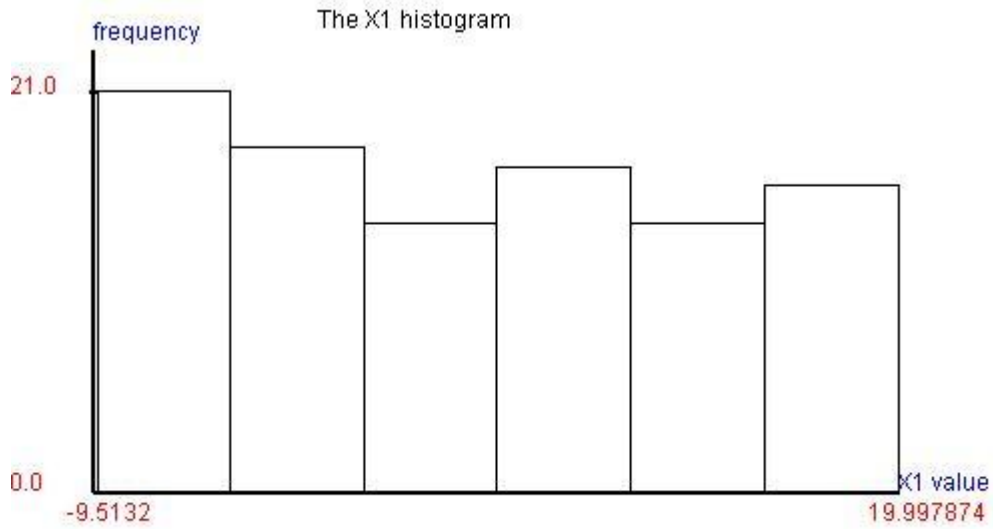
9.4.1)The population distribution is uniform distribution.

X1 is Uniform(alpha=-10.000000,beta=20.000000),

X1
16.0148324459
-0.1645587974
19.9978740573
13.7584293220
9.3681934442
19.4012105777
13.4644226164
-0.4809078048
-9.2801655751
14.0043112313
5.4704060214
-8.3619159609
7.7946828771
8.4871805833
0.2823806262
-3.8480125069
0.5548084042
-1.2458474689
19.1374332454
6.8375128782
17.5806037555
-6.4245674802
10.3140162125
7.0798608407
4.2832109062
6.2983046694
1.8448329834
-9.4413981740
13.3058418462
18.0056983145
4.6639613667
-0.2055973392
-7.6116737157
8.7254121406
-0.9729065583
16.6310603482
0.0378857597
18.2118121950
-7.9965132051
3.7245553168
4.2969466944
2.8840752397
-1.1452969076
-8.9276163471
-5.0682340250
-9.5132595670
-3.8902855826
-3.3176520022
14.3564851961
-0.9836435414
-0.8421376655
5.4131507221
-2.4156573393
11.4918888337
12.1573481959
5.0594178705
16.5600290999
13.1364799572
3.0862327619
3.7620222269
9.1319553498
-7.4920589484
11.7491885920
-2.2321907440
8.0266577772
3.1702891665
3.5276132567
-6.3639257616
10.1051685814
7.4473706764
-3.2806920100

-6.0612162319
 -8.7748775910
 18.8325729968
 16.2654005504
 -9.4801550919
 17.0305419511
 -5.3896848269
 11.6603609489
 -5.6304293585
 -5.0472095229
 -4.7394278507
 11.8968073299
 -3.8084519454
 7.7785085707
 2.1938834548
 -4.7856009719
 6.7125634413
 -5.9447943607
 0.3990877827
 16.4990206441
 10.7944801889
 15.6720121051
 17.5494032251
 10.1198659391
 14.4280215709
 -2.0862700456
 6.1335299486
 -8.9612776399
 18.6527049782

X1 is Uniform(alpha=-10.000000,beta=20.000000),



H0: $X_1 \sim \text{Uniform}(\alpha, \beta)$, α, β are unknown

alpha point estimated value = -9.513260 (MLE)

beta point estimated value = 19.997874 (MLE)

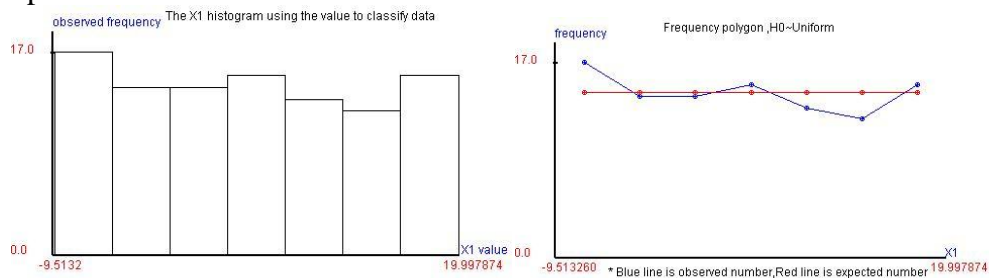
pearson goodness of fit

class	[1]	[2]	[3]	[4]
lower limit	-9.51326	-5.29738	-1.08151	3.13437
7.35025	11.56612	15.78200		
upper limit	-5.29738	-1.08151	3.13437	7.35025
11.56612	15.78200	19.99787		
observed no	17.00000	14.00000	14.00000	15.00000
13.00000	12.00000	15.00000		
probability	0.14286	0.14286	0.14286	0.14286
0.14286	0.14286	0.14286		
expected no	14.28571	14.28571	14.28571	14.28571
14.28571	14.28571	14.28571		
chi square	0.43337	0.00583	0.00583	0.03401
0.12716	0.43537	0.03401		

degree of freedom = 4

Likelihood ratio chi-square test statistic = 1.075595

p-value = 0.898100

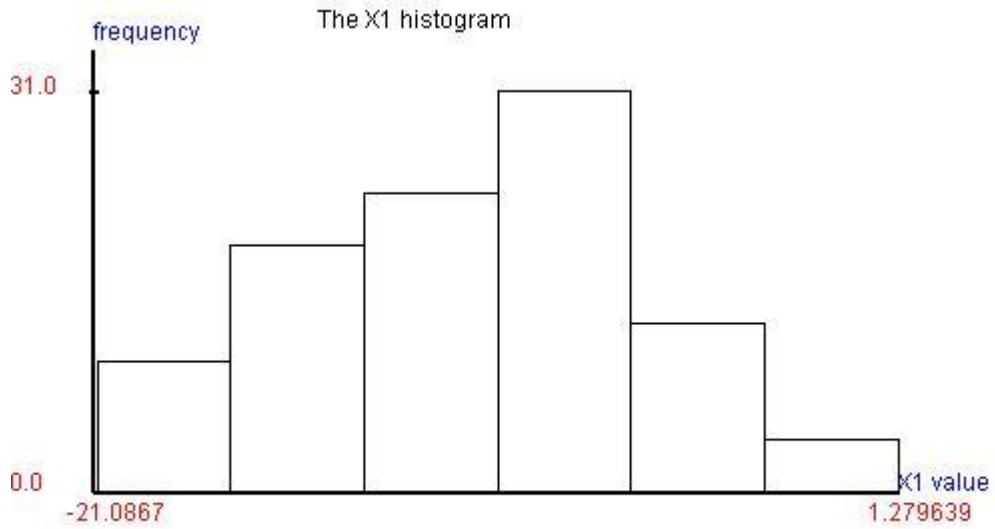


9.4.2)The population distribution is normal distribution.
X1 is Normal($\mu=-10.000000$, $\sigma^2=25.000000$),

X1
-13.3498035256
-8.7564925642
-11.9930564397
-5.1602962304
-16.6532964531
-7.8159210040
-9.7008664157
-13.1910828261
-21.0867245745
-10.8569787301
-16.1717283714
-13.5361015012
-8.4656334054
-6.4163098121
-16.0772773755
-17.2338190334
-8.0374096537
-0.0460707373
-5.5333169761
-18.2080634228
-3.8845509165
-14.8049987119
-20.2009897463
-13.1932352941
-14.7432016863
-6.3017701150
-8.1858612917
-5.3203727337
-15.3246357137
-8.0893862584
-10.3186918621
-8.2219614198
-5.3468579021
-5.0277244793
-6.3150405270
-5.2364908671
-15.9370825333
-19.6915810070
-6.2381979414
-13.9031370716
-15.1234297119
-8.9419105921
-2.5533480548
-7.1312349628
-18.0341523366
-10.4451840929
-15.2345740206
-11.9278117627
-9.7886500637
-15.2191778022
-12.4554910888
-8.5296446780
-15.1389888636
-7.9377651241
-7.9156283370
-10.8434543899
-4.0083921689
-7.7029526966
0.3272470669
-13.8176050424
-6.3867051161
-8.1070462225
-13.4121550404
-10.3335285860
-10.2928169430
-2.6170038623
-12.4755455148
-11.0338007143
-9.5948813967
-3.1902930898
-19.2764885941

-16.3497655773
 1.2796392301
 -4.2757703786
 -7.9839231163
 -9.0955518535
 -14.7568347580
 -13.4278556770
 -11.2606602444
 -17.5956649222
 -10.6686252845
 -9.4011303875
 -14.1500155906
 -18.7162528446
 -6.2772799767
 -14.0217474682
 -13.6646727262
 -9.5968363114
 -10.7781580612
 -11.1816312692
 -19.9213527096
 -6.6371585698
 -10.7916047555
 -7.1720584034
 -1.8961766187
 -19.1090035003
 -6.6818227885
 -7.2831954719
 -10.5695748427
 -5.5778377588

X1 is Normal($\mu=-10.000000$, $\sigma^2=25.000000$),



H0: $X_1 \sim \text{Normal}(\mu, \sigma^2)$, μ, σ are unknown
 population mean(μ) point estimated value=-10.512810 (MLE)
 population variance(σ^2) which point estimated value=24.417695 (MLE)
 pearson goodness of fit

class	[1]	[2]	[3]	[4]
lower limit	-21.08672	-17.89153	-14.69633	-11.50114
-5.11075	-1.91556			
upper limit	-17.89153	-14.69633	-11.50114	-8.30595
-1.91556	1.27964			
observed no	9.00000	15.00000	15.00000	23.00000
27.00000	7.00000	4.00000		
probability	0.06770	0.13100	0.22210	0.25170
0.19040	0.09620	0.04090		
expected no	6.77000	13.10000	22.21000	25.17000
19.04000	9.62000	4.09000		
chi square	0.55254	0.24067	3.46561	0.20473
2.34673	0.98063	0.00203		

Likelihood ratio chi square test statistic=7.792932
 degree of freedom=4,p-value=0.351200

correction:

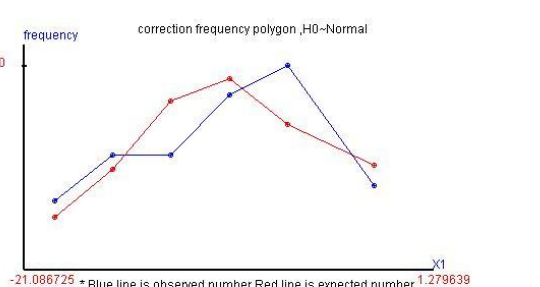
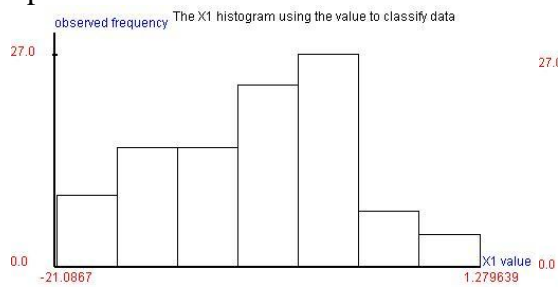
expected number ≥ 5 in each cell, the frequency table is adjusted

class	[1]	[2]	[3]	[4]
lower limit	-21.08672	-17.89153	-14.69633	-11.50114
-5.11075				
upper limit	-17.89153	-14.69633	-11.50114	-8.30595
1.27964				
observed no	9.00000	15.00000	15.00000	23.00000
27.00000	11.00000			
probability	0.06770	0.13100	0.22210	0.25170
0.19040	0.13710			
expected no	6.77000	13.10000	22.21000	25.17000
19.04000	13.71000			
chi square	0.55254	0.24067	3.46561	0.20473
2.34673	0.66765			

degree of freedom=3

Likelihood ratio chi-square test statistic =7.477924

p-value=0.058100



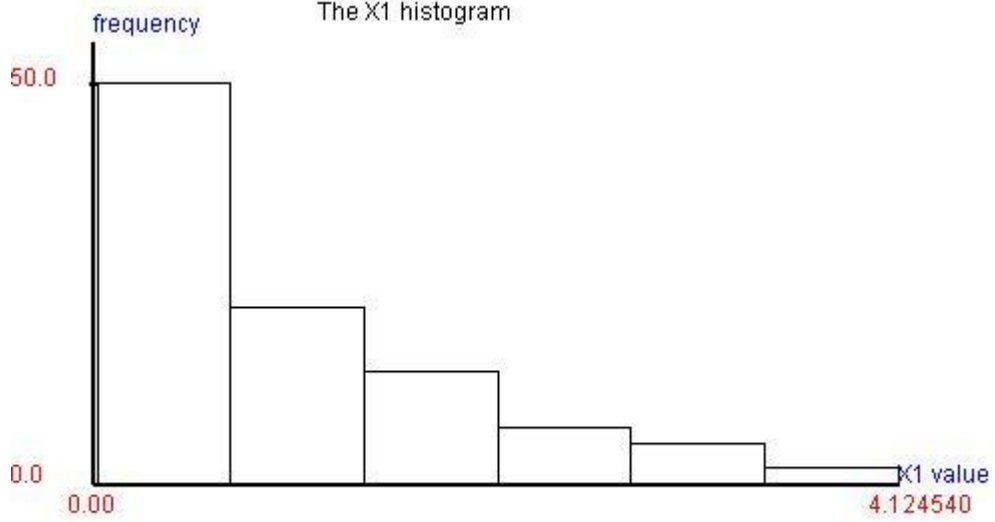
9.4.3)The population distribution is shifted exponential distribution.

X1 is Exponential($\lambda=1.000000, c=0.000000$),

X1
0.9394308799
1.1939298856
3.3357139264
0.1763734167
0.7291305607
0.1323043767
1.9541569692
1.8309962048
1.9715067083
0.3608083964
0.8800880650
0.1071981165
1.3441431557
0.3864180266
0.7827668744
0.0839891945
0.4194794595
1.6728135969
1.6982224203
1.9288081778
0.1245989351
0.6424459424
1.5487437635
2.2689170694
1.6541979775
0.5954085451
0.9374142590
0.5068469958
0.4796436791
0.8869881643
0.8289855425
0.6289117718
1.6507186170
0.8910973477
0.3522979015
0.2299708901
1.3578575089
0.0084183346
2.5061054170
1.0668609774
0.0612661825
0.1539395316
4.1245402724
0.9839829173
0.3173270280
1.5954470234
0.4319733855
0.4925599194
1.7519691457
2.5107373177
2.3956417893
0.2955109537
0.3150698969
0.1150163104
0.2050536624
0.0269912032
0.3470110899
0.0087913657
1.7465586683
0.3761705326
1.0414467645
0.8295547185
2.9620277066
1.3479720270
2.7789535070
0.1714413490
0.4719218244
0.0164585541
0.3482375179
0.9813527318
1.3530559063

2.1757202954
 2.3996371917
 0.1501357197
 0.6282781119
 3.0895100625
 0.6168465486
 3.5947554694
 0.9637356863
 0.4823659370
 0.5118185345
 0.1426203646
 1.0245169303
 0.6276803436
 0.1363295601
 2.0960649156
 3.1869906065
 0.0853614934
 0.0257708177
 0.1008286398
 0.6124579250
 0.0127625014
 0.4850542044
 0.8602314006
 1.8494975089
 0.1592436091
 0.2131732323
 0.1514629614
 1.9969725552
 0.8059895278

The X1 histogram



H0: $X_1 \sim \text{Shifted exponential}(\lambda, c)$, λ, c are unknown

λ point estimated value = 1.000033 (MLE)

c point estimated value = 0.008418 (MLE)

pearson goodness of fit

class	[1]	[2]	[3]	[4]
lower limit	0.00842	0.59644	1.18445	1.77247
2.36049	2.94851	3.53652		
upper limit	0.59644	1.18445	1.77247	2.36049
2.94851	3.53652	4.12454		
observed no	44.00000	23.00000	13.00000	9.00000
5.00000	4.00000	2.00000		
probability	0.44458	0.24693	0.13715	0.07617
0.04231	0.02350	0.02936		
expected no	44.45835	24.69290	13.71484	7.61745
4.23086	2.34989	2.93571		
chi square	0.00477	0.12460	0.03931	0.21238
0.11832	0.68072	0.43777		

Likelihood ratio chi square test statistic = 1.617876

degree of freedom = 4, p-value = 0.977900

correction:

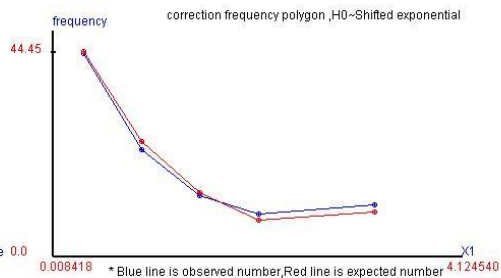
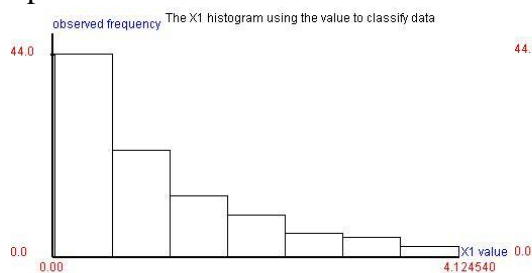
expected number ≥ 5 in each cell, the frequency table is adjusted

class	[1]	[2]	[3]	[4]
lower limit	0.00842	0.59644	1.18445	1.77247
2.36049				
upper limit	0.59644	1.18445	1.77247	2.36049
4.12454				
observed no	44.00000	23.00000	13.00000	9.00000
11.00000				
probability	0.44458	0.24693	0.13715	0.07617
0.09516				
expected no	44.45835	24.69290	13.71484	7.61745
9.51645				
chi square	0.00477	0.12460	0.03931	0.21238
0.20008				

degree of freedom = 2

Likelihood ratio chi-square test statistic = 0.581153

p-value = 0.747800



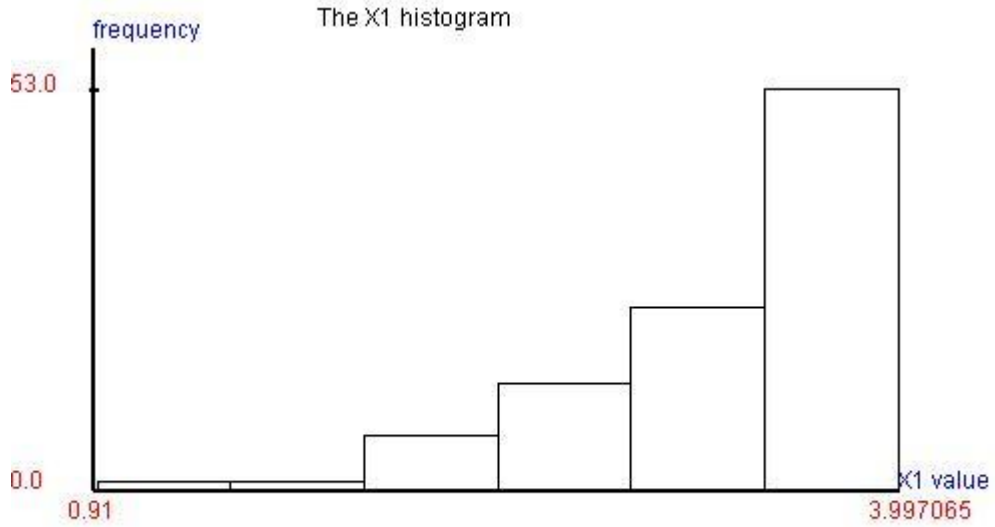
9.4.4)The population distribution is pareto1 distribution.

X1 is Pareto1(lamda=5.000000,c=4.000000),

X1
3.2119521586
3.775666509
3.8757758470
3.9588294732
3.9631402533
2.9961679685
2.9297826407
3.9863282569
3.8248803017
0.9182544777
3.9575599076
3.8423545433
3.7785554441
3.1251888791
3.9106674045
3.1750711360
3.1179085699
3.9327556774
2.9003338671
3.5186353276
3.8722181474
3.6910494465
2.1234110744
3.9323856405
3.8782176102
3.2182239002
3.9385039680
2.9462152212
3.1986454514
3.3092838036
3.6506029294
3.9019219074
2.6860158859
3.0062049661
3.8166090211
2.9286161540
3.9130359179
2.8317506415
2.2599835652
2.1522065318
3.4164640609
3.9043332593
3.9532627132
2.6856078227
1.9805469673
3.6218165822
3.0260888362
2.5626620528
3.9679312928
3.9469672298
2.6654197500
2.2438132281
2.4508646406
2.9321977095
3.8275353478
3.7963123267
3.7777404276
3.5786725913
3.8579687852
3.5931051723
3.3835640779
3.6195392363
2.5737384900
3.8709852892
3.9970654585
3.6524833067
3.3038590376
1.8862703493
3.5773310297
3.6425727234
3.4502245288

3.3396845279
 2.2923293652
 3.7571039667
 2.4774332813
 3.4487054322
 3.0683357865
 3.3954707467
 3.0319142883
 3.8915750407
 3.6546563777
 3.2845594149
 3.2761820861
 3.9848867738
 3.6812236536
 3.7069106505
 3.9516637503
 2.8105685972
 3.8783663341
 2.5142720292
 3.8803824016
 3.6487897988
 3.3506067198
 3.5819593890
 3.2485941792
 3.8708430744
 3.3798912640
 3.6026020926
 3.5603863198
 3.7276997734

X1 is Pareto1(lamda=5.000000,c=4.000000),



H0: $X_1 \sim \text{Pareto 1}(\lambda, c)$, λ, c are unknown

λ point estimated value = 5.153388 (MLE)

c point estimated value = 3.997065 (MLE)

pearson goodness of fit

class	[1]	[2]	[3]	[4]
lower limit	0.91825	1.35808	1.79791	2.23774
2.67758	3.11741	3.55724		
upper limit	1.35808	1.79791	2.23774	2.67758
3.11741	3.55724	3.99707		
observed no	1.00000	0.00000	4.00000	9.00000
14.00000	20.00000	52.00000		
probability	0.00384	0.01245	0.03403	0.07654
0.15092	0.27061	0.45161		
expected no	0.38372	1.24526	3.40258	7.65407
15.09242	27.06110	45.16083		
chi square	0.37980		0.08923	0.20128
0.08524	2.49296	0.89950		

Likelihood ratio chi square test statistic = 1.#INF00

degree of freedom = 4, p-value = 0.000000

correction:

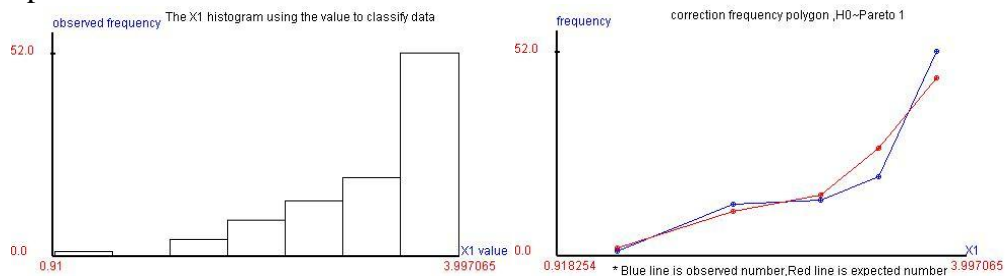
expected number ≥ 5 in each cell, the frequency table is adjusted

class	[1]	[2]	[3]	[4]
lower limit	0.91825	1.79791	2.67758	3.11741
3.55724				
upper limit	1.79791	2.67758	3.11741	3.55724
3.99707				
observed no	1.00000	13.00000	14.00000	20.00000
52.00000				
probability	0.01629	0.11057	0.15092	0.27061
0.45161				
expected no	1.62899	11.05665	15.09242	27.06110
45.16083				
chi square	0.39563	0.29051	0.08524	2.49296
0.89950				

degree of freedom = 2

Likelihood ratio chi-square test statistic = 4.163834

p-value = 0.124600



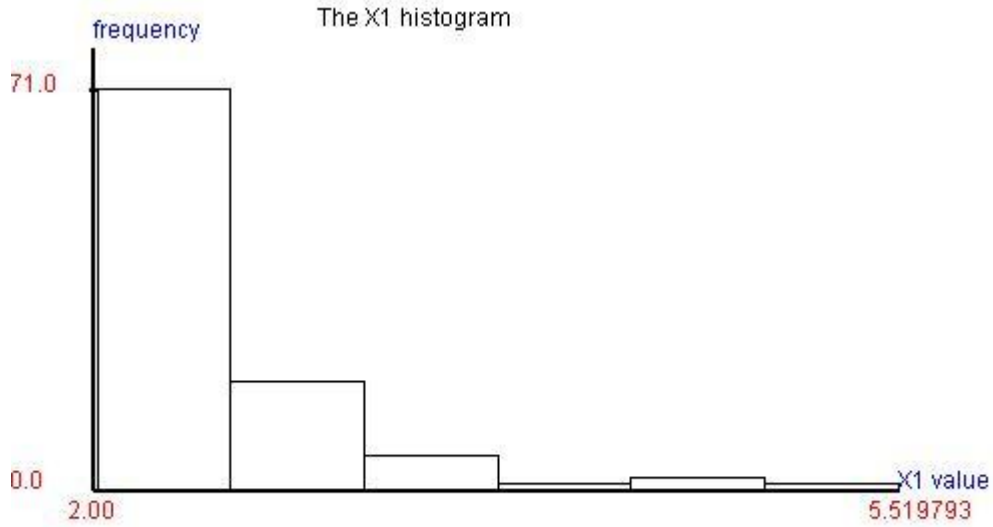
9.4.5)The population distribution is pareto2 distribution.

X1 is Pareto2(lamda=4.000000,c=2.000000),

X1
2.1298789478
2.3663163023
2.6802655086
2.1565038783
2.8269575210
2.2847688651
2.0569199256
2.2155795421
2.5698604110
2.1773780150
2.1633592813
2.1371300202
2.1744113863
4.5007584444
2.0670634620
2.3413390703
2.4573945912
2.0286229265
2.2451366635
2.0706763381
2.1384909284
2.0874218372
2.0660525800
2.8402611330
3.1081528236
2.2486457894
3.8700854052
2.4644941956
3.1260251491
2.1619318403
2.1024054727
2.0554145366
2.1817927098
2.0904512649
3.4499724645
2.1223372237
2.9276962944
3.1112259319
3.0903602545
2.2860375312
2.7701130466
2.4356235259
2.0359003652
2.6219083339
2.0765502513
2.4103472622
3.5065638427
2.3215178156
2.6798075882
2.7848203677
2.0271975950
2.1988448629
2.1043349929
2.0328772828
2.2967274646
2.7308167453
2.2710295520
3.5312920034
2.0251529827
2.1495691666
2.1827694107
2.0218278626
2.4034670414
2.2043638483
2.5338405906
2.0194252755
2.8973592235
2.0512256398
3.1006562366
3.1491690508
2.1333621560

3.4616195647
 2.3774155838
 3.3218030244
 2.2038928668
 3.6488214453
 5.5197936535
 2.3410428708
 2.2833279909
 2.2585617079
 2.3989844467
 2.6701062118
 2.0246385942
 2.0264536856
 2.6824873923
 2.3481749348
 2.5652712788
 2.0135491817
 2.1741077197
 2.1440524544
 2.3061885490
 2.0303455647
 2.3544709740
 2.9699602598
 2.0059853038
 2.0786694801
 2.3283837010
 2.5654472058
 4.8353498036
 2.4054551262

X1 is Pareto2(lamda=4.000000,c=2.000000),



H0: $X_1 \sim \text{Pareto } 2(\lambda, c)$, λ, c are unknown

λ point estimated value=4.953361 (MLE)

c point estimated value=2.005985 (MLE)

pearson goodness of fit

class	[1]	[2]	[3]	[4]	[5]	[6]	[7]
lower limit	2.00599	2.50796	3.00993	3.51190			
4.01388	4.51585	5.01782					
upper limit	2.50796	3.00993	3.51190	4.01388			
4.51585	5.01782	5.51979					
observed no	67.00000	17.00000	10.00000	3.00000			
1.00000	1.00000	1.00000					
probability	0.66920	0.19681	0.07158	0.03021			
0.01424	0.00731	0.01066					
expected no	66.92033	19.68057	7.15788	3.02112			
1.42380	0.73060	1.06570					
chi square	0.00009	0.42267	0.80777	0.00015			
0.17960	0.07258	0.00432					

Likelihood ratio chi square test statistic=1.487180

degree of freedom=4,p-value=0.982700

correction:

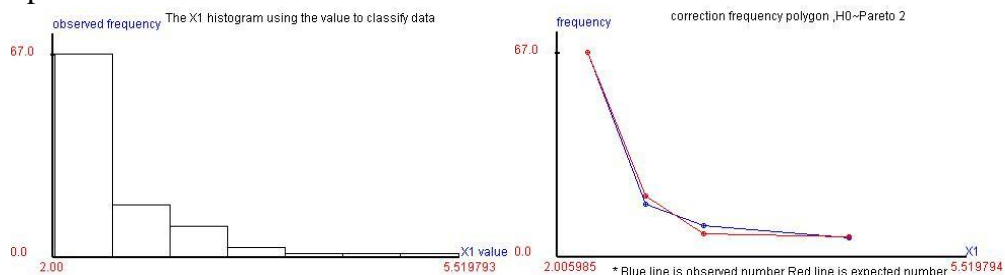
expected number ≥ 5 in each cell, the frequency table is adjusted

class	[1]	[2]	[3]	[4]
lower limit	2.00599	2.50796	3.00993	3.51190
upper limit	2.50796	3.00993	3.51190	5.51979
observed no	67.00000	17.00000	10.00000	6.00000
probability	0.66920	0.19681	0.07158	0.06241
expected no	66.92033	19.68057	7.15788	6.24122
chi square	0.00009	0.42267	0.80777	0.00970

degree of freedom=1

Likelihood ratio chi-square test statistic =1.240233

p-value=0.265400



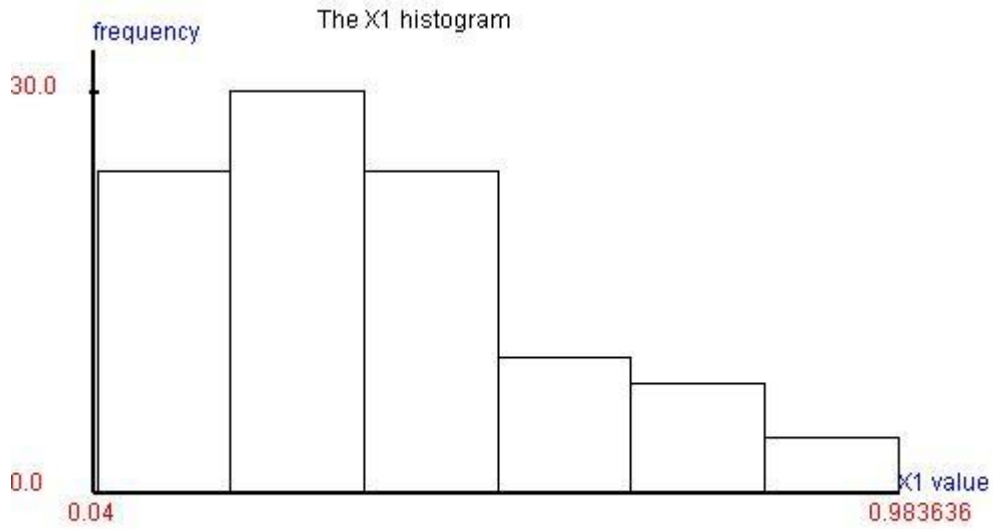
9.4.6)The population distribution is rayleigh distribution.

X1 is Rayleigh($\lambda=5.000000, c=0.000000$),

X1
0.1902216912
0.4399796827
0.8046690784
0.2724430820
0.3180975815
0.1784977768
0.7216416502
0.1417606088
0.2854055094
0.2397624432
0.4117626627
0.0632072597
0.1853933928
0.4114721714
0.6765697071
0.9577026641
0.1583056415
0.1740775774
0.1688123798
0.4823352664
0.2040232735
0.2370178778
0.4080862967
0.1657596538
0.1391774169
0.4734810335
0.5388810278
0.4249817946
0.4960177676
0.2894406642
0.1961885753
0.7599106181
0.6065917724
0.2586826371
0.1177209116
0.2639933291
0.3100096886
0.0737373599
0.6644682116
0.1285042534
0.5934100964
0.2849455918
0.8953278202
0.3646015123
0.2062733525
0.2818265065
0.4636776998
0.2449352515
0.2524013564
0.9297570007
0.4779160571
0.6639750998
0.2744329219
0.0535495003
0.1388401813
0.1922494729
0.0724350553
0.1341064595
0.3373860682
0.1626080434
0.9836367214
0.3567759517
0.2869633998
0.2812991976
0.3425307570
0.0735994800
0.3647903139
0.4148741220
0.3269214946
0.4120838821
0.7892707301

0.8055908186
 0.2474858186
 0.4471063950
 0.7307682984
 0.2588637425
 0.4196016468
 0.4042497340
 0.5615033437
 0.6552810918
 0.3610835873
 0.0742878911
 0.4207864075
 0.5705105725
 0.2144054734
 0.6929433256
 0.6268628900
 0.3008559073
 0.2190280448
 0.3810100125
 0.6440044482
 0.1349098775
 0.3949485449
 0.2233475076
 0.2121650764
 0.0405772931
 0.4261274497
 0.3482432261
 0.4802293330
 0.2986864520

X1 is Rayleigh(lamda=5.000000,c=0.000000),



H0: $X1 \sim \text{Rayleigh}(\lambda, c)$, λ, c are unknown

λ point estimated value = 6.253849 (MLE)

c point estimated value = 0.040577 (MLE)

pearson goodness of fit

class	[1]	[2]	[3]	[4]
lower limit	0.04058	0.17530	0.31002	0.44475
0.57947	0.71419	0.84891		
upper limit	0.17530	0.31002	0.44475	0.57947
0.71419	0.84891	0.98364		
observed no	19.00000	30.00000	22.00000	10.00000
9.00000	6.00000	4.00000		
probability	0.10730	0.25764	0.27503	0.19737
0.10409	0.04176	0.01680		
expected no	10.73036	25.76359	27.50344	19.73737
10.40933	4.17580	1.68010		
chi square	3.59931	0.59824	1.37672	9.48164
0.22069	0.55461	1.34548		

Likelihood ratio chi square test statistic = 17.176694

degree of freedom = 4, p-value = 0.016200

correction:

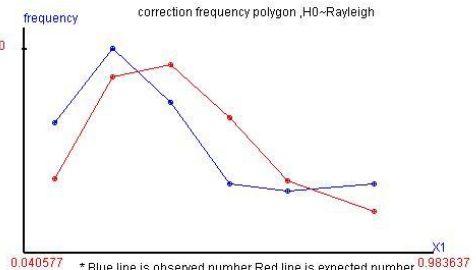
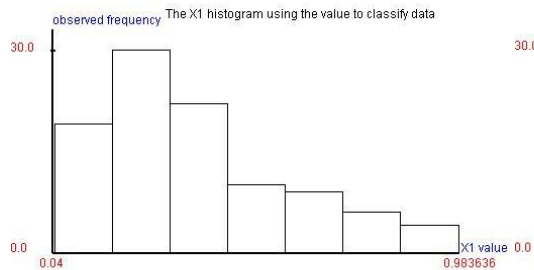
expected number ≥ 5 in each cell, the frequency table is adjusted

class	[1]	[2]	[3]	[4]
lower limit	0.04058	0.17530	0.31002	0.44475
0.57947	0.71419			
upper limit	0.17530	0.31002	0.44475	0.57947
0.71419	0.98364			
observed no	19.00000	30.00000	22.00000	10.00000
9.00000	10.00000			
probability	0.10730	0.25764	0.27503	0.19737
0.10409	0.05856			
expected no	10.73036	25.76359	27.50344	19.73737
10.40933	5.85591			
chi square	3.59931	0.59824	1.37672	9.48164
0.22069	1.71735			

degree of freedom = 3

Likelihood ratio chi-square test statistic = 16.993948

p-value = 0.000700



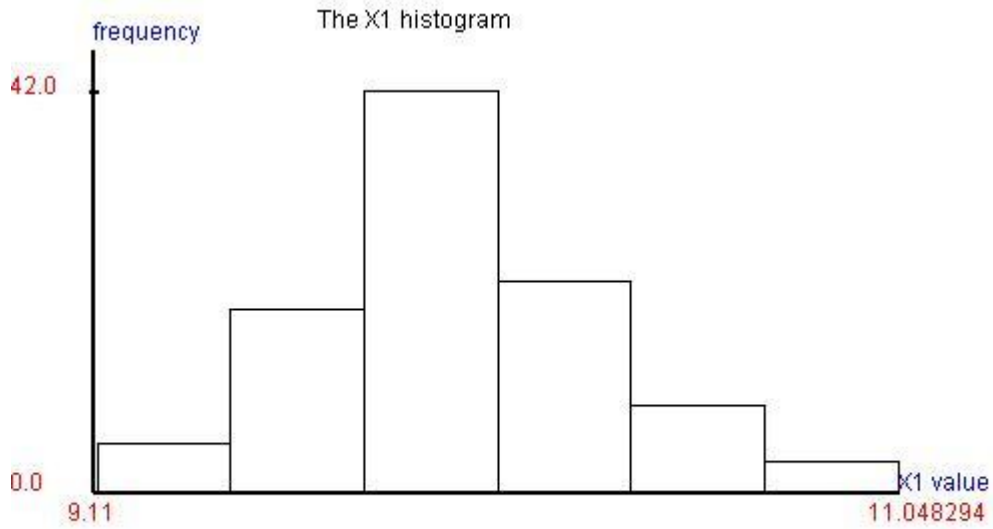
9.4.7)The population distribution is double exponential distribution.

X1 is Double Exponential($\lambda=4.000000, \mu=10.000000$),

X1
9.9877928684
9.7874309256
9.8815146634
9.5746131244
9.9870875990
9.9266982423
10.1274277461
9.9889553060
9.3772952287
10.0472671340
9.9132602306
9.8594001333
9.7444071562
9.8945327580
10.2487797882
9.9585265791
9.9707239225
10.1451370871
9.7448313521
10.0009718404
10.1293402739
9.5658173811
9.8207688041
10.3824279950
9.2487713389
9.9284051712
9.5063934533
9.9204627884
9.7366215464
10.3912684615
9.7301643514
10.4181206487
10.0720734937
10.3211068117
10.6781056586
9.5378553587
10.1036842915
10.1738964184
9.8148568609
10.1672916295
10.0959505801
9.9989257936
9.6548483670
9.6693538791
9.7264101297
9.2507254482
10.0211261145
10.4109110456
10.3475846014
10.4685223666
9.9548253989
10.2354523795
10.0274511065
9.8080343469
10.3229208522
9.1109590872
9.4138993876
10.1592655128
10.1183620536
9.9913525617
9.9281612355
9.9582115810
10.2477279791
10.0588858806
9.8242214207
10.9395371876
9.5657437529
9.6330353240
9.9767290395
9.7523998854
9.7055060223

10.4857688252
 11.0482941499
 9.8018604113
 10.1134765295
 9.8553957383
 10.4314653565
 10.3744453023
 10.0416478548
 10.1249152052
 10.0082111632
 10.6338604834
 9.7788204301
 9.4886458039
 9.8785407441
 10.7780538346
 9.7634102927
 9.9523677440
 9.9080081502
 9.8182467211
 9.9472003717
 9.6401903892
 10.0254446005
 10.4680944006
 10.5153284784
 10.1710073771
 9.7512299079
 9.8697697763
 9.6644485678
 10.2564598978

X1 is Double Exponential(lamda=4.000000,mu=10.000000),



H0: $X_1 \sim \text{Double exponential}(\lambda, \mu)$, λ, μ are unknown

λ point estimated value = 3.892332 (MLE)

μ point estimated value = 9.964625 (MLE)

pearson goodness of fit

class	[1]	[2]	[3]	[4]
[5]	[6]	[7]		
lower limit	9.11096	9.38772	9.66448	9.94125
10.21801	10.49477	10.77153		
upper limit	9.38772	9.66448	9.94125	10.21801
10.49477	10.77153	11.04829		
observed no	4.00000	11.00000	30.00000	33.00000
16.00000	3.00000	3.00000		
probability	0.05294	0.10252	0.30105	0.35701
0.12298	0.04188	0.02163		
expected no	5.29372	10.25178	30.10531	35.70052
12.29822	4.18792	2.16252		
chi square	0.41843	0.05089	0.00037	0.22099
0.85645	0.47039	0.23379		

Likelihood ratio chi square test statistic = 2.251310

degree of freedom = 4, p-value = 0.944600

correction:

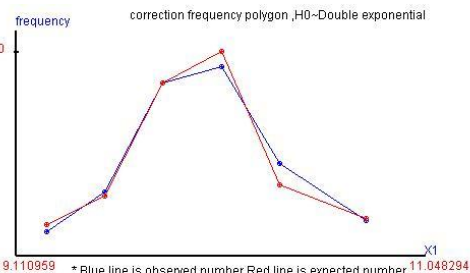
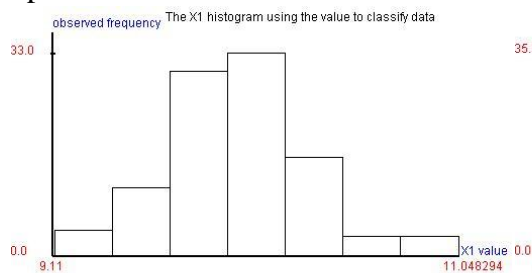
expected number ≥ 5 in each cell, the frequency table is adjusted

class	[1]	[2]	[3]	[4]
[5]	[6]			
lower limit	9.11096	9.38772	9.66448	9.94125
10.21801	10.49477			
upper limit	9.38772	9.66448	9.94125	10.21801
10.49477	11.04829			
observed no	4.00000	11.00000	30.00000	33.00000
16.00000	6.00000			
probability	0.05294	0.10252	0.30105	0.35701
0.12298	0.06350			
expected no	5.29372	10.25178	30.10531	35.70052
12.29822	6.35044			
chi square	0.41843	0.05089	0.00037	0.22099
0.85645	0.02047			

degree of freedom = 3

Likelihood ratio chi-square test statistic = 1.567602

p-value = 0.666700



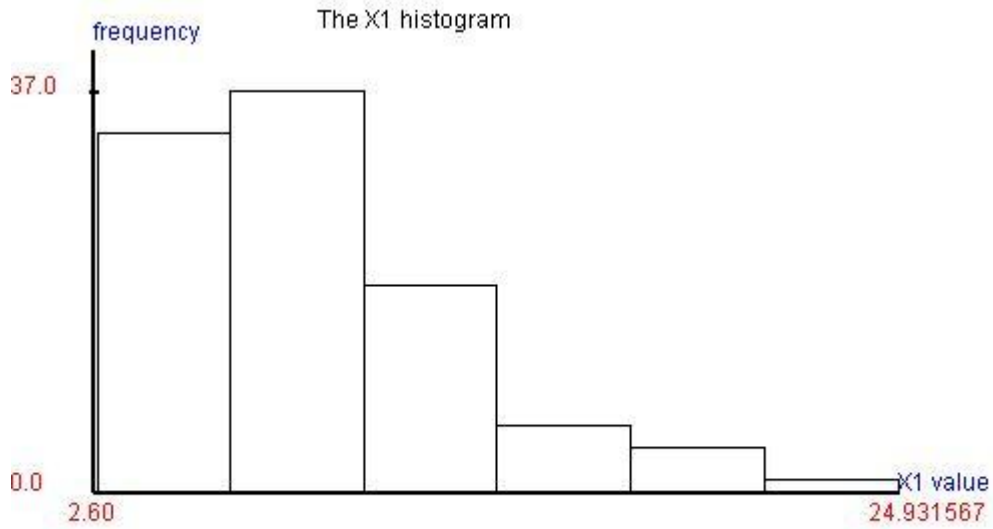
9.4.8)The population distribution is lognormal distribution.

X1 is Log normal($\mu=2.000000$, $\sigma=0.500000$),

X1
6.8193992940
2.6053065190
11.7993893899
7.5944743367
8.1587367940
7.1248684453
4.0968583977
15.9222989294
24.9315679364
4.3878542938
5.5720286824
4.7232615783
13.2529740956
5.5993154727
7.4099275065
8.6769914639
10.3051040364
7.5198486940
4.8925998355
7.9313715996
13.7424860095
2.9663808212
8.5114762548
8.3948551175
5.0016701060
10.4049146476
5.8943634495
8.2974384265
4.0459759289
7.9446081708
15.6605946321
5.5380768379
6.7197272101
9.8425491716
2.8708727902
8.6208287257
3.6985501037
6.7484318681
19.6500639062
6.4943325721
9.1540331666
3.8868451080
12.4135572556
5.4065415690
12.8982676809
16.9443733761
11.0961916376
9.3365155781
5.4286395732
21.1790331992
14.3696244629
9.9633757106
4.5574848841
8.7464037601
8.1711026999
11.7522830471
10.6946697902
10.4574118518
9.7474754425
9.0430873873
5.9016095054
11.7150837941
7.2570472365
20.0135500842
9.2020319483
8.7862190826
4.3118202744
10.3967722393
4.4070985216
3.4124804310
6.8019974951

7.6367155819
 8.0740858127
 13.2988824765
 7.2552875543
 2.8020861683
 17.8451201679
 5.4731927290
 5.0435980358
 17.3249521874
 7.6623507948
 15.3410812892
 12.1521433335
 7.7830798161
 8.2232542566
 5.8037668506
 9.6166356163
 12.2248829436
 3.4578402018
 3.9238996285
 8.5352323799
 12.7617763945
 11.2101575926
 5.7155854781
 7.5129077420
 11.9678913452
 4.2356432770
 4.5272147702
 6.3001559665
 5.4834672682

X1 is Log normal(mu=2.000000,sigma=0.500000),



H0: $X_1 \sim \text{Log_Normal}(\mu, \sigma^2)$, μ, σ are unknown
 population mean(μ) point estimated value=2.054501 (MLE,UMVUE)
 population variance(σ^2) which point estimated value=0.236971 (UMVUE)

pearson goodness of fit

class	[1]	[2]	[3]	[4]
lower limit	2.60531	5.79477	8.98424	12.17370
15.36317	18.55264	21.74210		
upper limit	5.79477	8.98424	12.17370	15.36317
18.55264	21.74210	24.93157		
observed no	29.00000	33.00000	20.00000	9.00000
5.00000	3.00000	1.00000		
probability	0.27060	0.34340	0.20560	0.09840
0.04440	0.02000	0.01760		
expected no	27.06000	34.34000	20.56000	9.84000
4.44000	2.00000	1.76000		
chi square	0.12978	0.05441	0.01568	0.07840
0.06272	0.33333	0.57760		

Likelihood ratio chi square test statistic=1.251925

degree of freedom=4,p-value=0.989600

correction:

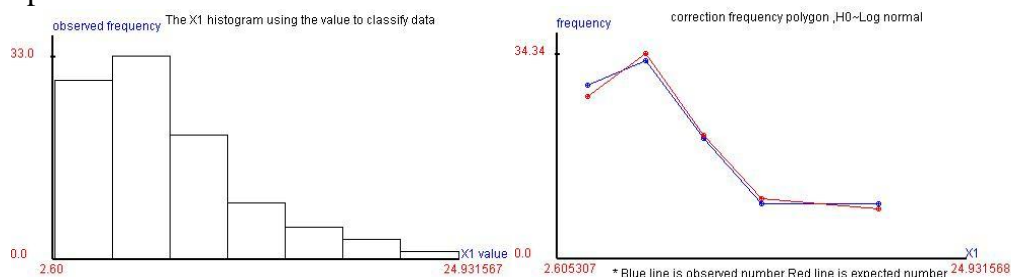
expected number ≥ 5 in each cell, the frequency table is adjusted

class	[1]	[2]	[3]	[4]
lower limit	2.60531	5.79477	8.98424	12.17370
15.36317				
upper limit	5.79477	8.98424	12.17370	15.36317
24.93157				
observed no	29.00000	33.00000	20.00000	9.00000
9.00000				
probability	0.27060	0.34340	0.20560	0.09840
0.08200				
expected no	27.06000	34.34000	20.56000	9.84000
8.20000				
chi square	0.12978	0.05441	0.01568	0.07840
0.07111				

degree of freedom=2

Likelihood ratio chi-square test statistic =0.349383

p-value=0.839700



9.4.9)The population distribution is gamma distribution.

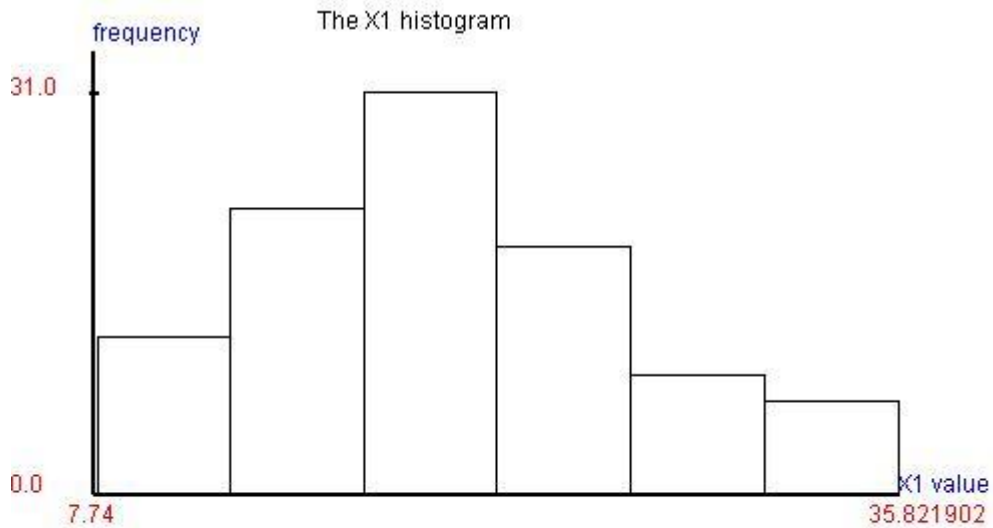
The alpha value is setting to 9 and beta is 2.

X1 is Gamma(alpha=10.000000,beta=2.000000),

X1
35.4421906648
21.2073421658
22.2770460847
10.9251027972
18.9502615190
14.6017264580
17.2595251104
14.4648249069
18.1571100551
21.8853096926
19.6087311569
19.9477542508
24.3147620841
25.8987535678
26.3764056248
27.5588073106
14.5485980418
23.0663919637
17.8561609377
14.4322288214
20.2492897829
12.1900056725
18.0160316694
21.1389929751
13.4433592313
13.2766644400
14.5622095530
22.0573134373
18.4398834425
21.5456712542
9.5110249187
23.3129227411
22.6791532833
23.6152641306
24.4889938041
22.9867703209
17.3741635667
22.6774302833
31.9366071237
20.3431884213
29.4037094615
28.5218876007
11.2807762274
7.7448608569
19.2705451964
11.2559567053
27.4508584674
15.0895541572
24.3168429946
10.4194703024
21.1701486904
21.4029173943
17.5042915071
22.3442312713
22.0564205510
18.7973617849
16.5908569515
19.6671920975
22.2956445481
29.6349301314
12.9601714725
17.6669101188
12.3595178092
29.3033431596
16.0893550412
21.2563763111
13.2816221913
35.8219028531
33.0032084923
22.0944543298

21.4177548307
 29.4332901367
 22.6502326037
 16.6705741190
 15.7476198354
 15.3527051999
 16.5203552088
 9.0297331079
 19.0813211894
 32.2991427333
 26.9237744531
 18.3148376012
 16.7268373727
 33.5625346879
 15.4842741154
 12.8208213928
 17.6697663208
 13.6204052920
 18.8632182023
 27.2281735964
 10.3047229752
 8.0946742301
 13.1689701869
 20.1202123572
 12.0439731258
 15.9814209656
 33.3754114796
 19.4000444227
 20.7085783011
 17.9394990840

X1 is Gamma(alpha=10.000000,beta=2.000000),



H0: $X1 \sim \text{Gamma}(\alpha=9.000000, \beta=2.000000)$,

pearson goodness of fit

class	[1]	[2]	[3]	[4]
[5]	[6]	[7]		
lower limit	7.74486	11.75587	15.76687	19.77788
23.78888	27.79989	31.81090		
upper limit	11.75587	15.76687	19.77788	23.78888
27.79989	31.81090	35.82190		
observed no	9.00000	19.00000	25.00000	26.00000
9.00000	5.00000	7.00000		
probability	0.14050	0.25070	0.26340	0.18340
0.09690	0.04200	0.02310		
expected no	14.05000	25.07000	26.34000	18.34000
9.69000	4.20000	2.31000		
chi square	2.83361	1.93921	0.07182	2.25675
0.05290	0.12800	3.14230		

Likelihood ratio chi square test statistic=10.424594

degree of freedom=6,p-value=0.165700

correction:

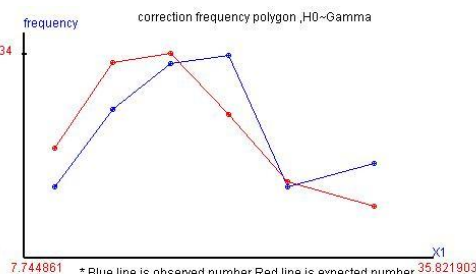
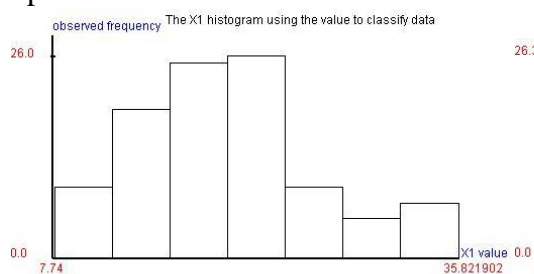
expected number ≥ 5 in each cell, the frequency table is adjusted

class	[1]	[2]	[3]	[4]
[5]	[6]			
lower limit	7.74486	11.75587	15.76687	19.77788
23.78888	27.79989			
upper limit	11.75587	15.76687	19.77788	23.78888
27.79989	35.82190			
observed no	9.00000	19.00000	25.00000	26.00000
9.00000	12.00000			
probability	0.14050	0.25070	0.26340	0.18340
0.09690	0.06510			
expected no	14.05000	25.07000	26.34000	18.34000
9.69000	6.51000			
chi square	2.83361	1.93921	0.07182	2.25675
0.05290	2.51167			

degree of freedom=5

Likelihood ratio chi-square test statistic =9.665969

p-value=0.085200



9.4.10)The population distribution is beta distribution.

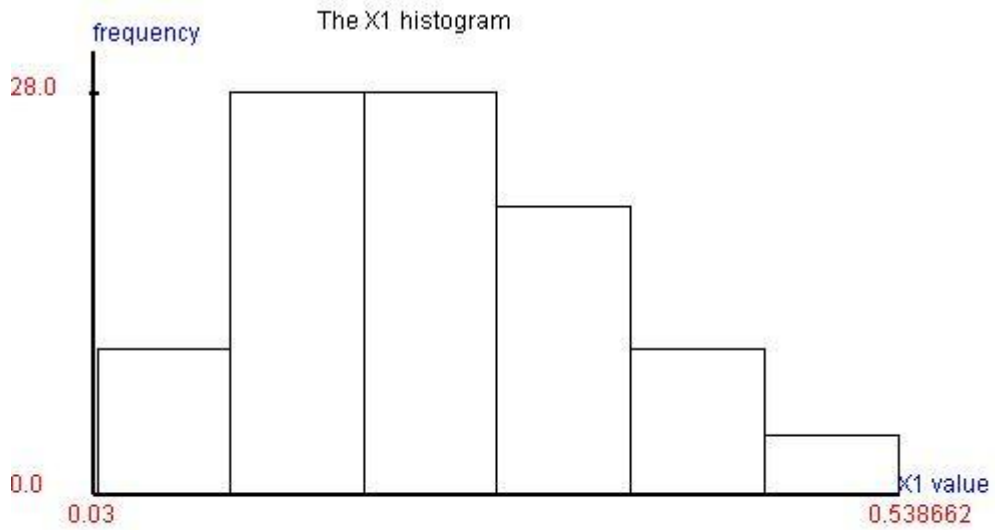
The alpha value is setting to 3 and beta is 13.

X1 is Beta(alpha=4.000000,beta=12.000000),

X1
0.1368891879
0.3721370450
0.2778241434
0.3315370704
0.2683607551
0.4770926270
0.5386623820
0.2773949598
0.2684251980
0.1265400482
0.3202271263
0.3372209637
0.1848069077
0.2630405037
0.3630950306
0.2287957497
0.1622864540
0.3805534899
0.2918754065
0.1311531562
0.3273531087
0.1990327257
0.3002543796
0.2255557434
0.0474953931
0.0647678477
0.2105781155
0.3472617019
0.3351871301
0.2102535400
0.2878301957
0.2452349587
0.1689348044
0.1405713404
0.3609295650
0.0752924668
0.1827863120
0.0895524131
0.2459988316
0.1769821105
0.3777603765
0.0705988756
0.1763868572
0.2814338862
0.2111477292
0.1877340214
0.2245954307
0.4310136677
0.2266833545
0.1313285566
0.1307822595
0.5205350919
0.1987344996
0.1047398293
0.1999981382
0.2123476244
0.1779533530
0.4415444260
0.2153212811
0.1533436195
0.3199880509
0.2045102225
0.3487435001
0.2069624178
0.1318865985
0.2884337807
0.3097856592
0.3946865027
0.2766183400
0.2002580643

0.3701900808
 0.1492669979
 0.4725252102
 0.2870950666
 0.2952835430
 0.3932417475
 0.2425746655
 0.4151725704
 0.0389123619
 0.2213581623
 0.3646190003
 0.2701321776
 0.4318638408
 0.2957461365
 0.3021020396
 0.1443818868
 0.1289560172
 0.1023846446
 0.3181852436
 0.2423142064
 0.2368854302
 0.2314884947
 0.1903556024
 0.0748681502
 0.3439876676
 0.0813770190
 0.1533191937
 0.4339662674
 0.2005611254
 0.1956260987

X1 is Beta(alpha=4.000000,beta=12.000000),



H0: $X_1 \sim \text{Beta}(\alpha=3.000000, \beta=13.000000)$,

pearson goodness of fit

class	[1]	[2]	[3]	[4]
[5]	[6]	[7]		
lower limit	0.03891	0.11031	0.18170	0.25309
0.32448	0.39588	0.46727		
upper limit	0.11031	0.18170	0.25309	0.32448
0.39588	0.46727	0.53866		
observed no	10.00000	17.00000	28.00000	20.00000
16.00000	5.00000	4.00000		
probability	0.22499	0.30537	0.24180	0.13758
0.06110	0.02164	0.00752		
expected no	22.49911	30.53724	24.17999	13.75790
6.11029	2.16389	0.75158		
chi square	15.62278	10.77981	0.52116	1.94819
6.11290	1.60871	2.63806		

Likelihood ratio chi square test statistic=39.231600

degree of freedom=6,p-value=0.000000

correction:

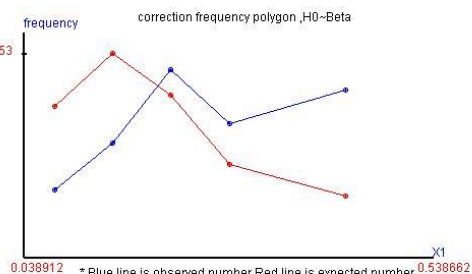
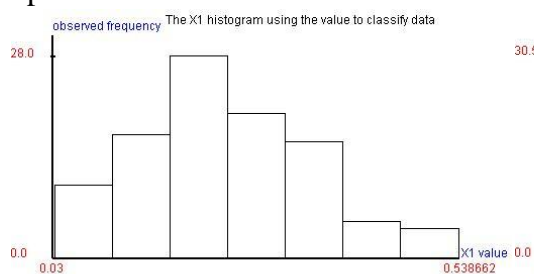
expected number ≥ 5 in each cell, the frequency table is adjusted

class	[1]	[2]	[3]	[4]
[5]				
lower limit	0.03891	0.11031	0.18170	0.25309
0.32448				
upper limit	0.11031	0.18170	0.25309	0.32448
0.53866				
observed no	10.00000	17.00000	28.00000	20.00000
25.00000				
probability	0.22499	0.30537	0.24180	0.13758
0.09026				
expected no	22.49911	30.53724	24.17999	13.75790
9.02576				
chi square	15.62278	10.77981	0.52116	1.94819
10.20705				

degree of freedom=4

Likelihood ratio chi-square test statistic =39.078995

p-value=0.000000



9.4.11)The population distribution is cauchy distribution.

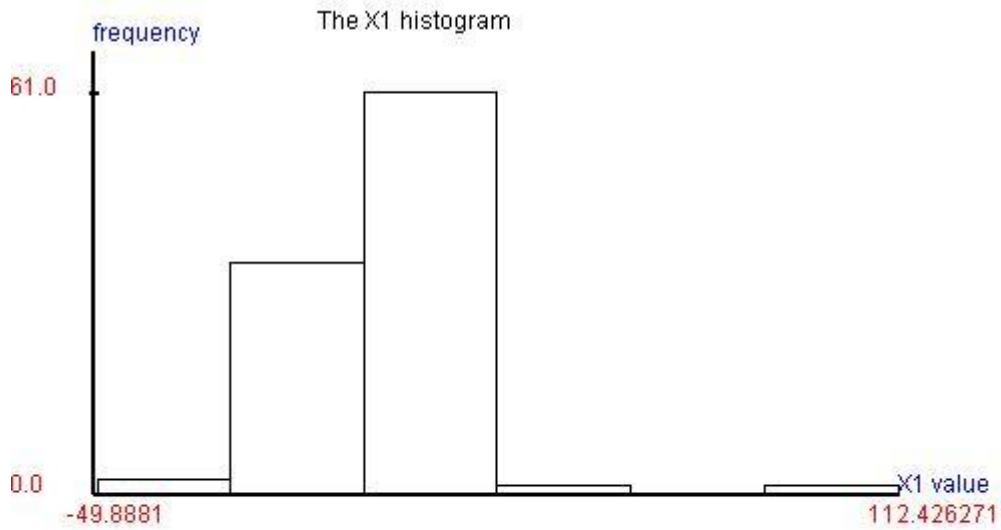
The mu value is setting to 6 and sigma value is 1.8.

X1 is Cauchy(mu=5.000000,sigma=2.000000),

```
X1
7.1527579564
51.9477518381
3.0401987730
2.1582878490
5.8377930191
7.1795423905
4.4695455816
28.1091450516
6.1292298768
3.5204315043
1.5138989327
5.4142182157
4.8209665816
4.2041963330
4.5704011412
5.6591151820
3.9301608712
4.2704050233
6.9722360169
4.7764290935
5.0451210391
5.1958938330
5.5358269954
6.3980766612
6.1894406224
5.8366882069
6.1752553789
6.7518966996
5.4147731683
5.5081683676
1.8603926898
4.8633518826
8.4740684048
0.1167547402
9.1827961624
4.8171388397
3.5827966680
5.1757953133
3.3183033240
4.0857343568
4.0295025336
-1.9432863036
4.9618600329
-49.8881912621
16.6867476846
2.1112979626
-1.2475362546
10.3098057566
0.8978460803
5.9056933046
7.3348742815
7.8098207823
5.9895354747
4.5947816189
13.6313742428
-1.9187083288
3.7315591693
4.4331428765
2.8529226538
4.8468874661
-0.9837554341
3.9249769275
4.9429372145
7.1961799573
7.1883748444
5.6921945810
0.5768160632
3.2204661377
-1.9231132667
6.5172101401
```

-2.5296967206
 4.7914787293
 5.1783196070
 6.6186723785
 4.8778501057
 6.2791544858
 0.4913919414
 -24.9805746226
 1.1439045235
 1.1848951882
 7.7889669906
 3.4828271717
 4.4360958769
 9.3486294815
 4.6976353934
 5.3178514200
 5.4927902139
 2.6506295330
 3.8621925558
 6.7202205260
 3.7608265518
 2.9216316949
 112.4262711298
 27.7119834226
 7.0964877860
 4.1260583222
 5.2585028290
 26.9199957200
 3.4462887854
 5.5904631936

X1 is Cauchy(mu=5.000000,sigma=2.000000),



H0: $X1 \sim \text{Cauchy}(\mu=6.000000, \sigma=1.800000)$,

pearson goodness of fit

class	[1]	[2]	[3]	[4]
[5]	[6]	[7]		
lower limit	-49.88819	-26.70041	-3.51263	19.67515
42.86293	66.05071	89.23849		
upper limit	-26.70041	-3.51263	19.67515	42.86293
66.05071	89.23849	112.42627		
observed no	1.00000	1.00000	93.00000	3.00000
1.00000	0.00000	1.00000		
probability	0.01750	0.04202	0.89881	0.02613
0.00599	0.00266	0.00688		
expected no	1.75038	4.20237	89.88143	2.61277
0.59922	0.26561	0.68823		
chi square	0.56306	10.25518	0.10458	0.04998
0.16062		0.09720		

Likelihood ratio chi square test statistic=1.#INF00

degree of freedom=6

p-value=0.000000

correction:

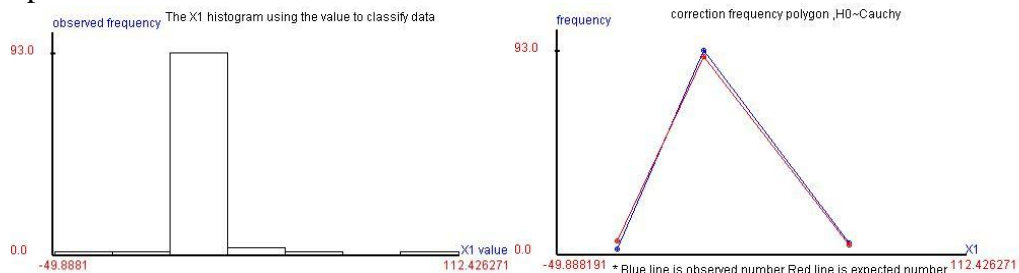
expected number ≥ 5 in each cell, the frequency table is adjusted

class	[1]	[2]	[3]
lower limit	-49.88819	-3.51263	19.67515
upper limit	-3.51263	19.67515	112.42627
observed no	2.00000	93.00000	5.00000
probability	0.05953	0.89881	0.04166
expected no	5.95275	89.88143	4.16583
chi square	7.81210	0.10458	0.13917

degree of freedom=2

Likelihood ratio chi-square test statistic =8.055847

p-value=0.017800



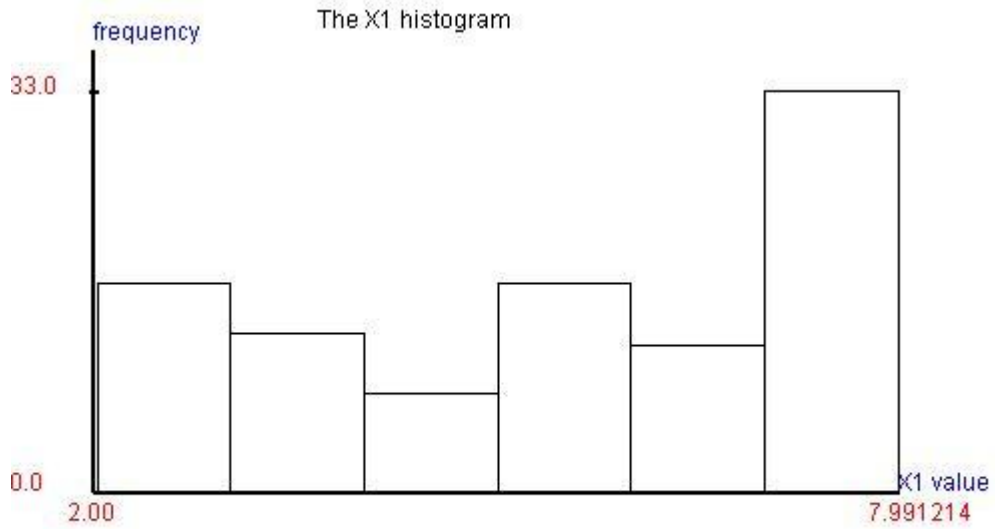
9.4.12) The population distribution is arcsin distribution.

X_1 is $\text{Arcsin}(\mu=5.000000, c=3.000000)$,

X_1
4.1993893668
7.9316262014
5.1032649139
7.0417293108
6.8429856792
7.7407392234
5.1685202505
7.9892712797
7.5780602474
2.0567159626
2.2563998812
5.4157219526
3.0419814902
3.8411702076
7.9488383091
6.5404780513
5.2792564354
7.0677060461
5.3425733466
7.8147501953
2.3112247778
7.9260470159
2.3852520793
7.6189269731
7.9524602452
3.9244301014
4.8741944516
2.6140406577
3.5792646875
2.7269886908
4.8871338000
7.2566760067
7.5703660391
3.7886057564
4.8447546384
5.6501926078
5.1960513222
7.0678370334
3.0077651032
2.6304061005
7.8394495594
6.5513512608
3.9734868879
3.1787057042
5.9684476415
3.7326851417
5.3032443064
7.9747715386
4.4348237639
4.9773624244
2.0379579793
5.6601711542
4.1777365511
2.0000490152
5.9611290018
5.0880984134
7.9743601524
7.8081724840
7.9244761817
2.1507426428
3.0635918520
2.8155389099
7.0054421604
6.5227072118
7.3081239114
3.6519146263
6.8953626235
6.8269625869
6.5859002565
7.9402162343
6.8986855095

2.4965468830
 6.6550385000
 5.0609708930
 7.9912144780
 7.9845080211
 7.7081310846
 7.7040710167
 5.2934062550
 3.8035537272
 5.3885026547
 6.6166566465
 7.0913068662
 7.5906840069
 4.6024246533
 3.2715723238
 7.9522193856
 6.9202431328
 7.8902928651
 5.4940109736
 2.4776943351
 2.9163708021
 2.2480033360
 7.0324203560
 2.3209635519
 5.5239430519
 6.7729383276
 7.2413871929
 2.4487802111
 7.5372092871

X1 is Arcsin(mu=5.000000,c=3.000000).



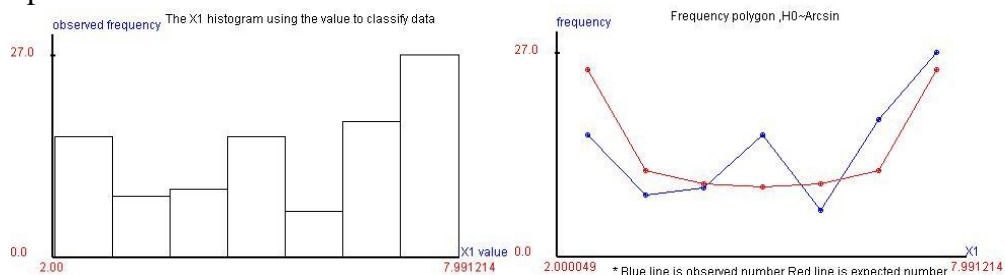
H0: $X_1 \sim \text{Arcsin}(\mu, c)$, μ, c are unknown
 μ point estimated value = 4.995632 (MLE)
 c point estimated value = 2.995583 (MLE)

pearson goodness of fit

class	[1]	[2]	[3]	[4]
lower limit	2.00005	2.85593	3.71181	4.56769
5.42357	6.27945	7.13533		
upper limit	2.85593	3.71181	4.56769	5.42357
6.27945	7.13533	7.99121		
observed no	16.00000	8.00000	9.00000	16.00000
6.00000	18.00000	27.00000		
probability	0.24675	0.11227	0.09535	0.09126
0.09535	0.11227	0.24675		
expected no	24.67517	11.22653	9.53540	9.12579
9.53540	11.22653	24.67517		
chi square	4.70366	1.30131	0.03185	2.95342
2.08318	2.54888	0.20018		

degree of freedom = 4

Likelihood ratio chi-square test statistic = 13.822487
p-value = 0.007800



9.4.13)The population distribution is gumbel distribution.

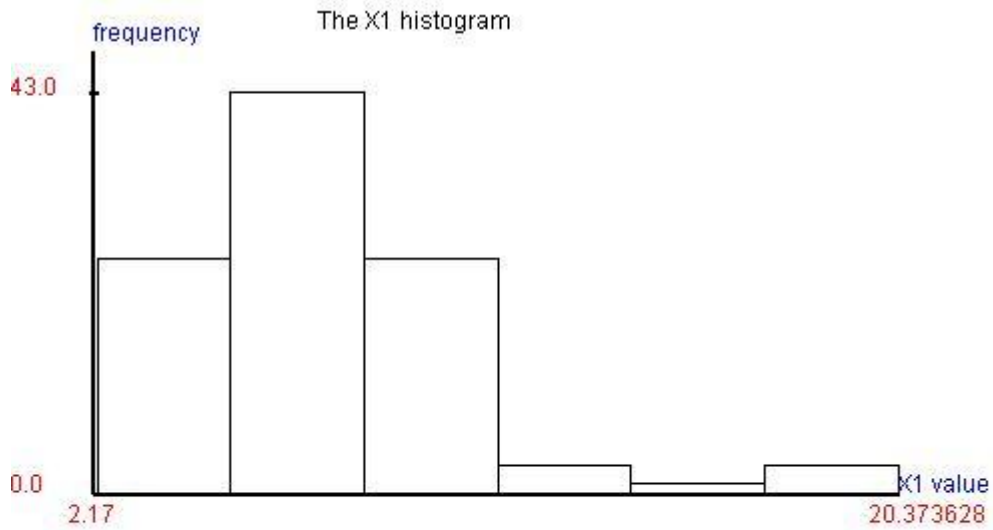
The mu value is setting to 6 and sigma value is 1.8.

X1 is Gumbel(mu=6.000000,sigma=2.000000),

X1
5.4303968401
9.0670650275
8.6757364624
6.4023797338
4.2980412486
7.8250758910
4.3154618006
7.9955881539
5.8950714948
6.5271514675
7.2933205566
18.0779554457
8.2322936170
6.1330395400
5.6550516146
10.0464940065
5.1378365044
7.5994667461
5.9546404344
8.3314505025
4.1184690051
9.4460396616
5.0818138863
4.6792742522
5.9920398112
4.8830595028
6.7048091297
8.7306395911
4.6689721663
4.4178592673
7.2699136392
10.5891541344
5.5520896247
8.2659176112
7.9270892310
7.2938363818
11.4406945409
7.9903990477
7.1199781881
6.5249652101
7.0884570019
10.0028598916
15.4922450691
4.4602331659
7.6241383843
7.0323193262
7.9991626508
4.7603215757
2.1755608709
5.4126087712
19.9886757924
9.8133271118
8.3483594773
5.5165114386
4.3352841794
6.5360863931
8.7674454749
8.1495960481
10.9473717733
3.9653391831
4.3780480775
5.1329873342
7.4579231245
4.7279028378
6.9529604470
5.9046766607
8.2652844380
8.8609183880
4.9716694322
7.0689440480

10.3465034306
 6.0288683655
 6.8982291976
 8.4075004411
 5.4775995593
 4.0082543115
 4.1446535857
 6.7697851570
 6.8656359247
 9.1887431024
 8.3052081948
 20.3736282763
 6.3492289020
 3.2954146503
 7.8354167373
 6.0693071309
 4.9048052807
 9.1855428179
 13.1219177135
 11.0929564950
 5.0689199245
 14.0800520932
 8.6441881930
 8.4091129511
 8.2525288177
 6.6367461463
 4.0113177775
 4.5160664809
 9.6274293618
 7.4887923732

X1 is Gumbel($\mu=6.000000$, $\sigma=2.000000$),



H0: $X1 \sim \text{Gumbel}(\mu=6.000000, \sigma=1.800000)$,

pearson goodness of fit

class	[1]	[2]	[3]	[4]
[5]	[6]	[7]		
lower limit	2.17556	4.77528	7.37501	9.97473
12.57446	15.17418	17.77390		
upper limit	4.77528	7.37501	9.97473	12.57446
15.17418	17.77390	20.37363		
observed no	18.00000	38.00000	31.00000	7.00000
2.00000	1.00000	3.00000		
probability	0.13881	0.48879	0.26832	0.07848
0.01950	0.00466	0.00144		
expected no	13.88082	48.87938	26.83217	7.84826
1.94958	0.46559	0.14419		
chi square	0.94265	3.11476	0.56035	0.10279
0.00127	0.28559	2.71855		

Likelihood ratio chi square test statistic=7.725962

degree of freedom=6,p-value=0.357300

correction:

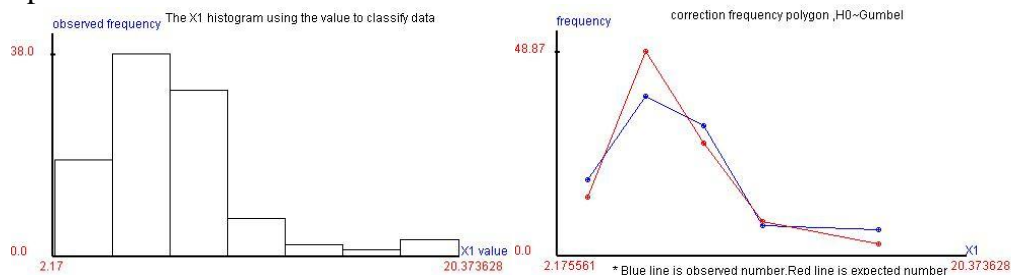
expected number ≥ 5 in each cell, the frequency table is adjusted

class	[1]	[2]	[3]	[4]
[5]				
lower limit	2.17556	4.77528	7.37501	9.97473
12.57446				
upper limit	4.77528	7.37501	9.97473	12.57446
20.37363				
observed no	18.00000	38.00000	31.00000	7.00000
6.00000				
probability	0.13881	0.48879	0.26832	0.07848
0.02559				
expected no	13.88082	48.87938	26.83217	7.84826
2.55936				
chi square	0.94265	3.11476	0.56035	0.10279
1.97300				

degree of freedom=4

Likelihood ratio chi-square test statistic =6.693546

p-value=0.152900



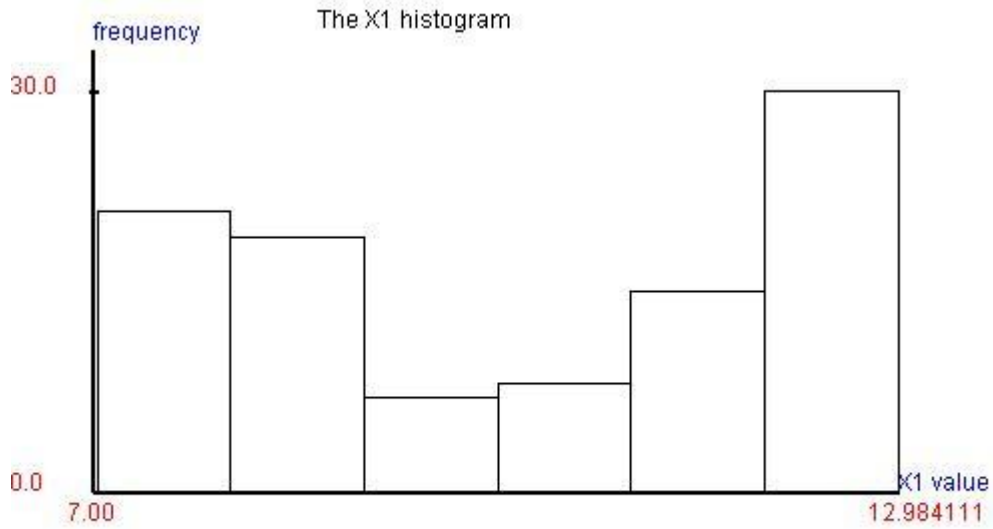
9.4.14)The population distribution is triangular 1 distribution.

X1 is Triangular1($\mu=10.000000,c=3.000000$),

X1
8.6375799808
12.6822343668
9.0364902069
8.6587103492
8.0426949603
7.1144190271
9.0947254770
7.3801314446
11.4935673975
12.7052845217
12.8834822566
7.3639389700
12.4217283771
8.9508471029
12.4432204119
8.8169548636
12.4065598516
8.7208253463
12.1831500302
12.6632401934
7.3762745926
7.1018611788
11.8103473091
11.3686975362
9.5195845517
7.6429869313
12.0231651830
12.6373044503
8.0519938097
7.9838414104
12.6725294787
12.9841119623
8.7138603510
11.6802550026
8.0669731416
8.5762249346
12.4034575433
12.3085149739
7.0042068396
11.5741420666
12.2784589393
12.8553096640
11.3402290693
12.9050723874
11.7571561777
10.8530954551
8.0162551932
8.7581602556
8.4878466551
7.3041143011
11.4178810722
11.3614843255
8.6807129790
12.6166905924
12.7062325347
7.5589211193
11.1442217027
7.3088782914
7.8264162614
12.2742186247
8.1364228743
12.1025651315
7.1788368142
12.4509862251
11.2698774995
7.2733952605
9.1425104569
11.7489359992
10.6986463078
12.1107288707
7.1509773481

10.9829001949
 10.8649912190
 8.9466898226
 9.1042860054
 7.5778994748
 7.6237801001
 8.1219635481
 9.0999552623
 12.3688559611
 11.2555383094
 12.6661725751
 12.0240771856
 7.3972289243
 10.4439971640
 12.1834866715
 7.8155490683
 10.9835812976
 7.1622378497
 11.8687044542
 7.7721369075
 8.4569677045
 12.8648009795
 10.8081303724
 12.5378614818
 8.1675227176
 10.9741285830
 11.8597961869
 9.2198717926
 12.7028897175

X1 is Triangular1(mu=10.000000,c=3.000000),



H0: $X_1 \sim \text{Triangular 1}(\mu, c)$, μ, c are unknown

μ point estimated value=9.994159 (MLE)

c point estimated value=2.989953 (MLE)

pearson goodness of fit

class	[1]	[2]	[3]	[4]
lower limit	7.00421	7.85848	8.71275	9.56702
10.42130	11.27557	12.12984		
upper limit	7.85848	8.71275	9.56702	10.42130
11.27557	12.12984	12.98411		
observed no	20.00000	14.00000	13.00000	0.00000
11.00000	16.00000	26.00000		
probability	0.24490	0.16327	0.08163	0.02041
0.08163	0.16327	0.24490		
expected no	24.48980	16.32653	8.16327	2.04082
8.16327	16.32653	24.48980		
chi square	1.00791	0.38662	1.79954	
0.73155	0.00666	0.08772		

Likelihood ratio chi square test statistic=1.#INF00

degree of freedom=4,p-value=0.000000

correction:

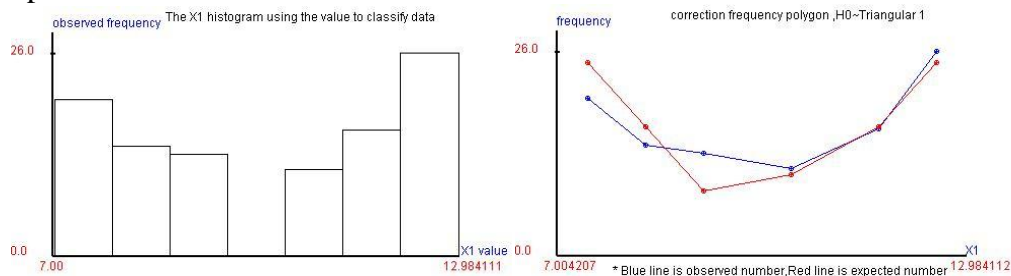
expected number ≥ 5 in each cell, the frequency table is adjusted

class	[1]	[2]	[3]	[4]
lower limit	7.00421	7.85848	8.71275	9.56702
11.27557	12.12984			
upper limit	7.85848	8.71275	9.56702	11.27557
12.12984	12.98411			
observed no	20.00000	14.00000	13.00000	11.00000
16.00000	26.00000			
probability	0.24490	0.16327	0.08163	0.10204
0.16327	0.24490			
expected no	24.48980	16.32653	8.16327	10.20408
16.32653	24.48980			
chi square	1.00791	0.38662	1.79954	0.05759
0.00666	0.08772			

degree of freedom=3

Likelihood ratio chi-square test statistic =3.346050

p-value=0.341200



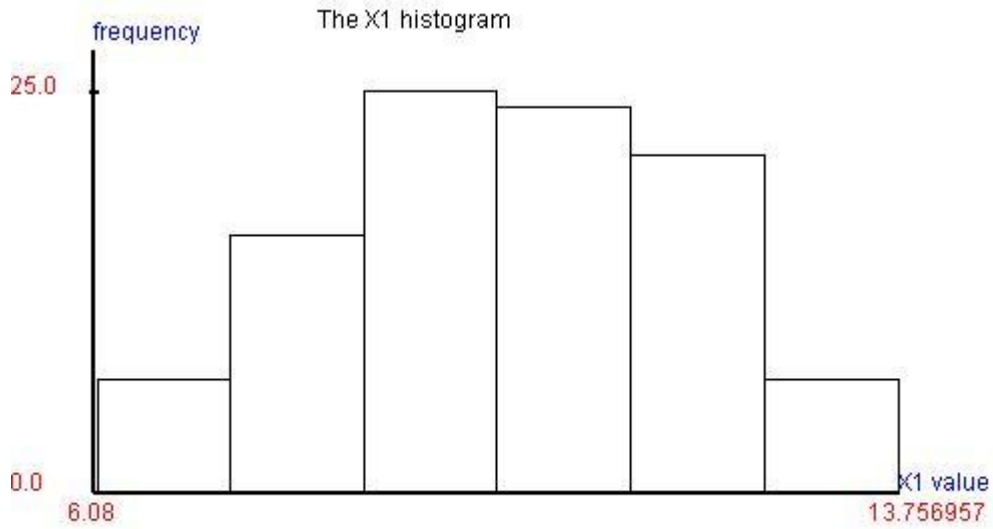
9.4.15)The population distribution is trapezoid distribution.

X1 is Trapezoid($\mu=10.000000,c=3.000000$),

X1
6.0883314089
11.1506676711
9.4316785001
9.8210977732
7.3793084475
9.5728784889
7.2448754838
8.1855820333
6.7613696959
11.6137445010
10.8695298002
9.7975191860
11.6360439420
8.7210744755
10.5289618489
11.0073289595
12.0072331446
9.4017166481
8.4067966436
11.9660677316
10.3138018530
13.1389069505
13.3264731222
11.2280662286
9.3227787831
12.6143141927
10.3985071659
7.6184781023
8.7739422427
8.1154304091
11.5258785463
8.8845766166
8.8956257904
10.9947346607
11.5149949341
9.6995486626
8.4162058232
8.0473130579
6.8137857462
8.7891102110
9.3739807353
11.7736332974
8.5061438951
13.7265477610
10.6603663323
7.4501010934
7.2311111488
12.4479700692
8.3311491378
11.2841159045
9.9459431734
10.8836899676
8.6799327288
11.7848443950
12.4533341575
9.0602132809
12.0460655661
11.8984207658
9.9813170420
11.3515770646
9.1663694503
9.0728205557
12.4301685161
10.7673648337
8.9563249403
6.8698545996
10.0842807128
9.9230421815
12.1379074597
11.4743038486
10.2415217712

13.7431423703
 9.9546480094
 10.6258343151
 10.8000259853
 11.9148860046
 8.3503972123
 10.0153942125
 13.7569572842
 10.4990269101
 9.6159000481
 9.8107415065
 9.4992559410
 11.9856078641
 7.6116931868
 8.5924126289
 10.7235484708
 7.7869464831
 8.1852389968
 7.3925107170
 10.1257586671
 9.9766470199
 12.7226214554
 9.7068677345
 11.5770348264
 10.3866055451
 9.3607507085
 6.5812713024
 8.9483757511
 8.7690272305

X1 is Trapezoid($\mu=10.000000, c=3.000000$),



H0: $X_1 \sim \text{Trapezoid}(\mu, c)$, μ, c are unknown

μ point estimated value=9.922644 (MLE)

c point estimated value=2.556209 (MLE)

pearson goodness of fit

class	[1]	[2]	[3]	[4]
lower limit	6.08833	7.18385	8.27937	9.37489
10.47040	11.56592	12.66144		
upper limit	7.18385	8.27937	9.37489	10.47040
11.56592	12.66144	13.75696		
observed no	5.00000	12.00000	21.00000	22.00000
18.00000	16.00000	6.00000		
probability	0.04592	0.13776	0.20918	0.21429
0.20918	0.13776	0.04592		
expected no	4.59184	13.77551	20.91837	21.42857
20.91837	13.77551	4.59184		
chi square	0.03332	0.26270	0.00032	0.01484
0.47316	0.30927	0.33049		

Likelihood ratio chi square test statistic=1.424101

degree of freedom=4, p-value=0.984800

correction:

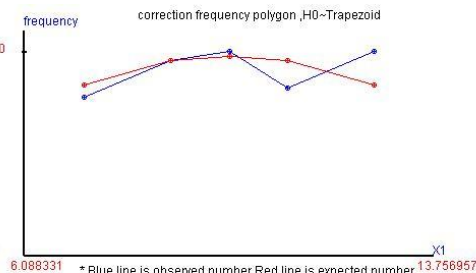
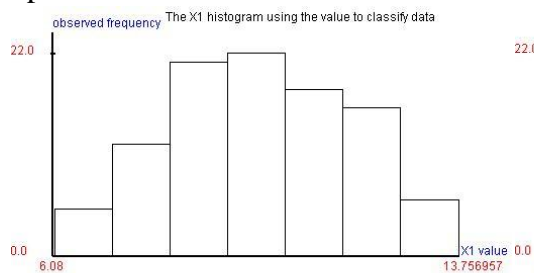
expected number ≥ 5 in each cell, the frequency table is adjusted

class	[1]	[2]	[3]	[4]
lower limit	6.08833	8.27937	9.37489	10.47040
11.56592				
upper limit	8.27937	9.37489	10.47040	11.56592
13.75696				
observed no	17.00000	21.00000	22.00000	18.00000
22.00000				
probability	0.18367	0.20918	0.21429	0.20918
0.18367				
expected no	18.36735	20.91837	21.42857	20.91837
18.36735				
chi square	0.10998	0.00032	0.01484	0.47316
0.59983				

degree of freedom=2

Likelihood ratio chi-square test statistic =1.198123

p-value=0.549300



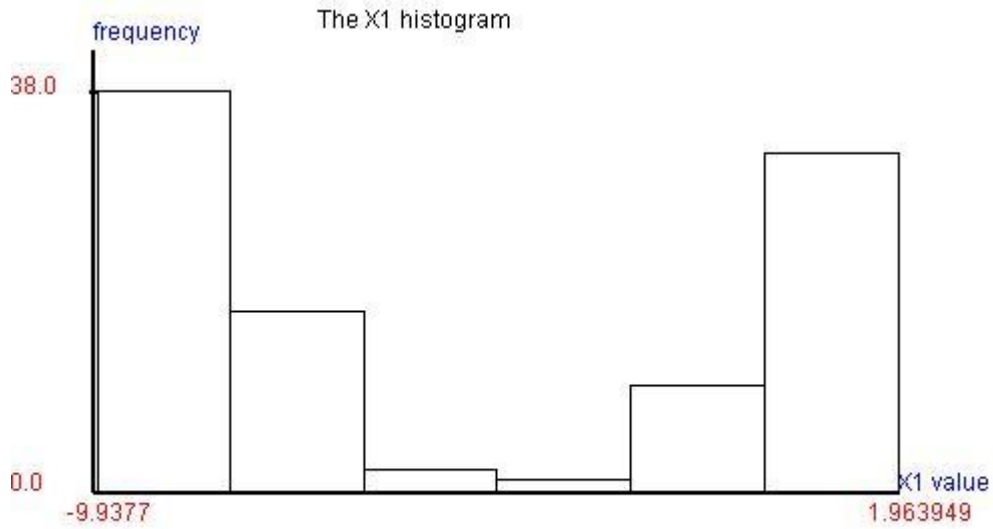
9.4.16) The population distribution is U quadratic distribution.

X1 is U-quadratic(a=-10.000000,b=2.000000),

X1
-9.5777311949
0.9067555874
-9.5154715952
-9.9288050056
-8.7590594010
0.5878117932
1.0719797715
0.6947258407
-9.6749596153
0.7071821191
-9.7174159076
1.9639493327
-9.0678246124
-8.6893689278
-9.9377901918
-8.8791701008
-7.1156425999
-1.8064991012
-6.7319001351
-5.8665013071
-8.1044218141
-0.5634790804
-9.9022765921
1.0720919961
-9.0530245329
-8.9636780522
-9.7966430959
-6.8651343718
0.7708875659
0.6727382594
-7.7901963212
0.8879341880
1.7608245830
-0.3941690874
-8.8323464699
-2.1640071059
-8.1579585821
-9.6007911285
-7.1176044613
1.3334832843
-7.2082510769
-5.2794964301
-8.8741775672
-9.6100750899
-7.2214499484
1.1737838460
-8.3976441584
-0.0826238759
0.8741077802
1.1740417653
1.0722173500
-9.5791709849
1.0541194143
-7.7920248231
-8.2123014035
-8.3381166310
1.7138407518
-8.3197081280
-7.1983444817
0.3335136979
1.0372874121
1.6118277790
0.8938784485
-7.3886432287
-7.6600983948
-8.4527270651
-0.5803648943
1.9491214563
-7.7074830466
1.0152198191
-9.0496903283

-8.4967323624
 -0.6593661263
 -7.9727403760
 0.7830172780
 -6.9542500015
 -9.5525155155
 -7.9259769002
 -0.6790053478
 -8.9170070456
 -8.9636048823
 1.7562880560
 1.5666242403
 1.9610082620
 -1.5160180293
 -6.6661551166
 -0.2655414044
 -9.5645023908
 -9.8119457131
 1.7411725263
 -6.0727731095
 -0.5194926184
 1.6028775799
 -8.8680925813
 1.3002668550
 0.7236759025
 -9.6325563864
 -9.4022225924
 -8.4299888718
 -7.9361821984

X1 is U-quadratic(a=-10.000000,b=2.000000),



H0: $X_1 \sim U_quadratic(a,b)$, a,b are unknown

a point estimated value=-9.937790 (MLE)

b point estimated value=1.963949 (MLE)

pearson goodness of fit

class	[1]	[2]	[3]	[4]	[5]
lower limit	-9.93779	-8.23754	-6.53729	-4.83704	-3.13680
upper limit	-1.43655	0.26370			
observed no	34.00000	20.00000	3.00000	0.00000	
3.00000	8.00000	32.00000			
probability	0.31780	0.14290	0.03790	0.00290	
0.03790	0.14290	0.31770			
expected no	31.78000	14.29000	3.79000	0.29000	
3.79000	14.29000	31.77000			
chi square	0.14495	1.63021	0.20803		
0.20803	4.94551	0.00165			

Likelihood ratio chi square test statistic=1.#INF00

degree of freedom=4,p-value=0.000000

correction:

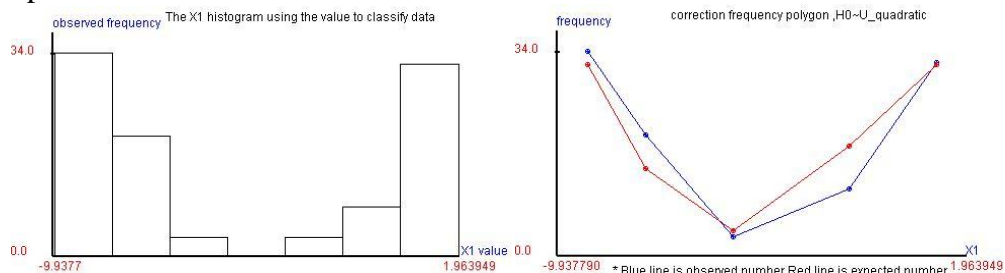
expected number ≥ 5 in each cell, the frequency table is adjusted

class	[1]	[2]	[3]	[4]	[5]
lower limit	-9.93779	-8.23754	-6.53729	-3.13680	0.26370
upper limit	-8.23754	-6.53729	-3.13680	0.26370	1.96395
observed no	34.00000	20.00000	3.00000	11.00000	
32.00000					
probability	0.31780	0.14290	0.04080	0.18080	
0.31770					
expected no	31.78000	14.29000	4.08000	18.08000	
31.77000					
chi square	0.14495	1.63021	0.38880	4.55695	
0.00165					

degree of freedom=2

Likelihood ratio chi-square test statistic =6.722557

p-value=0.034600



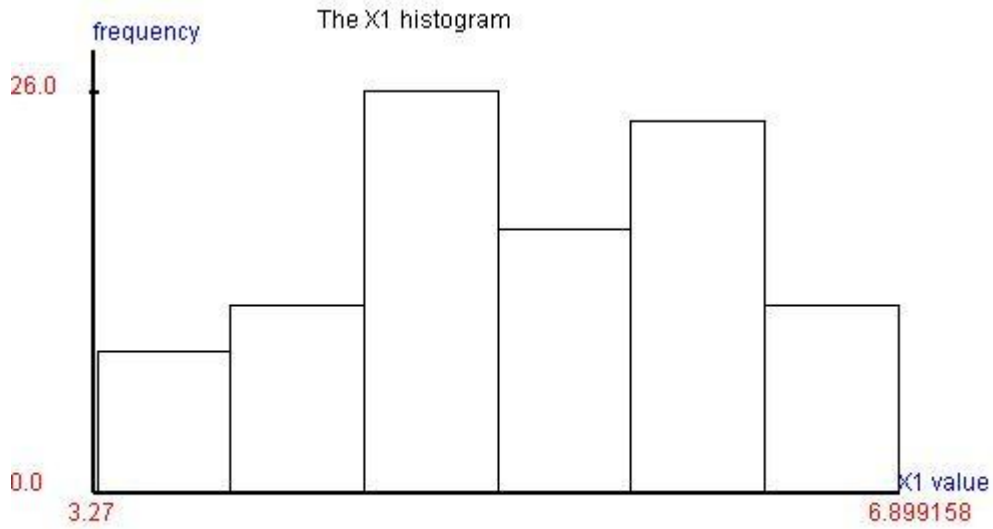
9.4.17)The population distribution is semi circle distribution.

X1 is Semi-circle($\mu=5.000000,R=2.000000$),

X1
6.3528366932
4.5864481187
5.2389891954
6.2777630972
4.6387667365
4.4331501991
4.7986294350
5.7340343807
5.5183432146
4.8936975583
4.4534506858
4.6280486256
5.0003157274
3.3392756418
5.3989019100
6.4416208471
3.3984266076
6.5202355290
6.4184617634
6.2978638509
6.0701435185
4.9012684428
5.1151655419
4.5517877717
4.8688092003
6.0349015511
4.1962521976
5.0470226898
3.9602319491
3.9646611958
5.8084003335
3.2777303146
6.2940040728
5.7315640811
5.6767632650
4.9876726765
4.6775816262
6.4469502191
5.0182900312
4.2230174260
4.9118748115
6.0191538442
5.1886694867
4.9282213272
4.6974678971
4.5936880176
5.4200211780
4.1069215866
4.2126406219
6.6780257351
6.0853714516
5.8273138579
4.6185729386
5.4447261167
6.0062458919
3.3039683210
5.6949835400
5.4893620467
4.2506246923
5.7116693206
4.2912813191
4.8519660634
6.2382089020
6.8065629897
5.8360041590
5.0458726511
5.8858218511
4.2906259010
4.8581832195
3.4113197358
6.6068955575

5.5897874871
 5.9816398185
 3.4066559696
 3.8527471177
 5.7581293589
 3.5433233948
 5.2931069909
 6.8991584424
 5.3530126786
 6.0601723370
 3.9734653701
 5.6482524897
 4.5963664747
 4.5191667880
 6.2121791635
 5.2734587564
 5.4077023789
 6.1495242617
 6.2754809628
 6.3012786897
 4.6377189022
 5.0871202104
 6.2017803275
 5.7460121034
 5.4598922854
 3.6437647752
 6.5901386794
 5.5664237491
 5.0248589728

X1 is Semi-circle(mu=5.000000,R=2.000000),



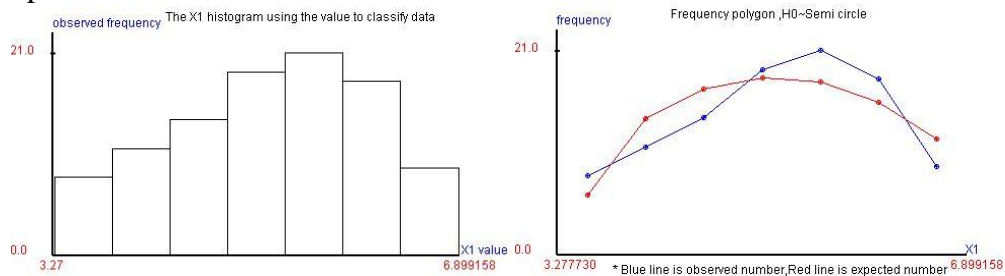
H0: $X_1 \sim \text{Semi-circle}(\mu, R)$, μ, R are unknown
 μ point estimated value=5.205861 (MLE)
 R point estimated value=1.810714 (MLE)

pearson goodness of fit

class	[1]	[2]	[3]	[4]
lower limit	3.27773	3.79508	4.31242	4.82977
5.34712	5.86446	6.38181		
upper limit	3.79508	4.31242	4.82977	5.34712
5.86446	6.38181	6.89916		
observed no	8.00000	11.00000	14.00000	19.00000
21.00000	18.00000	9.00000		
probability	0.06020	0.13900	0.16960	0.18080
0.17680	0.15580	0.11780		
expected no	6.02000	13.90000	16.96000	18.08000
17.68000	15.58000	11.78000		
chi square	0.49005	0.76455	0.62583	0.04455
0.52488	0.32536	0.85871		

degree of freedom=4

Likelihood ratio chi-square test statistic =3.633914
p-value=0.457800



9.4.18)The population distribution is logistic distribution.

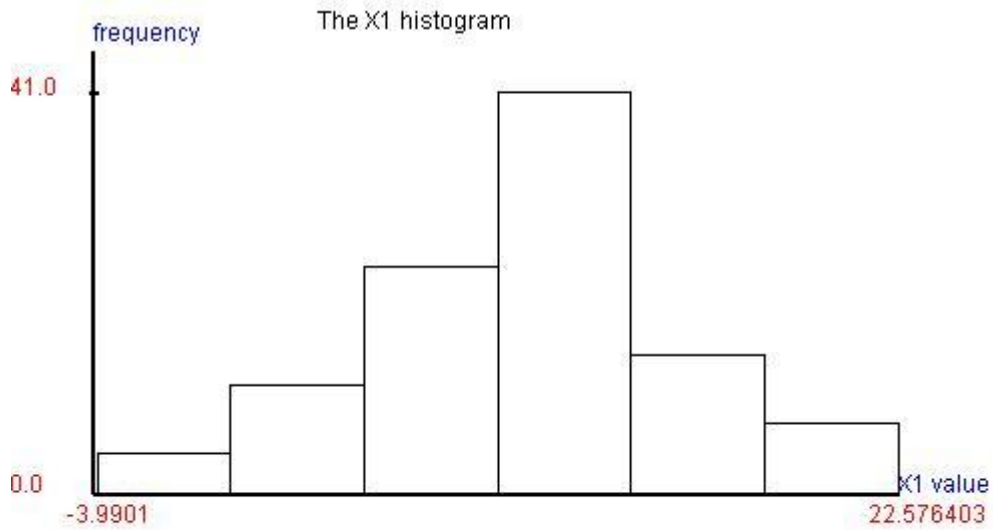
The mu value is setting to 10.5 and sigma value is 3.2.

X1 is Logistic(mu=10.000000,sigma=3.000000),

X1
7.5031168679
12.7140516439
8.0792426936
17.3069793276
10.3628497030
3.4165369494
13.6477279207
11.0876065833
4.8338725424
12.6400903769
12.3246668889
15.1533606517
8.8415348295
10.6005571156
5.7239033980
5.4926207692
6.7237529627
9.0325719834
11.3568443125
5.0136134972
11.4231319243
-0.9438318613
13.3421906503
4.3908483785
13.3208726984
6.8663983215
8.3370019949
2.5057254261
12.8701008759
11.2224012433
12.1755293222
10.4074088239
-1.2670291452
11.4984832720
1.8738892469
12.7098421043
7.6368083575
11.8248741299
15.7341941160
12.2326512883
7.1834630724
10.1506194939
7.7774778445
12.5133267784
11.6646936649
9.5550845017
10.8977955901
12.4075586145
6.0792714835
5.3156273086
16.3241586164
14.3413098341
6.2315065057
16.3299876475
18.9385900722
4.8348404084
11.2281051423
12.6716118932
6.9658011982
11.0680857555
10.1376420828
19.0375640050
13.0819221282
12.8550780353
18.7056844350
12.6775665893
7.7752289281
20.2413200344
9.7295716375
9.5738344353

15.2285008422
 13.5409069385
 3.5014931490
 15.8721533080
 18.0533805509
 16.6495821665
 6.7157285715
 12.0940734636
 -0.1805908839
 21.3201556000
 16.1256739609
 11.8235062214
 11.3493973646
 22.2624126581
 13.2228392535
 15.2803512212
 9.4858186973
 -3.9901959159
 12.6845733196
 14.5934679553
 3.4633870326
 2.4923907220
 7.2248028463
 22.5764035431
 8.3537573502
 3.0901658420
 6.0825333692
 5.5713264286
 0.6031055328
 17.9058503825

X1 is Logistic(mu=10.000000,sigma=3.000000),



H0: $X1 \sim \text{Logistic}(\mu=10.500000, \sigma=3.200000)$,

pearson goodness of fit

class	[1]	[2]	[3]	[4]
[5]	[6]	[7]		
lower limit	-3.99020	-0.19497	3.60026	7.39549
11.19072	14.98595	18.78118		
upper limit	-0.19497	3.60026	7.39549	11.19072
14.98595	18.78118	22.57640		
observed no	3.00000	9.00000	17.00000	21.00000
31.00000	13.00000	6.00000		
probability	0.03415	0.06960	0.17109	0.27891
0.24872	0.12760	0.06992		
expected no	3.41523	6.96039	17.10925	27.89051
24.87248	12.75980	6.99234		
chi square	0.05747	0.46222	0.00070	2.26091
1.21118	0.00444	0.16412		

Likelihood ratio chi square test statistic=4.161042

degree of freedom=6,p-value=0.761000

correction:

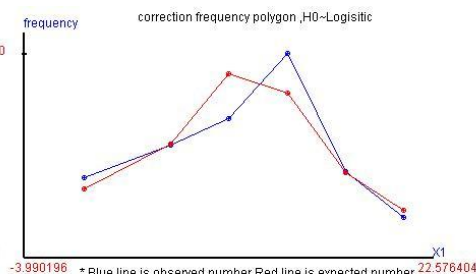
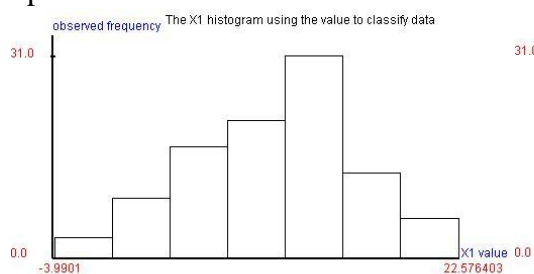
expected number ≥ 5 in each cell, the frequency table is adjusted

class	[1]	[2]	[3]	[4]
[5]	[6]			
lower limit	-3.99020	3.60026	7.39549	11.19072
14.98595	18.78118			
upper limit	3.60026	7.39549	11.19072	14.98595
18.78118	22.57640			
observed no	12.00000	17.00000	21.00000	31.00000
13.00000	6.00000			
probability	0.10376	0.17109	0.27891	0.24872
0.12760	0.06992			
expected no	10.37562	17.10925	27.89051	24.87248
12.75980	6.99234			
chi square	0.21988	0.00070	2.26091	1.21118
0.00444	0.16412			

degree of freedom=5

Likelihood ratio chi-square test statistic =3.861234

p-value=0.569500



9.4.19) The population distribution is Weibull distribution.

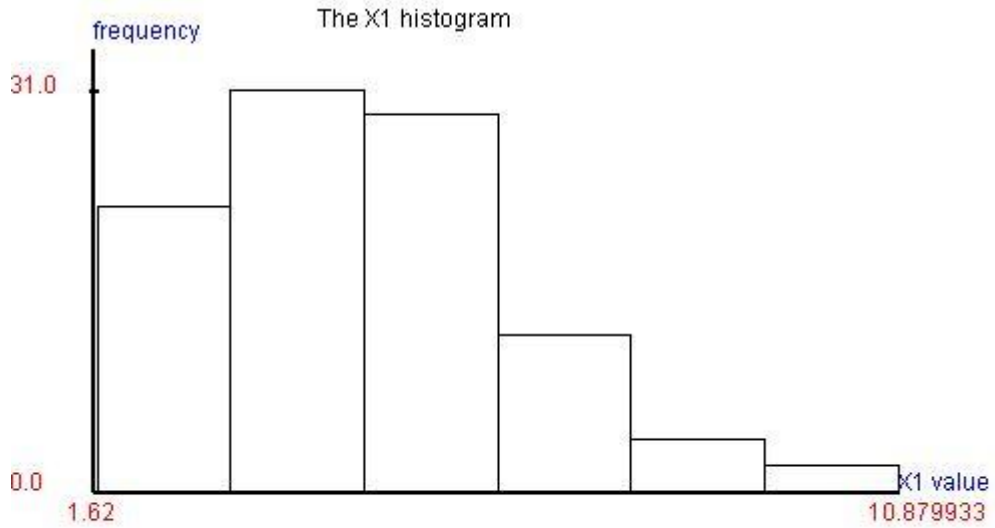
The alpha value is supposed to be 1 and the beta value is 3.8 and gamma value is 2.

X1 is Weibull(alpha=1.000000,beta=4.000000,gamma=2.000000),

X1
4.7555584608
6.7539452859
4.2333052024
2.4862689156
3.1675588583
5.2231901478
5.6367851034
4.6152261790
5.8274053710
3.3908052637
2.1954996590
6.1853785148
4.8769693049
4.0283218600
3.6584938594
2.4309454502
7.3603610441
3.7921899361
6.0856544882
2.5151657448
4.2000726944
7.1773923071
4.2955810175
5.4114382748
4.4687536969
7.2683133558
4.9649547546
4.3762135554
4.6712663262
1.8317013737
5.2837902804
4.6866283492
7.0879188263
2.3375755664
4.2168418046
5.0509968567
2.5342539466
3.9427711710
2.0736097476
4.1232267792
4.5682857886
9.7124824098
4.6556396946
6.8669958835
3.3975463348
4.8888840726
5.9244196264
7.4900242507
4.6112742827
5.9719882755
5.9110297850
4.9339316004
4.1139510163
4.6854371425
3.3804731678
5.8943232317
8.3224731331
3.2377740967
1.8165149212
5.0320040566
4.9551606303
10.8799338918
6.4080474795
3.7723946603
2.5243518227
2.6107937277
5.3952507453
4.3552087490
6.5323007716
5.4437420978

5.5549831647
 2.2384995620
 2.5125568969
 4.1041729478
 7.0926993221
 7.7719196328
 4.6808811090
 1.6280155188
 5.8208846313
 4.8047115498
 2.3000755373
 6.8851421721
 8.0582070736
 2.9024294514
 8.7033652673
 2.5280423704
 4.9390323440
 3.3857775184
 4.9813259707
 2.9778283975
 5.5127664952
 4.0384409700
 3.8526250878
 4.2488472302
 2.8172323689
 5.3970013399
 2.5351381733
 5.1092800188
 8.5796522108
 2.1950884769

X1 is Weibull(alpha=1.000000,beta=4.000000,gamma=2.000000),



H0: $X1 \sim \text{Weibull}(\alpha=1.000000, \beta=3.800000, \gamma=2.000000)$,

pearson goodness of fit

class	[1]	[2]	[3]	[4]
[5]	[6]	[7]		
lower limit	1.62802	2.94972	4.27142	5.59312
6.91483	8.23653	9.55823		
upper limit	2.94972	4.27142	5.59312	6.91483
8.23653	9.55823	10.87993		
observed no	20.00000	21.00000	32.00000	14.00000
8.00000	3.00000	2.00000		
probability	0.23145	0.29198	0.24456	0.14333
0.06207	0.02034	0.00627		
expected no	23.14540	29.19800	24.45609	14.33317
6.20655	2.03396	0.62683		
chi square	0.49468	3.20034	1.77846	0.00793
0.40206	0.31108	0.94280		

Likelihood ratio chi square test statistic=7.137343

degree of freedom=7,p-value=0.414700

correction:

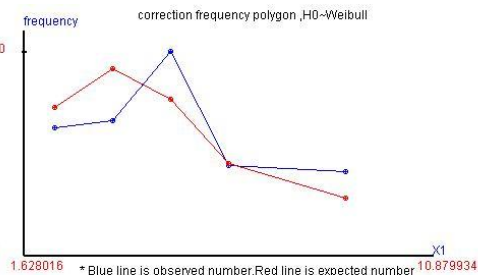
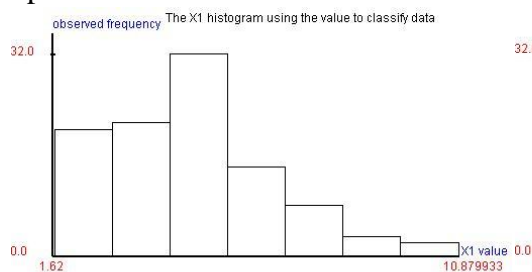
expected number ≥ 5 in each cell, the frequency table is adjusted

class	[1]	[2]	[3]	[4]
[5]				
lower limit	1.62802	2.94972	4.27142	5.59312
6.91483				
upper limit	2.94972	4.27142	5.59312	6.91483
10.87993				
observed no	20.00000	21.00000	32.00000	14.00000
13.00000				
probability	0.23145	0.29198	0.24456	0.14333
0.08867				
expected no	23.14540	29.19800	24.45609	14.33317
8.86734				
chi square	0.49468	3.20034	1.77846	0.00793
1.31376				

degree of freedom=5

Likelihood ratio chi-square test statistic =6.795169

p-value=0.236300



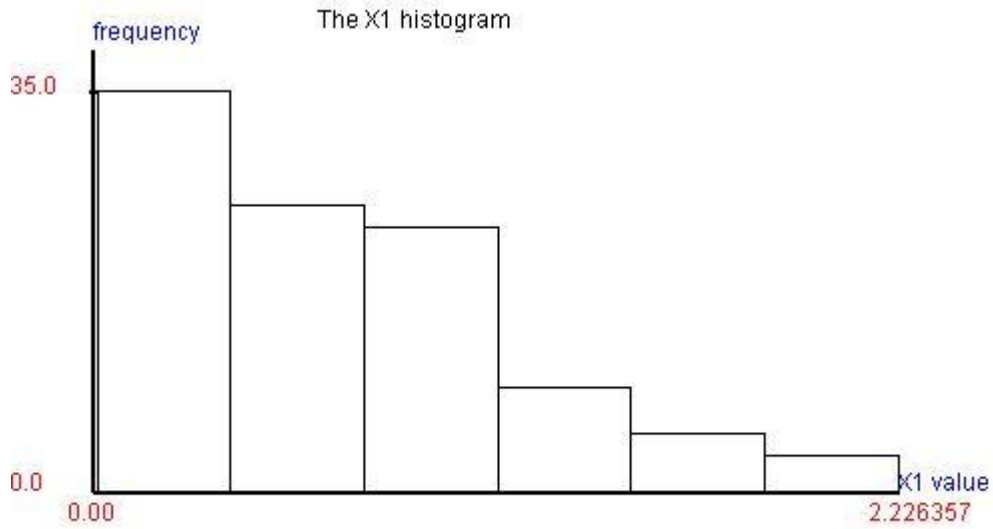
9.4.20)The population distribution is pareto 3 distribution.

X1 is Pareto3(lamda=5.000000,c=4.000000),

X1
1.5983418516
0.2100529889
0.0843554830
1.0998783957
1.0571027027
2.0164323515
1.6512135671
0.7824749298
0.3987885329
0.3446071707
0.4269488196
0.9333337712
1.2507872630
0.0277081823
0.5429672552
0.2329862303
0.5247162181
0.5742099887
0.2415577922
0.8759587963
0.6840262124
0.5917243297
0.6295935244
0.7768511416
0.0331388620
0.4123012155
0.1550902858
0.0273252877
0.6038226631
0.0888068843
0.0536572933
0.8981690956
0.3678883329
0.5056181718
0.2154752562
0.6244401062
0.1378728992
0.0473075400
1.4476275093
1.0466392735
0.2855528305
1.8089346743
1.5339378559
0.6970002278
0.9162170380
0.4002729552
1.0628104278
0.5935122123
1.4140899586
0.6346750631
1.4688532845
0.9941381173
0.2719851652
0.8306120791
0.1371996548
2.2263578599
0.3227058392
0.6934984934
0.0386702750
1.0371501377
0.5252932337
1.1418098470
0.0011918661
0.1163604056
1.7487634233
0.7228241749
0.1805080748
0.5052672488
0.4706802892
2.1971661507
0.9450348298

0.0572778306
1.3190320576
0.4022869111
0.0871338840
0.7665564739
0.7351613058
0.5617307140
0.1046233965
0.8550525984
0.0587659001
1.1676401049
0.2008007540
1.174222576
0.8749244866
1.1721989147
0.0788575750
0.8292236703
0.1648937207
0.8314687011
0.8366000779
0.2053971362
0.4695273226
0.0985832036
0.7798925022
0.2829724505
0.3690098275
0.7938131412
0.0984265092
0.7834361761

X1 is Pareto3(lamda=5.000000,c=4.000000),



H0: $X_1 \sim \text{Pareto } 3(\lambda, c)$, λ, c are unknown

λ point estimated value = 2.311893 (MLE)

c point estimated value = 2.226358 (MLE)

pearson goodness of fit

class	[1]	[2]	[3]	[4]
lower limit	0.00119	0.31907	0.63695	0.95483
1.27272	1.59060	1.90848		
upper limit	0.31907	0.63695	0.95483	1.27272
1.59060	1.90848	2.22636		
observed no	31.00000	24.00000	22.00000	11.00000
5.00000	4.00000	3.00000		
probability	0.30066	0.24053	0.18491	0.13305
0.08568	0.04405	0.01111		
expected no	30.06598	24.05333	18.49117	13.30508
8.56829	4.40521	1.11093		
chi square	0.02814	0.00012	0.55963	0.48304
2.54654	0.04105	1.18953		

Likelihood ratio chi square test statistic = 4.848050

degree of freedom = 4, p-value = 0.678500

correction:

expected number ≥ 5 in each cell, the frequency table is adjusted

class	[1]	[2]	[3]	[4]
lower limit	0.00119	0.31907	0.63695	0.95483
1.27272	1.59060			
upper limit	0.31907	0.63695	0.95483	1.27272
1.59060	2.22636			
observed no	31.00000	24.00000	22.00000	11.00000
5.00000	7.00000			
probability	0.30066	0.24053	0.18491	0.13305
0.08568	0.05516			
expected no	30.06598	24.05333	18.49117	13.30508
8.56829	5.51614			
chi square	0.02814	0.00012	0.55963	0.48304
2.54654	0.31455			

degree of freedom = 3

Likelihood ratio chi-square test statistic = 3.932020

p-value = 0.268800

