

Chapter six The goodness of fit test

9)MLR test 2:

9.1)The multi-nominal distribution is the sampling distribution, each category has a probability and the summation of probability is 1.

| | | | | |
|-------------|-------|-------|-------|-------|
| category | 1 | 2 | | k |
| probability | p_1 | p_2 | | p_k |

$$p_1 + \dots + p_k = 1,$$

$$f(x_1, \dots, x_k) = \frac{n!}{x_1! \times \dots \times x_k!} \times p_1^{x_1} \times \dots \times p_k^{x_k}, x_1 + \dots + x_k = n \quad \text{that is trial number.}$$

9.2)

$$X_i = np_i + \varepsilon_i, E(\varepsilon_i) = 0, E[(\varepsilon_i)^2] = np_i(1 - p_i), i = 1, 2, \dots, k$$

$$X_i - np_i = \varepsilon_i, E\left[\frac{(\varepsilon_i)^2}{np_i}\right] = E\left[\frac{(X_i - np_i)^2}{np_i}\right] = 1 - p_i,$$

$$\sum_{i=1}^k E\left[\frac{(X_i - np_i)^2}{np_i}\right] = E\left[\sum_{i=1}^k \left(\frac{(X_i - np_i)^2}{np_i}\right)\right] = k - \sum_{i=1}^k p_i = k - 1,$$

$$\sum_{i=1}^k \left(\frac{(X_i - np_i)^2}{np_i}\right) \rightarrow \chi_{k-1}^2$$

$$H_0 : p_1 = p_{01}, p_2 = p_{02}, \dots, p_k = p_{0k} \quad H_1 : \text{against } H_0$$

$p_{01}, p_{02}, \dots, p_{0k}$ are known value and $p_{01} + p_{02} + \dots + p_{0k} = 1,$

$$E_i = np_{i0}, i = 1, 2, \dots, k, O_i = x_i,$$

O_i : The observed sample number of cell i,

E_i : The expected sample number of cell i,

$$\chi_v^2 = \sum_{i=1}^k \frac{(O_i - E_i)^2}{O_i}, \quad v = k - 1 - (\text{number of point estimator}) \text{ is degree of freedom of}$$

pearsson chi square test.

$$\chi_v^2 > \chi_{\alpha, v}^2 \Rightarrow \text{reject } H_0.$$

9.3)The process of test

H_0 : Population distribution is a continuous probability distribution,

H_1 : against H_0

The sample size is n.

The process: i)The class number of frequency distribution: $k = \log_2(n) + 1.$

ii)The class limit of frequency distribution:

The class limit will be found by the a general frequency distribution table.

iii)The class sample number of frequency distribution(O_i):

The frequency distribution is done and getting the O_i of each class.

iv) The class expected number of frequency distribution(E_i):

$$E_i = n \times \text{the probability of each class.}$$

Note: There are 20 kinds of continuous probability distribution that is can be assigned to null hypothesis.

9.4) Example (The simulated sample data and computing the result by the P_S_CCC)

The Likelihood ratio chi square test (goodness of fit), the traditional frequency distribution, please select the population distribution

| | |
|---------------------------------------|------------------------------------|
| 1.H0:Uniform distribution | 13.H0:Gumbel distribution |
| 2.H0:Normal distribution | 14.H0:Triangular 1 distribution |
| 3.H0:Shifted exponential distribution | 15.H0:Trapezoid distribution |
| 4.H0:Pareto 1 distribution | 16.H0:U-quadratic distribution |
| 5.H0:Pareto 2 distribution | 17.H0:Semi-circle distribution |
| 6.H0:Rayleigh distribution | 18.H0:Logistic distribution |
| 7.H0:Double exponential distribution | 19.H0:Weibull distribution |
| 8.H0:Log normal distribution | 20.H0:Pareto 3 distribution |
| 9.H0:Gamma distribution | ** Above H0 population all do once |
| 10.H0:Beta distribution | |
| 11.H0:Cauchy distribution | |
| 12.H0:Arcsin distribution | |

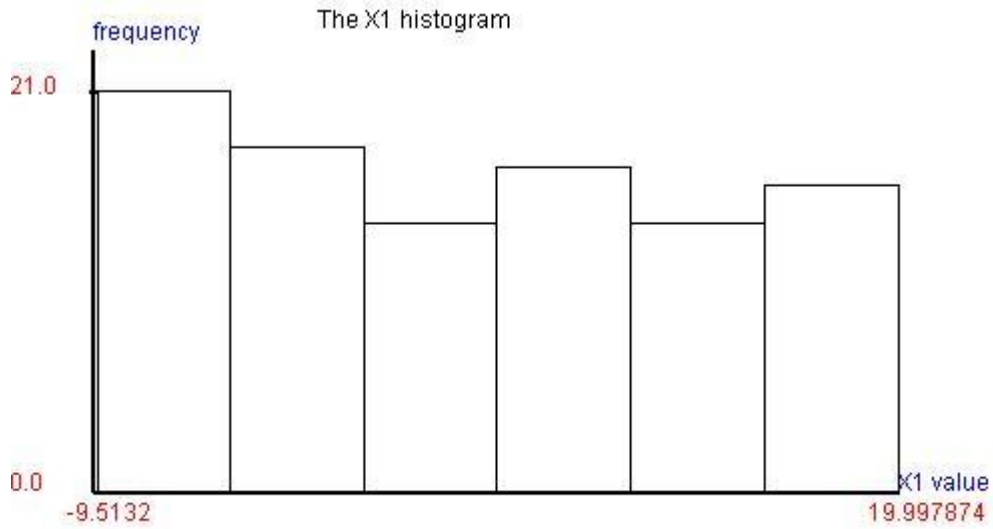
9.4.1)The population distribution is uniform distribution.

X1 is Uniform(alpha=-10.000000,beta=20.000000),

| X1 |
|---------------|
| 16.0148324459 |
| -0.1645587974 |
| 19.9978740573 |
| 13.7584293220 |
| 9.3681934442 |
| 19.4012105777 |
| 13.4644226164 |
| -0.4809078048 |
| -9.2801655751 |
| 14.0043112313 |
| 5.4704060214 |
| -8.3619159609 |
| 7.7946828771 |
| 8.4871805833 |
| 0.2823806262 |
| -3.8480125069 |
| 0.5548084042 |
| -1.2458474689 |
| 19.1374332454 |
| 6.8375128782 |
| 17.5806037555 |
| -6.4245674802 |
| 10.3140162125 |
| 7.0798608407 |
| 4.2832109062 |
| 6.2983046694 |
| 1.8448329834 |
| -9.4413981740 |
| 13.3058418462 |
| 18.0056983145 |
| 4.6639613667 |
| -0.2055973392 |
| -7.6116737157 |
| 8.7254121406 |
| -0.9729065583 |
| 16.6310603482 |
| 0.0378857597 |
| 18.2118121950 |
| -7.9965132051 |
| 3.7245553168 |
| 4.2969466944 |
| 2.8840752397 |
| -1.1452969076 |
| -8.9276163471 |
| -5.0682340250 |
| -9.5132595670 |
| -3.8902855826 |
| -3.3176520022 |
| 14.3564851961 |
| -0.9836435414 |
| -0.8421376655 |
| 5.4131507221 |
| -2.4156573393 |
| 11.4918888337 |
| 12.1573481959 |
| 5.0594178705 |
| 16.5600290999 |
| 13.1364799572 |
| 3.0862327619 |
| 3.7620222269 |
| 9.1319553498 |
| -7.4920589484 |
| 11.7491885920 |
| -2.2321907440 |
| 8.0266577772 |
| 3.1702891665 |
| 3.5276132567 |
| -6.3639257616 |
| 10.1051685814 |
| 7.4473706764 |
| -3.2806920100 |

-6.0612162319
 -8.7748775910
 18.8325729968
 16.2654005504
 -9.4801550919
 17.0305419511
 -5.3896848269
 11.6603609489
 -5.6304293585
 -5.0472095229
 -4.7394278507
 11.8968073299
 -3.8084519454
 7.7785085707
 2.1938834548
 -4.7856009719
 6.7125634413
 -5.9447943607
 0.3990877827
 16.4990206441
 10.7944801889
 15.6720121051
 17.5494032251
 10.1198659391
 14.4280215709
 -2.0862700456
 6.1335299486
 -8.9612776399
 18.6527049782

X1 is Uniform(alpha=-10.000000,beta=20.000000),



H0: $X_1 \sim \text{Uniform}(\alpha, \beta)$, α, β are unknown

alpha point estimated value = -9.513260 (MLE)

beta point estimated value = 19.997874 (MLE)

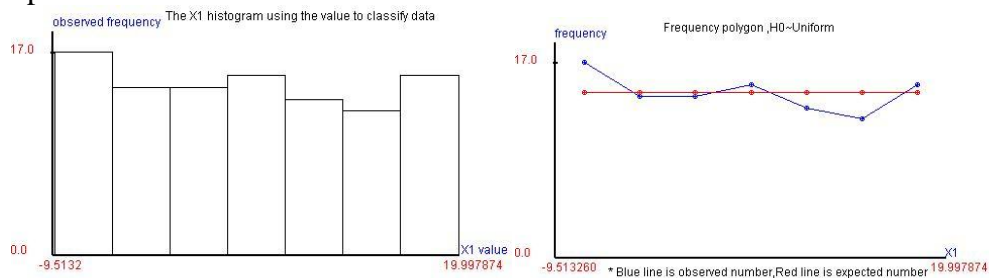
pearson goodness of fit

| class | [1] | [2] | [3] | [4] |
|-------------|----------|----------|----------|----------|
| lower limit | -9.51326 | -5.29738 | -1.08151 | 3.13437 |
| 7.35025 | 11.56612 | 15.78200 | | |
| upper limit | -5.29738 | -1.08151 | 3.13437 | 7.35025 |
| 11.56612 | 15.78200 | 19.99787 | | |
| observed no | 17.00000 | 14.00000 | 14.00000 | 15.00000 |
| 13.00000 | 12.00000 | 15.00000 | | |
| probability | 0.14286 | 0.14286 | 0.14286 | 0.14286 |
| 0.14286 | 0.14286 | 0.14286 | | |
| expected no | 14.28571 | 14.28571 | 14.28571 | 14.28571 |
| 14.28571 | 14.28571 | 14.28571 | | |
| chi square | 0.43337 | 0.00583 | 0.00583 | 0.03401 |
| 0.12716 | 0.43537 | 0.03401 | | |

degree of freedom = 4

Likelihood ratio chi-square test statistic = 1.075595

p-value = 0.898100

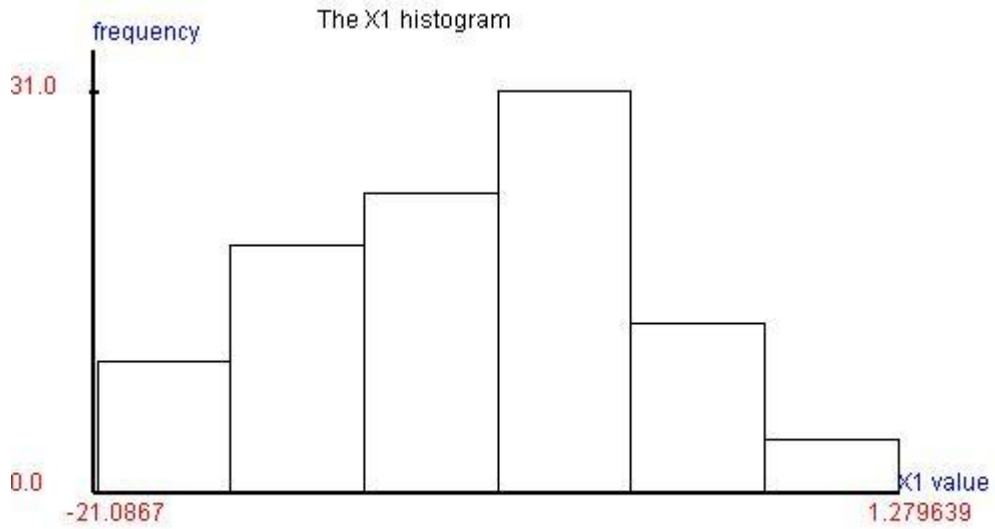


9.4.2)The population distribution is normal distribution.
X1 is Normal($\mu=-10.000000$, $\sigma^2=25.000000$),

X1
-13.3498035256
-8.7564925642
-11.9930564397
-5.1602962304
-16.6532964531
-7.8159210040
-9.7008664157
-13.1910828261
-21.0867245745
-10.8569787301
-16.1717283714
-13.5361015012
-8.4656334054
-6.4163098121
-16.0772773755
-17.2338190334
-8.0374096537
-0.0460707373
-5.5333169761
-18.2080634228
-3.8845509165
-14.8049987119
-20.2009897463
-13.1932352941
-14.7432016863
-6.3017701150
-8.1858612917
-5.3203727337
-15.3246357137
-8.0893862584
-10.3186918621
-8.2219614198
-5.3468579021
-5.0277244793
-6.3150405270
-5.2364908671
-15.9370825333
-19.6915810070
-6.2381979414
-13.9031370716
-15.1234297119
-8.9419105921
-2.5533480548
-7.1312349628
-18.0341523366
-10.4451840929
-15.2345740206
-11.9278117627
-9.7886500637
-15.2191778022
-12.4554910888
-8.5296446780
-15.1389888636
-7.9377651241
-7.9156283370
-10.8434543899
-4.0083921689
-7.7029526966
0.3272470669
-13.8176050424
-6.3867051161
-8.1070462225
-13.4121550404
-10.3335285860
-10.2928169430
-2.6170038623
-12.4755455148
-11.0338007143
-9.5948813967
-3.1902930898
-19.2764885941

-16.3497655773
 1.2796392301
 -4.2757703786
 -7.9839231163
 -9.0955518535
 -14.7568347580
 -13.4278556770
 -11.2606602444
 -17.5956649222
 -10.6686252845
 -9.4011303875
 -14.1500155906
 -18.7162528446
 -6.2772799767
 -14.0217474682
 -13.6646727262
 -9.5968363114
 -10.7781580612
 -11.1816312692
 -19.9213527096
 -6.6371585698
 -10.7916047555
 -7.1720584034
 -1.8961766187
 -19.1090035003
 -6.6818227885
 -7.2831954719
 -10.5695748427
 -5.5778377588

X1 is Normal($\mu=-10.000000$, $\sigma^2=25.000000$),



H0: $X_1 \sim \text{Normal}(\mu, \sigma^2)$, μ, σ are unknown
 population mean(μ) point estimated value = -10.512810 (MLE)
 population variance(σ^2) which point estimated value = 24.417695 (MLE)
 pearson goodness of fit

| class | [1] | [2] | [3] | [4] |
|-------------|-----------|-----------|-----------|-----------|
| lower limit | -21.08672 | -17.89153 | -14.69633 | -11.50114 |
| -5.11075 | -1.91556 | | | |
| upper limit | -17.89153 | -14.69633 | -11.50114 | -8.30595 |
| -1.91556 | 1.27964 | | | |
| observed no | 9.00000 | 15.00000 | 15.00000 | 23.00000 |
| 27.00000 | 7.00000 | 4.00000 | | |
| probability | 0.06770 | 0.13100 | 0.22210 | 0.25170 |
| 0.19040 | 0.09620 | 0.04090 | | |
| expected no | 6.77000 | 13.10000 | 22.21000 | 25.17000 |
| 19.04000 | 9.62000 | 4.09000 | | |
| chi square | 0.55254 | 0.24067 | 3.46561 | 0.20473 |
| 2.34673 | 0.98063 | 0.00203 | | |

Likelihood ratio chi square test statistic = 7.792932
 degree of freedom = 4, p-value = 0.351200

correction:

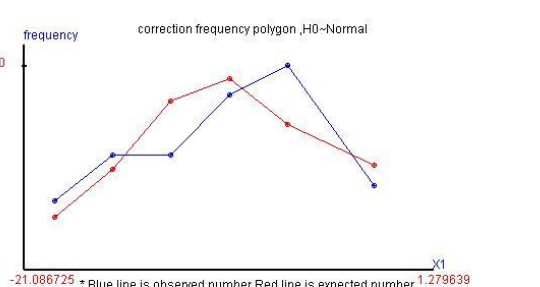
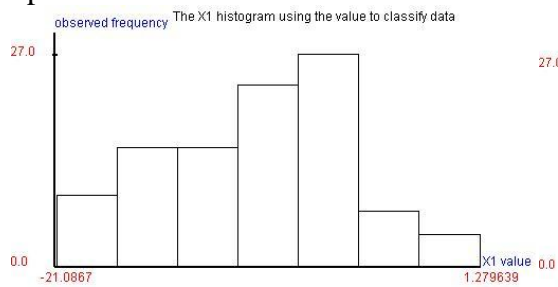
expected number ≥ 5 in each cell, the frequency table is adjusted

| class | [1] | [2] | [3] | [4] |
|-------------|-----------|-----------|-----------|-----------|
| lower limit | -21.08672 | -17.89153 | -14.69633 | -11.50114 |
| -5.11075 | | | | |
| upper limit | -17.89153 | -14.69633 | -11.50114 | -8.30595 |
| 1.27964 | | | | |
| observed no | 9.00000 | 15.00000 | 15.00000 | 23.00000 |
| 27.00000 | 11.00000 | | | |
| probability | 0.06770 | 0.13100 | 0.22210 | 0.25170 |
| 0.19040 | 0.13710 | | | |
| expected no | 6.77000 | 13.10000 | 22.21000 | 25.17000 |
| 19.04000 | 13.71000 | | | |
| chi square | 0.55254 | 0.24067 | 3.46561 | 0.20473 |
| 2.34673 | 0.66765 | | | |

degree of freedom = 3

Likelihood ratio chi-square test statistic = 7.477924

p-value = 0.058100

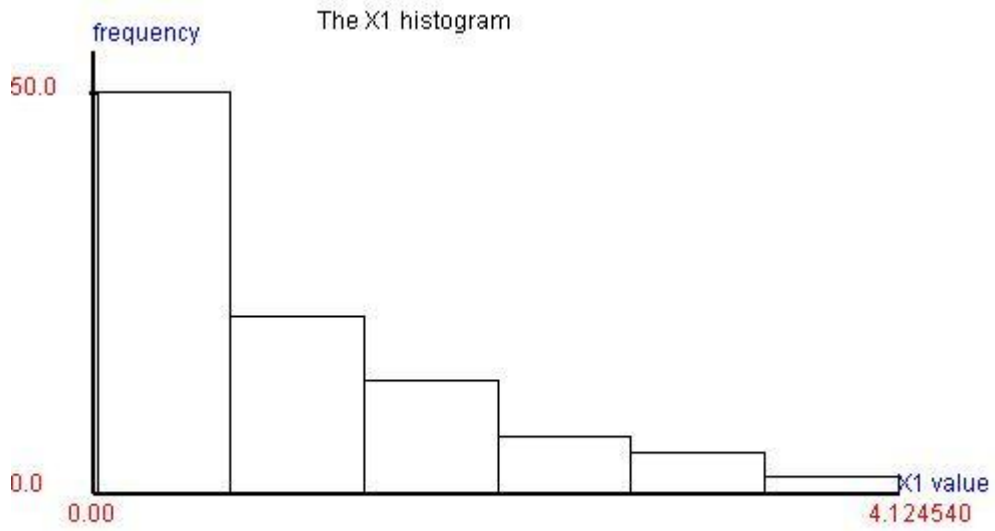


9.4.3)The population distribution is shifted exponential distribution.

X1 is Exponential($\lambda=1.000000,c=0.000000$),

| X1 |
|--------------|
| 0.9394308799 |
| 1.1939298856 |
| 3.3357139264 |
| 0.1763734167 |
| 0.7291305607 |
| 0.1323043767 |
| 1.9541569692 |
| 1.8309962048 |
| 1.9715067083 |
| 0.3608083964 |
| 0.8800880650 |
| 0.1071981165 |
| 1.3441431557 |
| 0.3864180266 |
| 0.7827668744 |
| 0.0839891945 |
| 0.4194794595 |
| 1.6728135969 |
| 1.6982224203 |
| 1.9288081778 |
| 0.1245989351 |
| 0.6424459424 |
| 1.5487437635 |
| 2.2689170694 |
| 1.6541979775 |
| 0.5954085451 |
| 0.9374142590 |
| 0.5068469958 |
| 0.4796436791 |
| 0.8869881643 |
| 0.8289855425 |
| 0.6289117718 |
| 1.6507186170 |
| 0.8910973477 |
| 0.3522979015 |
| 0.2299708901 |
| 1.3578575089 |
| 0.0084183346 |
| 2.5061054170 |
| 1.0668609774 |
| 0.0612661825 |
| 0.1539395316 |
| 4.1245402724 |
| 0.9839829173 |
| 0.3173270280 |
| 1.5954470234 |
| 0.4319733855 |
| 0.4925599194 |
| 1.7519691457 |
| 2.5107373177 |
| 2.3956417893 |
| 0.2955109537 |
| 0.3150698969 |
| 0.1150163104 |
| 0.2050536624 |
| 0.0269912032 |
| 0.3470110899 |
| 0.0087913657 |
| 1.7465586683 |
| 0.3761705326 |
| 1.0414467645 |
| 0.8295547185 |
| 2.9620277066 |
| 1.3479720270 |
| 2.7789535070 |
| 0.1714413490 |
| 0.4719218244 |
| 0.0164585541 |
| 0.3482375179 |
| 0.9813527318 |
| 1.3530559063 |

2.1757202954
2.3996371917
0.1501357197
0.6282781119
3.0895100625
0.6168465486
3.5947554694
0.9637356863
0.4823659370
0.5118185345
0.1426203646
1.0245169303
0.6276803436
0.1363295601
2.0960649156
3.1869906065
0.0853614934
0.0257708177
0.1008286398
0.6124579250
0.0127625014
0.4850542044
0.8602314006
1.8494975089
0.1592436091
0.2131732323
0.1514629614
1.9969725552
0.8059895278



H0: $X_1 \sim \text{Shifted exponential}(\lambda, c)$, λ, c are unknown

λ point estimated value = 1.000033 (MLE)

c point estimated value = 0.008418 (MLE)

pearson goodness of fit

| class | [1] | [2] | [3] | [4] |
|-------------|----------|----------|----------|---------|
| lower limit | 0.00842 | 0.59644 | 1.18445 | 1.77247 |
| 2.36049 | 2.94851 | 3.53652 | | |
| upper limit | 0.59644 | 1.18445 | 1.77247 | 2.36049 |
| 2.94851 | 3.53652 | 4.12454 | | |
| observed no | 44.00000 | 23.00000 | 13.00000 | 9.00000 |
| 5.00000 | 4.00000 | 2.00000 | | |
| probability | 0.44458 | 0.24693 | 0.13715 | 0.07617 |
| 0.04231 | 0.02350 | 0.02936 | | |
| expected no | 44.45835 | 24.69290 | 13.71484 | 7.61745 |
| 4.23086 | 2.34989 | 2.93571 | | |
| chi square | 0.00477 | 0.12460 | 0.03931 | 0.21238 |
| 0.11832 | 0.68072 | 0.43777 | | |

Likelihood ratio chi square test statistic = 1.617876

degree of freedom = 4, p-value = 0.977900

correction:

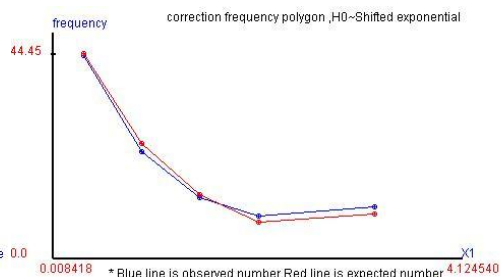
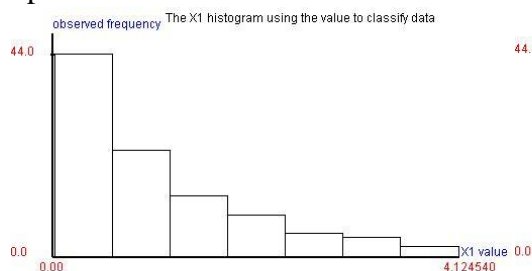
expected number ≥ 5 in each cell, the frequency table is adjusted

| class | [1] | [2] | [3] | [4] |
|-------------|----------|----------|----------|---------|
| lower limit | 0.00842 | 0.59644 | 1.18445 | 1.77247 |
| 2.36049 | | | | |
| upper limit | 0.59644 | 1.18445 | 1.77247 | 2.36049 |
| 4.12454 | | | | |
| observed no | 44.00000 | 23.00000 | 13.00000 | 9.00000 |
| 11.00000 | | | | |
| probability | 0.44458 | 0.24693 | 0.13715 | 0.07617 |
| 0.09516 | | | | |
| expected no | 44.45835 | 24.69290 | 13.71484 | 7.61745 |
| 9.51645 | | | | |
| chi square | 0.00477 | 0.12460 | 0.03931 | 0.21238 |
| 0.20008 | | | | |

degree of freedom = 2

Likelihood ratio chi-square test statistic = 0.581153

p-value = 0.747800



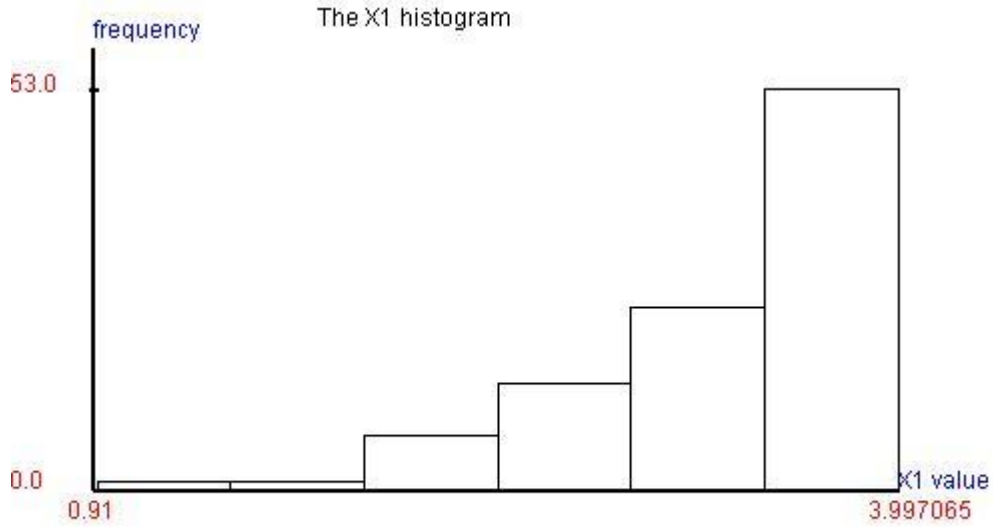
9.4.4)The population distribution is pareto1 distribution.

X1 is Pareto1(lamda=5.000000,c=4.000000),

| X1 |
|--------------|
| 3.2119521586 |
| 3.775666509 |
| 3.8757758470 |
| 3.9588294732 |
| 3.9631402533 |
| 2.9961679685 |
| 2.9297826407 |
| 3.9863282569 |
| 3.8248803017 |
| 0.9182544777 |
| 3.9575599076 |
| 3.8423545433 |
| 3.7785554441 |
| 3.1251888791 |
| 3.9106674045 |
| 3.1750711360 |
| 3.1179085699 |
| 3.9327556774 |
| 2.9003338671 |
| 3.5186353276 |
| 3.8722181474 |
| 3.6910494465 |
| 2.1234110744 |
| 3.9323856405 |
| 3.8782176102 |
| 3.2182239002 |
| 3.9385039680 |
| 2.9462152212 |
| 3.1986454514 |
| 3.3092838036 |
| 3.6506029294 |
| 3.9019219074 |
| 2.6860158859 |
| 3.0062049661 |
| 3.8166090211 |
| 2.9286161540 |
| 3.9130359179 |
| 2.8317506415 |
| 2.2599835652 |
| 2.1522065318 |
| 3.4164640609 |
| 3.9043332593 |
| 3.9532627132 |
| 2.6856078227 |
| 1.9805469673 |
| 3.6218165822 |
| 3.0260888362 |
| 2.5626620528 |
| 3.9679312928 |
| 3.9469672298 |
| 2.6654197500 |
| 2.2438132281 |
| 2.4508646406 |
| 2.9321977095 |
| 3.8275353478 |
| 3.7963123267 |
| 3.7777404276 |
| 3.5786725913 |
| 3.8579687852 |
| 3.5931051723 |
| 3.3835640779 |
| 3.6195392363 |
| 2.5737384900 |
| 3.8709852892 |
| 3.9970654585 |
| 3.6524833067 |
| 3.3038590376 |
| 1.8862703493 |
| 3.5773310297 |
| 3.6425727234 |
| 3.4502245288 |

3.3396845279
 2.2923293652
 3.7571039667
 2.4774332813
 3.4487054322
 3.0683357865
 3.3954707467
 3.0319142883
 3.8915750407
 3.6546563777
 3.2845594149
 3.2761820861
 3.9848867738
 3.6812236536
 3.7069106505
 3.9516637503
 2.8105685972
 3.8783663341
 2.5142720292
 3.8803824016
 3.6487897988
 3.3506067198
 3.5819593890
 3.2485941792
 3.8708430744
 3.3798912640
 3.6026020926
 3.5603863198
 3.7276997734

X1 is Pareto1(lamda=5.000000,c=4.000000),



H0: $X_1 \sim \text{Pareto 1}(\lambda, c)$, λ, c are unknown

λ point estimated value=5.153388 (MLE)

c point estimated value=3.997065 (MLE)

pearson goodness of fit

| class | [1] | [2] | [3] | [4] |
|-------------|----------|----------|---------|---------|
| lower limit | 0.91825 | 1.35808 | 1.79791 | 2.23774 |
| 2.67758 | 3.11741 | 3.55724 | | |
| upper limit | 1.35808 | 1.79791 | 2.23774 | 2.67758 |
| 3.11741 | 3.55724 | 3.99707 | | |
| observed no | 1.00000 | 0.00000 | 4.00000 | 9.00000 |
| 14.00000 | 20.00000 | 52.00000 | | |
| probability | 0.00384 | 0.01245 | 0.03403 | 0.07654 |
| 0.15092 | 0.27061 | 0.45161 | | |
| expected no | 0.38372 | 1.24526 | 3.40258 | 7.65407 |
| 15.09242 | 27.06110 | 45.16083 | | |
| chi square | 0.37980 | | 0.08923 | 0.20128 |
| 0.08524 | 2.49296 | 0.89950 | | |

Likelihood ratio chi square test statistic=1.#INF00

degree of freedom=4,p-value=0.000000

correction:

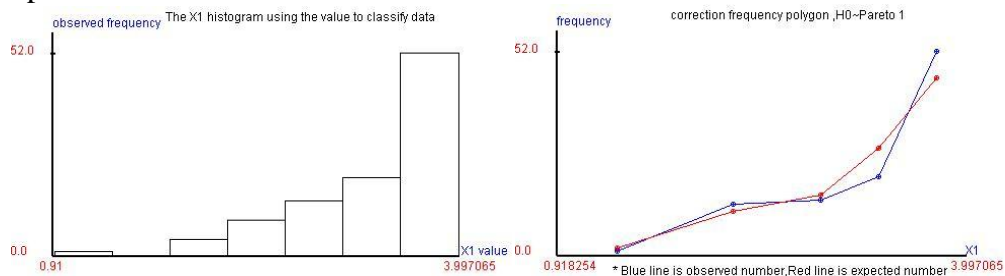
expected number ≥ 5 in each cell, the frequency table is adjusted

| class | [1] | [2] | [3] | [4] |
|-------------|---------|----------|----------|----------|
| lower limit | 0.91825 | 1.79791 | 2.67758 | 3.11741 |
| 3.55724 | | | | |
| upper limit | 1.79791 | 2.67758 | 3.11741 | 3.55724 |
| 3.99707 | | | | |
| observed no | 1.00000 | 13.00000 | 14.00000 | 20.00000 |
| 52.00000 | | | | |
| probability | 0.01629 | 0.11057 | 0.15092 | 0.27061 |
| 0.45161 | | | | |
| expected no | 1.62899 | 11.05665 | 15.09242 | 27.06110 |
| 45.16083 | | | | |
| chi square | 0.39563 | 0.29051 | 0.08524 | 2.49296 |
| 0.89950 | | | | |

degree of freedom=2

Likelihood ratio chi-square test statistic =4.163834

p-value=0.124600



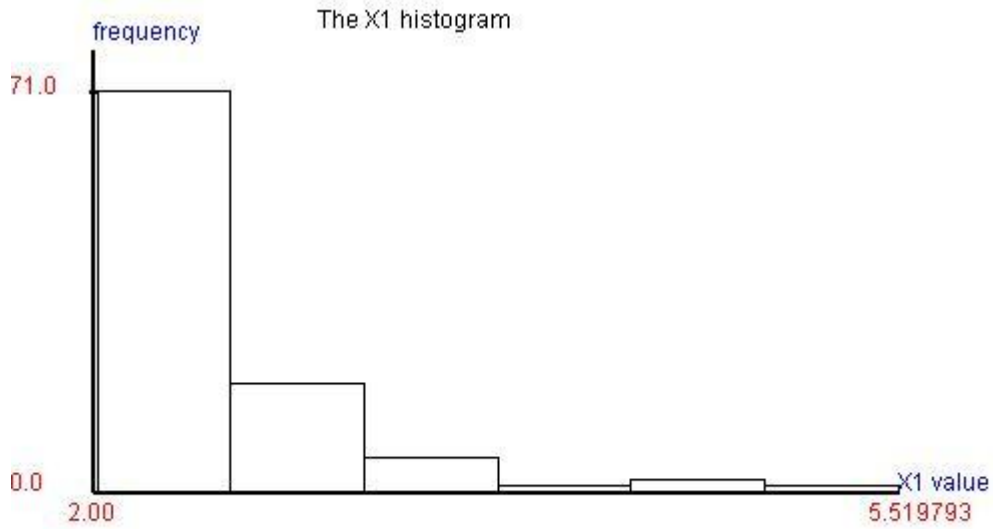
9.4.5)The population distribution is pareto2 distribution.

X1 is Pareto2(lamda=4.000000,c=2.000000),

| X1 |
|--------------|
| 2.1298789478 |
| 2.3663163023 |
| 2.6802655086 |
| 2.1565038783 |
| 2.8269575210 |
| 2.2847688651 |
| 2.0569199256 |
| 2.2155795421 |
| 2.5698604110 |
| 2.1773780150 |
| 2.1633592813 |
| 2.1371300202 |
| 2.1744113863 |
| 4.5007584444 |
| 2.0670634620 |
| 2.3413390703 |
| 2.4573945912 |
| 2.0286229265 |
| 2.2451366635 |
| 2.0706763381 |
| 2.1384909284 |
| 2.0874218372 |
| 2.0660525800 |
| 2.8402611330 |
| 3.1081528236 |
| 2.2486457894 |
| 3.8700854052 |
| 2.4644941956 |
| 3.1260251491 |
| 2.1619318403 |
| 2.1024054727 |
| 2.0554145366 |
| 2.1817927098 |
| 2.0904512649 |
| 3.4499724645 |
| 2.1223372237 |
| 2.9276962944 |
| 3.1112259319 |
| 3.0903602545 |
| 2.2860375312 |
| 2.7701130466 |
| 2.4356235259 |
| 2.0359003652 |
| 2.6219083339 |
| 2.0765502513 |
| 2.4103472622 |
| 3.5065638427 |
| 2.3215178156 |
| 2.6798075882 |
| 2.7848203677 |
| 2.0271975950 |
| 2.1988448629 |
| 2.1043349929 |
| 2.0328772828 |
| 2.2967274646 |
| 2.7308167453 |
| 2.2710295520 |
| 3.5312920034 |
| 2.0251529827 |
| 2.1495691666 |
| 2.1827694107 |
| 2.0218278626 |
| 2.4034670414 |
| 2.2043638483 |
| 2.5338405906 |
| 2.0194252755 |
| 2.8973592235 |
| 2.0512256398 |
| 3.1006562366 |
| 3.1491690508 |
| 2.1333621560 |

3.4616195647
 2.3774155838
 3.3218030244
 2.2038928668
 3.6488214453
 5.5197936535
 2.3410428708
 2.2833279909
 2.2585617079
 2.3989844467
 2.6701062118
 2.0246385942
 2.0264536856
 2.6824873923
 2.3481749348
 2.5652712788
 2.0135491817
 2.1741077197
 2.1440524544
 2.3061885490
 2.0303455647
 2.3544709740
 2.9699602598
 2.0059853038
 2.0786694801
 2.3283837010
 2.5654472058
 4.8353498036
 2.4054551262

X1 is Pareto2(lamda=4.000000,c=2.000000),



H0: $X_1 \sim \text{Pareto } 2(\lambda, c)$, λ, c are unknown

λ point estimated value=4.953361 (MLE)

c point estimated value=2.005985 (MLE)

pearson goodness of fit

| class | [1] | [2] | [3] | [4] |
|-------------|----------|----------|----------|---------|
| lower limit | 2.00599 | 2.50796 | 3.00993 | 3.51190 |
| 4.01388 | 4.51585 | 5.01782 | | |
| upper limit | 2.50796 | 3.00993 | 3.51190 | 4.01388 |
| 4.51585 | 5.01782 | 5.51979 | | |
| observed no | 67.00000 | 17.00000 | 10.00000 | 3.00000 |
| 1.00000 | 1.00000 | 1.00000 | | |
| probability | 0.66920 | 0.19681 | 0.07158 | 0.03021 |
| 0.01424 | 0.00731 | 0.01066 | | |
| expected no | 66.92033 | 19.68057 | 7.15788 | 3.02112 |
| 1.42380 | 0.73060 | 1.06570 | | |
| chi square | 0.00009 | 0.42267 | 0.80777 | 0.00015 |
| 0.17960 | 0.07258 | 0.00432 | | |

Likelihood ratio chi square test statistic=1.487180

degree of freedom=4,p-value=0.982700

correction:

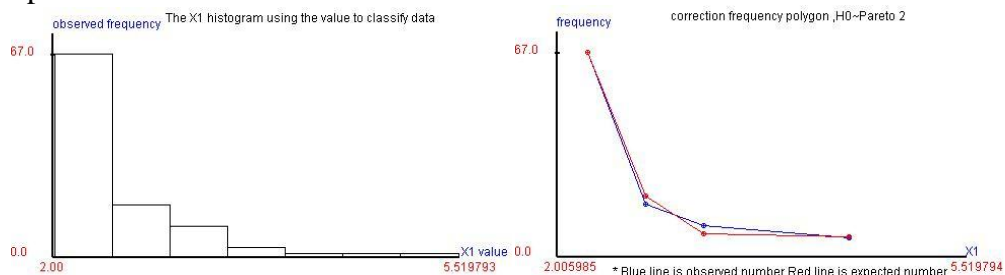
expected number ≥ 5 in each cell, the frequency table is adjusted

| class | [1] | [2] | [3] | [4] |
|-------------|----------|----------|----------|---------|
| lower limit | 2.00599 | 2.50796 | 3.00993 | 3.51190 |
| upper limit | 2.50796 | 3.00993 | 3.51190 | 5.51979 |
| observed no | 67.00000 | 17.00000 | 10.00000 | 6.00000 |
| probability | 0.66920 | 0.19681 | 0.07158 | 0.06241 |
| expected no | 66.92033 | 19.68057 | 7.15788 | 6.24122 |
| chi square | 0.00009 | 0.42267 | 0.80777 | 0.00970 |

degree of freedom=1

Likelihood ratio chi-square test statistic =1.240233

p-value=0.265400



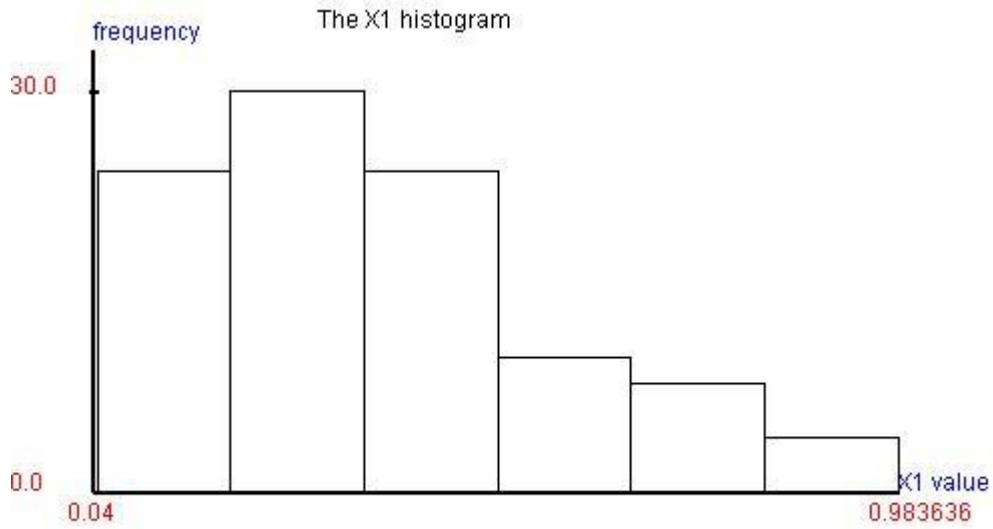
9.4.6)The population distribution is rayleigh distribution.

X1 is Rayleigh($\lambda=5.000000, c=0.000000$),

| X1 |
|--------------|
| 0.1902216912 |
| 0.4399796827 |
| 0.8046690784 |
| 0.2724430820 |
| 0.3180975815 |
| 0.1784977768 |
| 0.7216416502 |
| 0.1417606088 |
| 0.2854055094 |
| 0.2397624432 |
| 0.4117626627 |
| 0.0632072597 |
| 0.1853933928 |
| 0.4114721714 |
| 0.6765697071 |
| 0.9577026641 |
| 0.1583056415 |
| 0.1740775774 |
| 0.1688123798 |
| 0.4823352664 |
| 0.2040232735 |
| 0.2370178778 |
| 0.4080862967 |
| 0.1657596538 |
| 0.1391774169 |
| 0.4734810335 |
| 0.5388810278 |
| 0.4249817946 |
| 0.4960177676 |
| 0.2894406642 |
| 0.1961885753 |
| 0.7599106181 |
| 0.6065917724 |
| 0.2586826371 |
| 0.1177209116 |
| 0.2639933291 |
| 0.3100096886 |
| 0.0737373599 |
| 0.6644682116 |
| 0.1285042534 |
| 0.5934100964 |
| 0.2849455918 |
| 0.8953278202 |
| 0.3646015123 |
| 0.2062733525 |
| 0.2818265065 |
| 0.4636776998 |
| 0.2449352515 |
| 0.2524013564 |
| 0.9297570007 |
| 0.4779160571 |
| 0.6639750998 |
| 0.2744329219 |
| 0.0535495003 |
| 0.1388401813 |
| 0.1922494729 |
| 0.0724350553 |
| 0.1341064595 |
| 0.3373860682 |
| 0.1626080434 |
| 0.9836367214 |
| 0.3567759517 |
| 0.2869633998 |
| 0.2812991976 |
| 0.3425307570 |
| 0.0735994800 |
| 0.3647903139 |
| 0.4148741220 |
| 0.3269214946 |
| 0.4120838821 |
| 0.7892707301 |

0.8055908186
 0.2474858186
 0.4471063950
 0.7307682984
 0.2588637425
 0.4196016468
 0.4042497340
 0.5615033437
 0.6552810918
 0.3610835873
 0.0742878911
 0.4207864075
 0.5705105725
 0.2144054734
 0.6929433256
 0.6268628900
 0.3008559073
 0.2190280448
 0.3810100125
 0.6440044482
 0.1349098775
 0.3949485449
 0.2233475076
 0.2121650764
 0.0405772931
 0.4261274497
 0.3482432261
 0.4802293330
 0.2986864520

X1 is Rayleigh(lamda=5.000000,c=0.000000),



H0: $X1 \sim \text{Rayleigh}(\lambda, c)$, λ, c are unknown

λ point estimated value = 6.253849 (MLE)

c point estimated value = 0.040577 (MLE)

pearson goodness of fit

| class | [1] | [2] | [3] | [4] |
|-------------|----------|----------|----------|----------|
| lower limit | 0.04058 | 0.17530 | 0.31002 | 0.44475 |
| 0.57947 | 0.71419 | 0.84891 | | |
| upper limit | 0.17530 | 0.31002 | 0.44475 | 0.57947 |
| 0.71419 | 0.84891 | 0.98364 | | |
| observed no | 19.00000 | 30.00000 | 22.00000 | 10.00000 |
| 9.00000 | 6.00000 | 4.00000 | | |
| probability | 0.10730 | 0.25764 | 0.27503 | 0.19737 |
| 0.10409 | 0.04176 | 0.01680 | | |
| expected no | 10.73036 | 25.76359 | 27.50344 | 19.73737 |
| 10.40933 | 4.17580 | 1.68010 | | |
| chi square | 3.59931 | 0.59824 | 1.37672 | 9.48164 |
| 0.22069 | 0.55461 | 1.34548 | | |

Likelihood ratio chi square test statistic = 17.176694

degree of freedom = 4, p-value = 0.016200

correction:

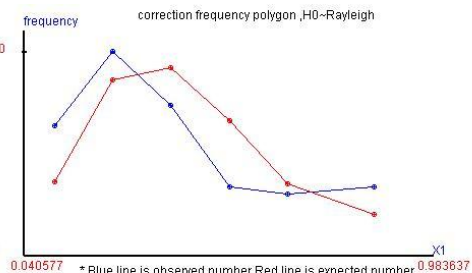
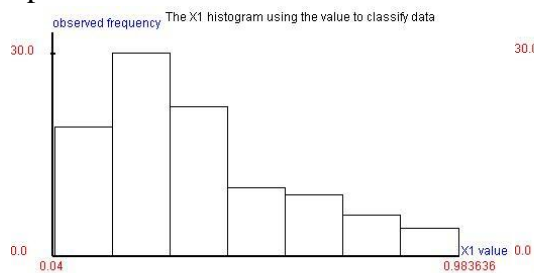
expected number ≥ 5 in each cell, the frequency table is adjusted

| class | [1] | [2] | [3] | [4] |
|-------------|----------|----------|----------|----------|
| lower limit | 0.04058 | 0.17530 | 0.31002 | 0.44475 |
| 0.57947 | 0.71419 | | | |
| upper limit | 0.17530 | 0.31002 | 0.44475 | 0.57947 |
| 0.71419 | 0.98364 | | | |
| observed no | 19.00000 | 30.00000 | 22.00000 | 10.00000 |
| 9.00000 | 10.00000 | | | |
| probability | 0.10730 | 0.25764 | 0.27503 | 0.19737 |
| 0.10409 | 0.05856 | | | |
| expected no | 10.73036 | 25.76359 | 27.50344 | 19.73737 |
| 10.40933 | 5.85591 | | | |
| chi square | 3.59931 | 0.59824 | 1.37672 | 9.48164 |
| 0.22069 | 1.71735 | | | |

degree of freedom = 3

Likelihood ratio chi-square test statistic = 16.993948

p-value = 0.000700



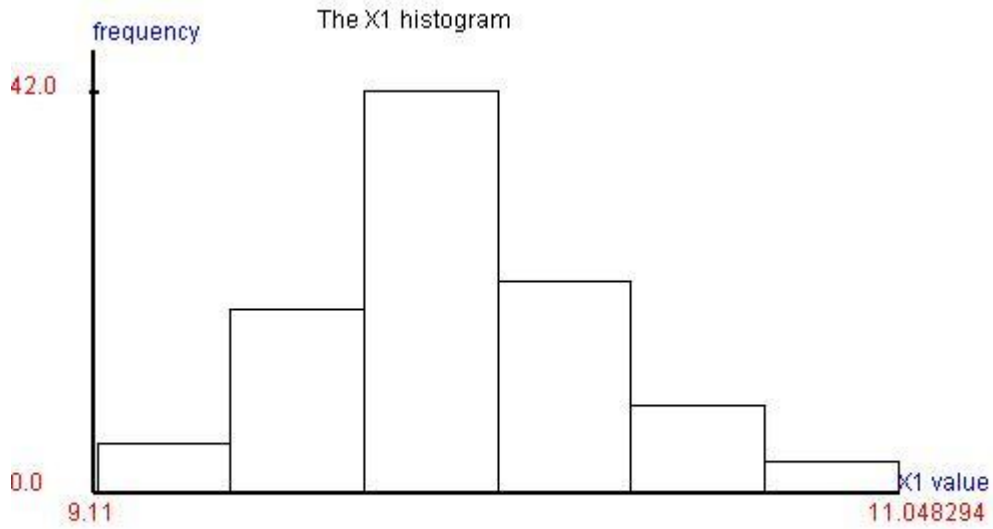
9.4.7)The population distribution is double exponential distribution.

X1 is Double Exponential($\lambda=4.000000, \mu=10.000000$),

| X1 |
|---------------|
| 9.9877928684 |
| 9.7874309256 |
| 9.8815146634 |
| 9.5746131244 |
| 9.9870875990 |
| 9.9266982423 |
| 10.1274277461 |
| 9.9889553060 |
| 9.3772952287 |
| 10.0472671340 |
| 9.9132602306 |
| 9.8594001333 |
| 9.7444071562 |
| 9.8945327580 |
| 10.2487797882 |
| 9.9585265791 |
| 9.9707239225 |
| 10.1451370871 |
| 9.7448313521 |
| 10.0009718404 |
| 10.1293402739 |
| 9.5658173811 |
| 9.8207688041 |
| 10.3824279950 |
| 9.2487713389 |
| 9.9284051712 |
| 9.5063934533 |
| 9.9204627884 |
| 9.7366215464 |
| 10.3912684615 |
| 9.7301643514 |
| 10.4181206487 |
| 10.0720734937 |
| 10.3211068117 |
| 10.6781056586 |
| 9.5378553587 |
| 10.1036842915 |
| 10.1738964184 |
| 9.8148568609 |
| 10.1672916295 |
| 10.0959505801 |
| 9.9989257936 |
| 9.6548483670 |
| 9.6693538791 |
| 9.7264101297 |
| 9.2507254482 |
| 10.0211261145 |
| 10.4109110456 |
| 10.3475846014 |
| 10.4685223666 |
| 9.9548253989 |
| 10.2354523795 |
| 10.0274511065 |
| 9.8080343469 |
| 10.3229208522 |
| 9.1109590872 |
| 9.4138993876 |
| 10.1592655128 |
| 10.1183620536 |
| 9.9913525617 |
| 9.9281612355 |
| 9.9582115810 |
| 10.2477279791 |
| 10.0588858806 |
| 9.8242214207 |
| 10.9395371876 |
| 9.5657437529 |
| 9.6330353240 |
| 9.9767290395 |
| 9.7523998854 |
| 9.7055060223 |

10.4857688252
 11.0482941499
 9.8018604113
 10.1134765295
 9.8553957383
 10.4314653565
 10.3744453023
 10.0416478548
 10.1249152052
 10.0082111632
 10.6338604834
 9.7788204301
 9.4886458039
 9.8785407441
 10.7780538346
 9.7634102927
 9.9523677440
 9.9080081502
 9.8182467211
 9.9472003717
 9.6401903892
 10.0254446005
 10.4680944006
 10.5153284784
 10.1710073771
 9.7512299079
 9.8697697763
 9.6644485678
 10.2564598978

X1 is Double Exponential(lamda=4.000000,mu=10.000000),



H0: $X_1 \sim \text{Double exponential}(\lambda, \mu)$, λ, μ are unknown

λ point estimated value = 3.892332 (MLE)

μ point estimated value = 9.964625 (MLE)

pearson goodness of fit

| class | [1] | [2] | [3] | [4] |
|-------------|----------|----------|----------|----------|
| lower limit | 9.11096 | 9.38772 | 9.66448 | 9.94125 |
| 10.21801 | 10.49477 | 10.77153 | | |
| upper limit | 9.38772 | 9.66448 | 9.94125 | 10.21801 |
| 10.49477 | 10.77153 | 11.04829 | | |
| observed no | 4.00000 | 11.00000 | 30.00000 | 33.00000 |
| 16.00000 | 3.00000 | 3.00000 | | |
| probability | 0.05294 | 0.10252 | 0.30105 | 0.35701 |
| 0.12298 | 0.04188 | 0.02163 | | |
| expected no | 5.29372 | 10.25178 | 30.10531 | 35.70052 |
| 12.29822 | 4.18792 | 2.16252 | | |
| chi square | 0.41843 | 0.05089 | 0.00037 | 0.22099 |
| 0.85645 | 0.47039 | 0.23379 | | |

Likelihood ratio chi square test statistic = 2.251310

degree of freedom = 4, p-value = 0.944600

correction:

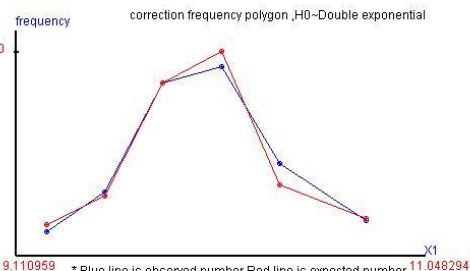
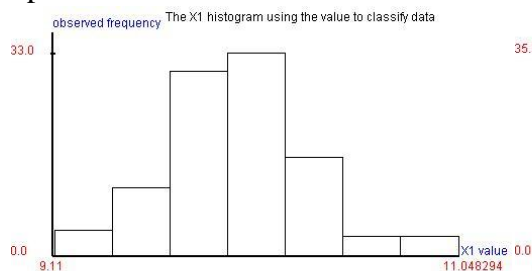
expected number ≥ 5 in each cell, the frequency table is adjusted

| class | [1] | [2] | [3] | [4] |
|-------------|----------|----------|----------|----------|
| lower limit | 9.11096 | 9.38772 | 9.66448 | 9.94125 |
| 10.21801 | 10.49477 | | | |
| upper limit | 9.38772 | 9.66448 | 9.94125 | 10.21801 |
| 10.49477 | 11.04829 | | | |
| observed no | 4.00000 | 11.00000 | 30.00000 | 33.00000 |
| 16.00000 | 6.00000 | | | |
| probability | 0.05294 | 0.10252 | 0.30105 | 0.35701 |
| 0.12298 | 0.06350 | | | |
| expected no | 5.29372 | 10.25178 | 30.10531 | 35.70052 |
| 12.29822 | 6.35044 | | | |
| chi square | 0.41843 | 0.05089 | 0.00037 | 0.22099 |
| 0.85645 | 0.02047 | | | |

degree of freedom = 3

Likelihood ratio chi-square test statistic = 1.567602

p-value = 0.666700



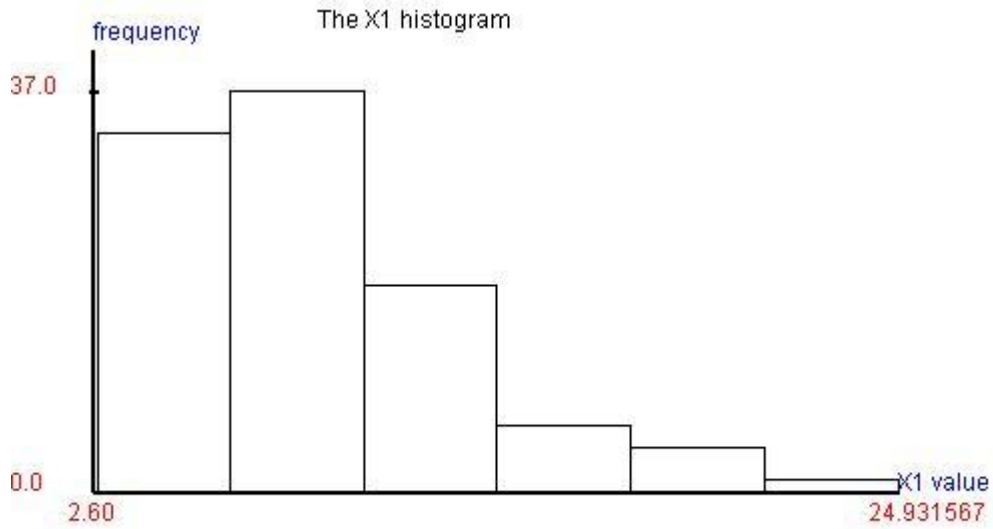
9.4.8)The population distribution is lognormal distribution.

X1 is Log normal($\mu=2.000000$, $\sigma=0.500000$),

| X1 |
|---------------|
| 6.8193992940 |
| 2.6053065190 |
| 11.7993893899 |
| 7.5944743367 |
| 8.1587367940 |
| 7.1248684453 |
| 4.0968583977 |
| 15.9222989294 |
| 24.9315679364 |
| 4.3878542938 |
| 5.5720286824 |
| 4.7232615783 |
| 13.2529740956 |
| 5.5993154727 |
| 7.4099275065 |
| 8.6769914639 |
| 10.3051040364 |
| 7.5198486940 |
| 4.8925998355 |
| 7.9313715996 |
| 13.7424860095 |
| 2.9663808212 |
| 8.5114762548 |
| 8.3948551175 |
| 5.0016701060 |
| 10.4049146476 |
| 5.8943634495 |
| 8.2974384265 |
| 4.0459759289 |
| 7.9446081708 |
| 15.6605946321 |
| 5.5380768379 |
| 6.7197272101 |
| 9.8425491716 |
| 2.8708727902 |
| 8.6208287257 |
| 3.6985501037 |
| 6.7484318681 |
| 19.6500639062 |
| 6.4943325721 |
| 9.1540331666 |
| 3.8868451080 |
| 12.4135572556 |
| 5.4065415690 |
| 12.8982676809 |
| 16.9443733761 |
| 11.0961916376 |
| 9.3365155781 |
| 5.4286395732 |
| 21.1790331992 |
| 14.3696244629 |
| 9.9633757106 |
| 4.5574848841 |
| 8.7464037601 |
| 8.1711026999 |
| 11.7522830471 |
| 10.6946697902 |
| 10.4574118518 |
| 9.7474754425 |
| 9.0430873873 |
| 5.9016095054 |
| 11.7150837941 |
| 7.2570472365 |
| 20.0135500842 |
| 9.2020319483 |
| 8.7862190826 |
| 4.3118202744 |
| 10.3967722393 |
| 4.4070985216 |
| 3.4124804310 |
| 6.8019974951 |

7.6367155819
 8.0740858127
 13.2988824765
 7.2552875543
 2.8020861683
 17.8451201679
 5.4731927290
 5.0435980358
 17.3249521874
 7.6623507948
 15.3410812892
 12.1521433335
 7.7830798161
 8.2232542566
 5.8037668506
 9.6166356163
 12.2248829436
 3.4578402018
 3.9238996285
 8.5352323799
 12.7617763945
 11.2101575926
 5.7155854781
 7.5129077420
 11.9678913452
 4.2356432770
 4.5272147702
 6.3001559665
 5.4834672682

X1 is Log normal(mu=2.000000,sigma=0.500000),



H0: $X_1 \sim \text{Log_Normal}(\mu, \sigma^2)$, μ, σ are unknown
 population mean(μ) point estimated value=2.054501 (MLE,UMVUE)
 population variance(σ^2) which point estimated value=0.236971 (UMVUE)

pearson goodness of fit

| class | [1] | [2] | [3] | [4] | [5] | [6] | [7] |
|-------------|----------|----------|----------|----------|----------|----------|----------|
| lower limit | 2.60531 | 5.79477 | 8.98424 | 12.17370 | 15.36317 | 18.55264 | 21.74210 |
| upper limit | 5.79477 | 8.98424 | 12.17370 | 15.36317 | 18.55264 | 21.74210 | 24.93157 |
| observed no | 29.00000 | 33.00000 | 20.00000 | 9.00000 | 5.00000 | 3.00000 | 1.00000 |
| probability | 0.27060 | 0.34340 | 0.20560 | 0.09840 | 0.04440 | 0.02000 | 0.01760 |
| expected no | 27.06000 | 34.34000 | 20.56000 | 9.84000 | 4.44000 | 2.00000 | 1.76000 |
| chi square | 0.12978 | 0.05441 | 0.01568 | 0.07840 | 0.06272 | 0.33333 | 0.57760 |

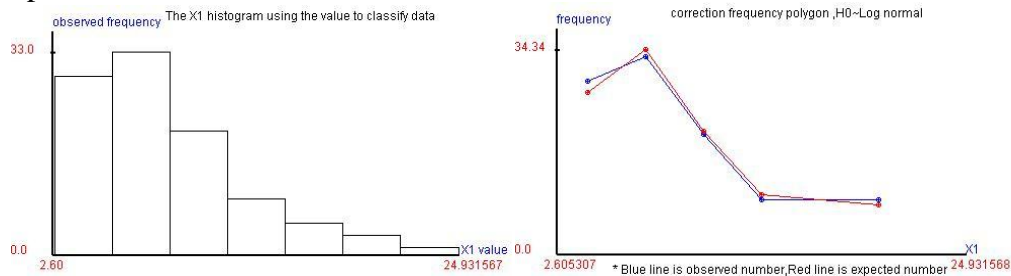
Likelihood ratio chi square test statistic=1.251925
 degree of freedom=4,p-value=0.989600

correction:

expected number ≥ 5 in each cell, the frequency table is adjusted

| class | [1] | [2] | [3] | [4] | [5] |
|-------------|----------|----------|----------|----------|----------|
| lower limit | 2.60531 | 5.79477 | 8.98424 | 12.17370 | 15.36317 |
| upper limit | 5.79477 | 8.98424 | 12.17370 | 15.36317 | 24.93157 |
| observed no | 29.00000 | 33.00000 | 20.00000 | 9.00000 | 9.00000 |
| probability | 0.27060 | 0.34340 | 0.20560 | 0.09840 | 0.08200 |
| expected no | 27.06000 | 34.34000 | 20.56000 | 9.84000 | 8.20000 |
| chi square | 0.12978 | 0.05441 | 0.01568 | 0.07840 | 0.07111 |

degree of freedom=2
 Likelihood ratio chi-square test statistic =0.349383
 p-value=0.839700



9.4.9)The population distribution is gamma distribution.

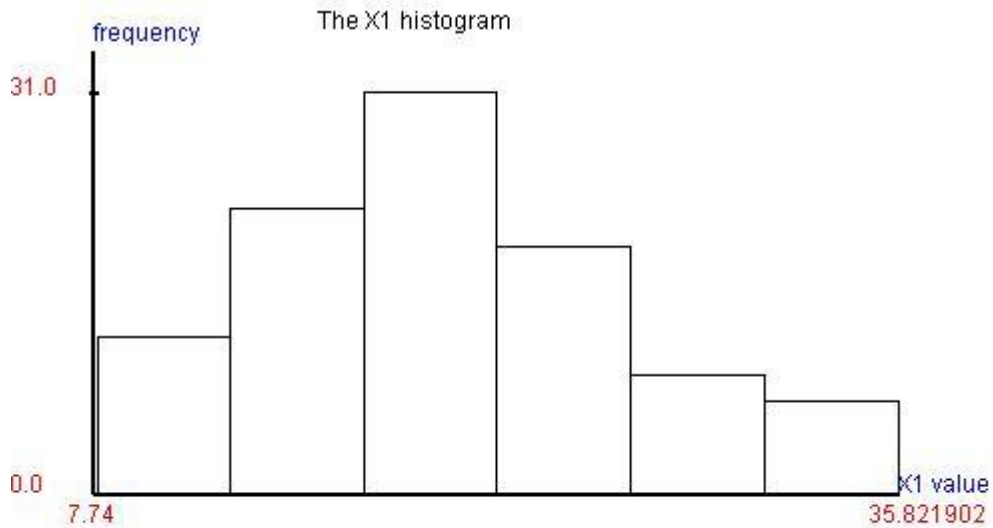
The alpha value is setting to 9 and beta is 2.

X1 is Gamma(alpha=10.000000,beta=2.000000),

| X1 |
|---------------|
| 35.4421906648 |
| 21.2073421658 |
| 22.2770460847 |
| 10.9251027972 |
| 18.9502615190 |
| 14.6017264580 |
| 17.2595251104 |
| 14.4648249069 |
| 18.1571100551 |
| 21.8853096926 |
| 19.6087311569 |
| 19.9477542508 |
| 24.3147620841 |
| 25.8987535678 |
| 26.3764056248 |
| 27.5588073106 |
| 14.5485980418 |
| 23.0663919637 |
| 17.8561609377 |
| 14.4322288214 |
| 20.2492897829 |
| 12.1900056725 |
| 18.0160316694 |
| 21.1389929751 |
| 13.4433592313 |
| 13.2766644400 |
| 14.5622095530 |
| 22.0573134373 |
| 18.4398834425 |
| 21.5456712542 |
| 9.5110249187 |
| 23.3129227411 |
| 22.6791532833 |
| 23.6152641306 |
| 24.4889938041 |
| 22.9867703209 |
| 17.3741635667 |
| 22.6774302833 |
| 31.9366071237 |
| 20.3431884213 |
| 29.4037094615 |
| 28.5218876007 |
| 11.2807762274 |
| 7.7448608569 |
| 19.2705451964 |
| 11.2559567053 |
| 27.4508584674 |
| 15.0895541572 |
| 24.3168429946 |
| 10.4194703024 |
| 21.1701486904 |
| 21.4029173943 |
| 17.5042915071 |
| 22.3442312713 |
| 22.0564205510 |
| 18.7973617849 |
| 16.5908569515 |
| 19.6671920975 |
| 22.2956445481 |
| 29.6349301314 |
| 12.9601714725 |
| 17.6669101188 |
| 12.3595178092 |
| 29.3033431596 |
| 16.0893550412 |
| 21.2563763111 |
| 13.2816221913 |
| 35.8219028531 |
| 33.0032084923 |
| 22.0944543298 |

21.4177548307
 29.4332901367
 22.6502326037
 16.6705741190
 15.7476198354
 15.3527051999
 16.5203552088
 9.0297331079
 19.0813211894
 32.2991427333
 26.9237744531
 18.3148376012
 16.7268373727
 33.5625346879
 15.4842741154
 12.8208213928
 17.6697663208
 13.6204052920
 18.8632182023
 27.2281735964
 10.3047229752
 8.0946742301
 13.1689701869
 20.1202123572
 12.0439731258
 15.9814209656
 33.3754114796
 19.4000444227
 20.7085783011
 17.9394990840

X1 is Gamma(alpha=10.000000,beta=2.000000),



H0: $X_1 \sim \text{Gamma}(\alpha=9.000000, \beta=2.000000)$,

pearson goodness of fit

| class | [1] | [2] | [3] | [4] |
|-------------|----------|----------|----------|----------|
| [5] | [6] | [7] | | |
| lower limit | 7.74486 | 11.75587 | 15.76687 | 19.77788 |
| 23.78888 | 27.79989 | 31.81090 | | |
| upper limit | 11.75587 | 15.76687 | 19.77788 | 23.78888 |
| 27.79989 | 31.81090 | 35.82190 | | |
| observed no | 9.00000 | 19.00000 | 25.00000 | 26.00000 |
| 9.00000 | 5.00000 | 7.00000 | | |
| probability | 0.14050 | 0.25070 | 0.26340 | 0.18340 |
| 0.09690 | 0.04200 | 0.02310 | | |
| expected no | 14.05000 | 25.07000 | 26.34000 | 18.34000 |
| 9.69000 | 4.20000 | 2.31000 | | |
| chi square | 2.83361 | 1.93921 | 0.07182 | 2.25675 |
| 0.05290 | 0.12800 | 3.14230 | | |

Likelihood ratio chi square test statistic=10.424594

degree of freedom=6,p-value=0.165700

correction:

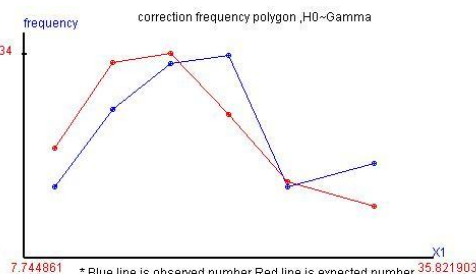
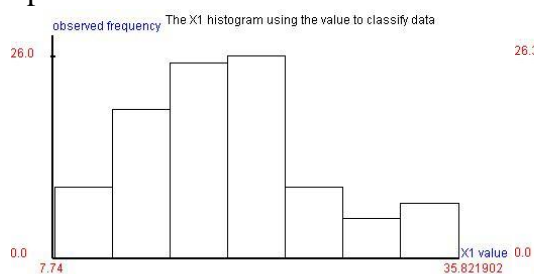
expected number ≥ 5 in each cell, the frequency table is adjusted

| class | [1] | [2] | [3] | [4] |
|-------------|----------|----------|----------|----------|
| [5] | [6] | | | |
| lower limit | 7.74486 | 11.75587 | 15.76687 | 19.77788 |
| 23.78888 | 27.79989 | | | |
| upper limit | 11.75587 | 15.76687 | 19.77788 | 23.78888 |
| 27.79989 | 35.82190 | | | |
| observed no | 9.00000 | 19.00000 | 25.00000 | 26.00000 |
| 9.00000 | 12.00000 | | | |
| probability | 0.14050 | 0.25070 | 0.26340 | 0.18340 |
| 0.09690 | 0.06510 | | | |
| expected no | 14.05000 | 25.07000 | 26.34000 | 18.34000 |
| 9.69000 | 6.51000 | | | |
| chi square | 2.83361 | 1.93921 | 0.07182 | 2.25675 |
| 0.05290 | 2.51167 | | | |

degree of freedom=5

Likelihood ratio chi-square test statistic =9.665969

p-value=0.085200



9.4.10)The population distribution is beta distribution.

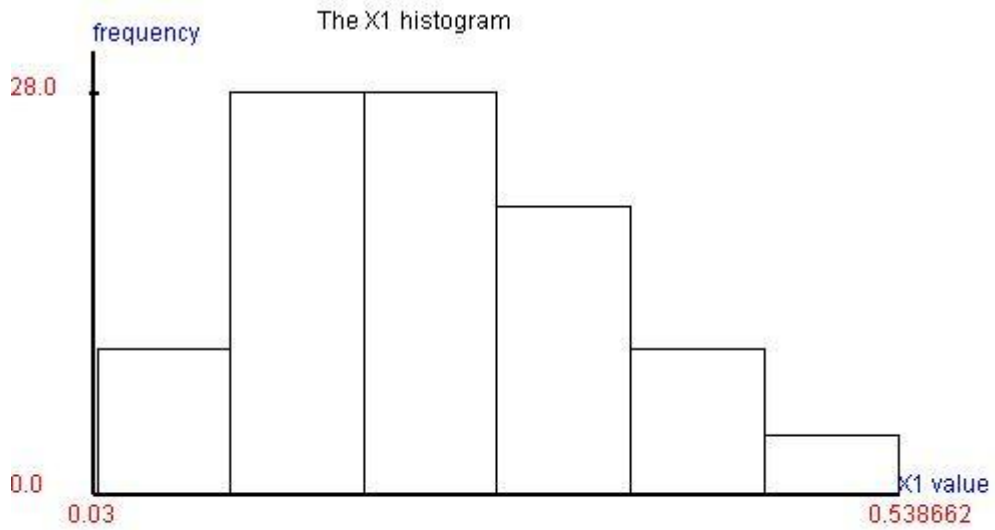
The alpha value is setting to 3 and beta is 13.

X1 is Beta(alpha=4.000000,beta=12.000000),

| X1 |
|--------------|
| 0.1368891879 |
| 0.3721370450 |
| 0.2778241434 |
| 0.3315370704 |
| 0.2683607551 |
| 0.4770926270 |
| 0.5386623820 |
| 0.2773949598 |
| 0.2684251980 |
| 0.1265400482 |
| 0.3202271263 |
| 0.3372209637 |
| 0.1848069077 |
| 0.2630405037 |
| 0.3630950306 |
| 0.2287957497 |
| 0.1622864540 |
| 0.3805534899 |
| 0.2918754065 |
| 0.1311531562 |
| 0.3273531087 |
| 0.1990327257 |
| 0.3002543796 |
| 0.2255557434 |
| 0.0474953931 |
| 0.0647678477 |
| 0.2105781155 |
| 0.3472617019 |
| 0.3351871301 |
| 0.2102535400 |
| 0.2878301957 |
| 0.2452349587 |
| 0.1689348044 |
| 0.1405713404 |
| 0.3609295650 |
| 0.0752924668 |
| 0.1827863120 |
| 0.0895524131 |
| 0.2459988316 |
| 0.1769821105 |
| 0.3777603765 |
| 0.0705988756 |
| 0.1763868572 |
| 0.2814338862 |
| 0.2111477292 |
| 0.1877340214 |
| 0.2245954307 |
| 0.4310136677 |
| 0.2266833545 |
| 0.1313285566 |
| 0.1307822595 |
| 0.5205350919 |
| 0.1987344996 |
| 0.1047398293 |
| 0.1999981382 |
| 0.2123476244 |
| 0.1779533530 |
| 0.4415444260 |
| 0.2153212811 |
| 0.1533436195 |
| 0.3199880509 |
| 0.2045102225 |
| 0.3487435001 |
| 0.2069624178 |
| 0.1318865985 |
| 0.2884337807 |
| 0.3097856592 |
| 0.3946865027 |
| 0.2766183400 |
| 0.2002580643 |

0.3701900808
 0.1492669979
 0.4725252102
 0.2870950666
 0.2952835430
 0.3932417475
 0.2425746655
 0.4151725704
 0.0389123619
 0.2213581623
 0.3646190003
 0.2701321776
 0.4318638408
 0.2957461365
 0.3021020396
 0.1443818868
 0.1289560172
 0.1023846446
 0.3181852436
 0.2423142064
 0.2368854302
 0.2314884947
 0.1903556024
 0.0748681502
 0.3439876676
 0.0813770190
 0.1533191937
 0.4339662674
 0.2005611254
 0.1956260987

X1 is Beta(alpha=4.000000,beta=12.000000),



H0: $X_1 \sim \text{Beta}(\alpha=3.000000, \beta=13.000000)$,

pearson goodness of fit

| class | [1] | [2] | [3] | [4] |
|-------------|----------|----------|----------|----------|
| [5] | [6] | [7] | | |
| lower limit | 0.03891 | 0.11031 | 0.18170 | 0.25309 |
| 0.32448 | 0.39588 | 0.46727 | | |
| upper limit | 0.11031 | 0.18170 | 0.25309 | 0.32448 |
| 0.39588 | 0.46727 | 0.53866 | | |
| observed no | 10.00000 | 17.00000 | 28.00000 | 20.00000 |
| 16.00000 | 5.00000 | 4.00000 | | |
| probability | 0.22499 | 0.30537 | 0.24180 | 0.13758 |
| 0.06110 | 0.02164 | 0.00752 | | |
| expected no | 22.49911 | 30.53724 | 24.17999 | 13.75790 |
| 6.11029 | 2.16389 | 0.75158 | | |
| chi square | 15.62278 | 10.77981 | 0.52116 | 1.94819 |
| 6.11290 | 1.60871 | 2.63806 | | |

Likelihood ratio chi square test statistic=39.231600

degree of freedom=6,p-value=0.000000

correction:

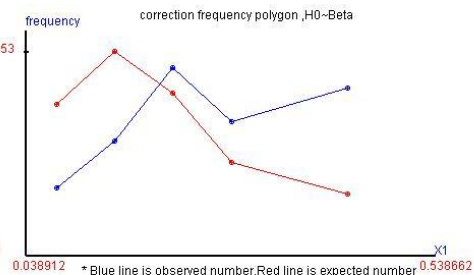
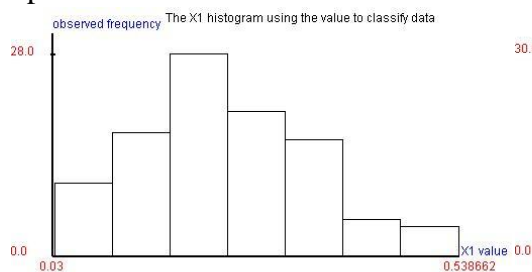
expected number ≥ 5 in each cell, the frequency table is adjusted

| class | [1] | [2] | [3] | [4] |
|-------------|----------|----------|----------|----------|
| [5] | | | | |
| lower limit | 0.03891 | 0.11031 | 0.18170 | 0.25309 |
| 0.32448 | | | | |
| upper limit | 0.11031 | 0.18170 | 0.25309 | 0.32448 |
| 0.53866 | | | | |
| observed no | 10.00000 | 17.00000 | 28.00000 | 20.00000 |
| 25.00000 | | | | |
| probability | 0.22499 | 0.30537 | 0.24180 | 0.13758 |
| 0.09026 | | | | |
| expected no | 22.49911 | 30.53724 | 24.17999 | 13.75790 |
| 9.02576 | | | | |
| chi square | 15.62278 | 10.77981 | 0.52116 | 1.94819 |
| 10.20705 | | | | |

degree of freedom=4

Likelihood ratio chi-square test statistic =39.078995

p-value=0.000000



9.4.11)The population distribution is cauchy distribution.

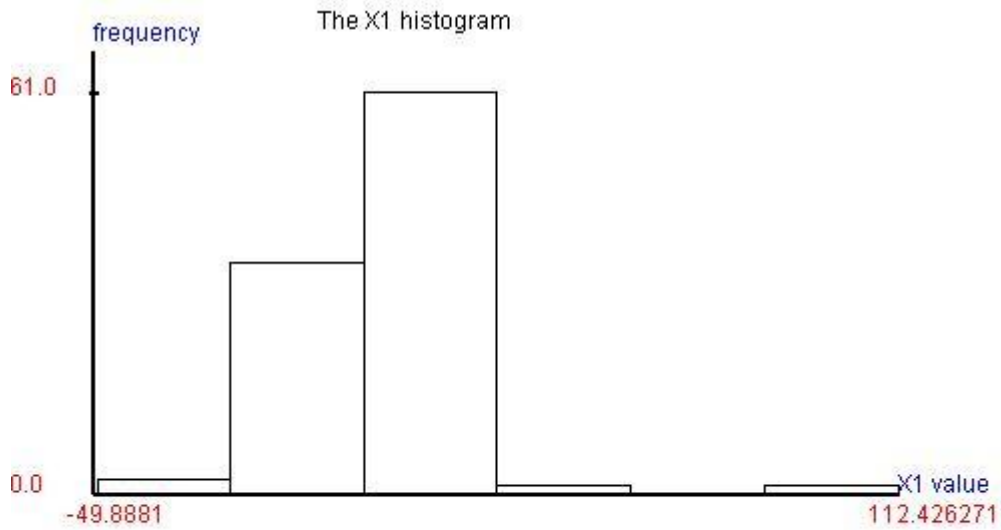
The mu value is setting to 6 and sigma value is 1.8.

X1 is Cauchy(mu=5.000000,sigma=2.000000),

```
X1
7.1527579564
51.9477518381
3.0401987730
2.1582878490
5.8377930191
7.1795423905
4.4695455816
28.1091450516
6.1292298768
3.5204315043
1.5138989327
5.4142182157
4.8209665816
4.2041963330
4.5704011412
5.6591151820
3.9301608712
4.2704050233
6.9722360169
4.7764290935
5.0451210391
5.1958938330
5.5358269954
6.3980766612
6.1894406224
5.8366882069
6.1752553789
6.7518966996
5.4147731683
5.5081683676
1.8603926898
4.8633518826
8.4740684048
0.1167547402
9.1827961624
4.8171388397
3.5827966680
5.1757953133
3.3183033240
4.0857343568
4.0295025336
-1.9432863036
4.9618600329
-49.8881912621
16.6867476846
2.1112979626
-1.2475362546
10.3098057566
0.8978460803
5.9056933046
7.3348742815
7.8098207823
5.9895354747
4.5947816189
13.6313742428
-1.9187083288
3.7315591693
4.4331428765
2.8529226538
4.8468874661
-0.9837554341
3.9249769275
4.9429372145
7.1961799573
7.1883748444
5.6921945810
0.5768160632
3.2204661377
-1.9231132667
6.5172101401
```

-2.5296967206
 4.7914787293
 5.1783196070
 6.6186723785
 4.8778501057
 6.2791544858
 0.4913919414
 -24.9805746226
 1.1439045235
 1.1848951882
 7.7889669906
 3.4828271717
 4.4360958769
 9.3486294815
 4.6976353934
 5.3178514200
 5.4927902139
 2.6506295330
 3.8621925558
 6.7202205260
 3.7608265518
 2.9216316949
 112.4262711298
 27.7119834226
 7.0964877860
 4.1260583222
 5.2585028290
 26.9199957200
 3.4462887854
 5.5904631936

X1 is Cauchy(mu=5.000000,sigma=2.000000),



H0: $X_1 \sim \text{Cauchy}(\mu=6.000000, \sigma=1.800000)$,

pearson goodness of fit

| class | [1] | [2] | [3] | [4] |
|-------------|-----------|-----------|----------|----------|
| lower limit | -49.88819 | -26.70041 | -3.51263 | 19.67515 |
| 42.86293 | 66.05071 | 89.23849 | | |
| upper limit | -26.70041 | -3.51263 | 19.67515 | 42.86293 |
| 66.05071 | 89.23849 | 112.42627 | | |
| observed no | 1.00000 | 1.00000 | 93.00000 | 3.00000 |
| 1.00000 | 0.00000 | 1.00000 | | |
| probability | 0.01750 | 0.04202 | 0.89881 | 0.02613 |
| 0.00599 | 0.00266 | 0.00688 | | |
| expected no | 1.75038 | 4.20237 | 89.88143 | 2.61277 |
| 0.59922 | 0.26561 | 0.68823 | | |
| chi square | 0.56306 | 10.25518 | 0.10458 | 0.04998 |
| 0.16062 | | 0.09720 | | |

Likelihood ratio chi square test statistic=1.#INF00

degree of freedom=6

p-value=0.000000

correction:

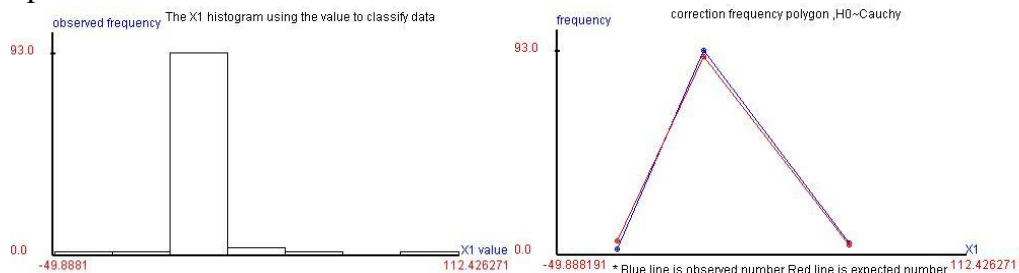
expected number ≥ 5 in each cell, the frequency table is adjusted

| class | [1] | [2] | [3] |
|-------------|-----------|----------|-----------|
| lower limit | -49.88819 | -3.51263 | 19.67515 |
| upper limit | -3.51263 | 19.67515 | 112.42627 |
| observed no | 2.00000 | 93.00000 | 5.00000 |
| probability | 0.05953 | 0.89881 | 0.04166 |
| expected no | 5.95275 | 89.88143 | 4.16583 |
| chi square | 7.81210 | 0.10458 | 0.13917 |

degree of freedom=2

Likelihood ratio chi-square test statistic =8.055847

p-value=0.017800



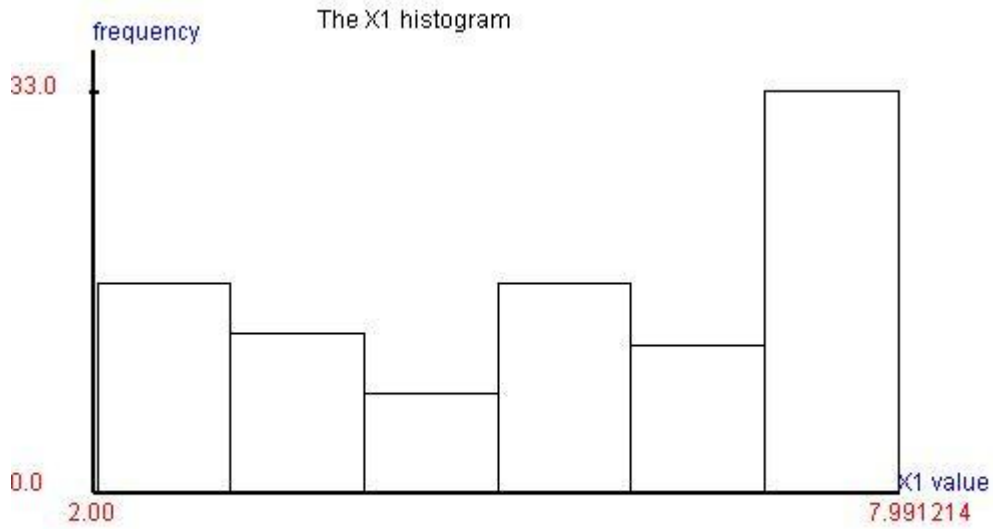
9.4.12) The population distribution is arcsin distribution.

X_1 is $\text{Arcsin}(\mu=5.000000, c=3.000000)$,

| X_1 |
|--------------|
| 4.1993893668 |
| 7.9316262014 |
| 5.1032649139 |
| 7.0417293108 |
| 6.8429856792 |
| 7.7407392234 |
| 5.1685202505 |
| 7.9892712797 |
| 7.5780602474 |
| 2.0567159626 |
| 2.2563998812 |
| 5.4157219526 |
| 3.0419814902 |
| 3.8411702076 |
| 7.9488383091 |
| 6.5404780513 |
| 5.2792564354 |
| 7.0677060461 |
| 5.3425733466 |
| 7.8147501953 |
| 2.3112247778 |
| 7.9260470159 |
| 2.3852520793 |
| 7.6189269731 |
| 7.9524602452 |
| 3.9244301014 |
| 4.8741944516 |
| 2.6140406577 |
| 3.5792646875 |
| 2.7269886908 |
| 4.8871338000 |
| 7.2566760067 |
| 7.5703660391 |
| 3.7886057564 |
| 4.8447546384 |
| 5.6501926078 |
| 5.1960513222 |
| 7.0678370334 |
| 3.0077651032 |
| 2.6304061005 |
| 7.8394495594 |
| 6.5513512608 |
| 3.9734868879 |
| 3.1787057042 |
| 5.9684476415 |
| 3.7326851417 |
| 5.3032443064 |
| 7.9747715386 |
| 4.4348237639 |
| 4.9773624244 |
| 2.0379579793 |
| 5.6601711542 |
| 4.1777365511 |
| 2.0000490152 |
| 5.9611290018 |
| 5.0880984134 |
| 7.9743601524 |
| 7.8081724840 |
| 7.9244761817 |
| 2.1507426428 |
| 3.0635918520 |
| 2.8155389099 |
| 7.0054421604 |
| 6.5227072118 |
| 7.3081239114 |
| 3.6519146263 |
| 6.8953626235 |
| 6.8269625869 |
| 6.5859002565 |
| 7.9402162343 |
| 6.8986855095 |

2.4965468830
 6.6550385000
 5.0609708930
 7.9912144780
 7.9845080211
 7.7081310846
 7.7040710167
 5.2934062550
 3.8035537272
 5.3885026547
 6.6166566465
 7.0913068662
 7.5906840069
 4.6024246533
 3.2715723238
 7.9522193856
 6.9202431328
 7.8902928651
 5.4940109736
 2.4776943351
 2.9163708021
 2.2480033360
 7.0324203560
 2.3209635519
 5.5239430519
 6.7729383276
 7.2413871929
 2.4487802111
 7.5372092871

X1 is Arcsin(mu=5.000000,c=3.000000).



H0: $X_1 \sim \text{Arcsin}(\mu, c)$, μ, c are unknown
 μ point estimated value=4.995632 (MLE)
 c point estimated value=2.995583 (MLE)

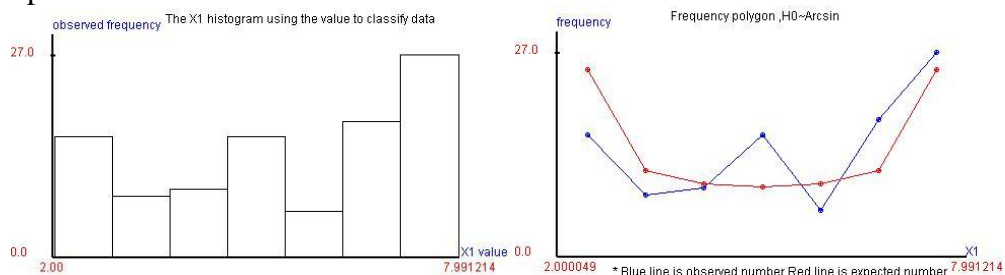
pearson goodness of fit

| class | [1] | [2] | [3] | [4] |
|-------------|----------|----------|---------|----------|
| lower limit | 2.00005 | 2.85593 | 3.71181 | 4.56769 |
| 5.42357 | 6.27945 | 7.13533 | | |
| upper limit | 2.85593 | 3.71181 | 4.56769 | 5.42357 |
| 6.27945 | 7.13533 | 7.99121 | | |
| observed no | 16.00000 | 8.00000 | 9.00000 | 16.00000 |
| 6.00000 | 18.00000 | 27.00000 | | |
| probability | 0.24675 | 0.11227 | 0.09535 | 0.09126 |
| 0.09535 | 0.11227 | 0.24675 | | |
| expected no | 24.67517 | 11.22653 | 9.53540 | 9.12579 |
| 9.53540 | 11.22653 | 24.67517 | | |
| chi square | 4.70366 | 1.30131 | 0.03185 | 2.95342 |
| 2.08318 | 2.54888 | 0.20018 | | |

degree of freedom=4

Likelihood ratio chi-square test statistic =13.822487

p-value=0.007800



9.4.13)The population distribution is gumbel distribution.

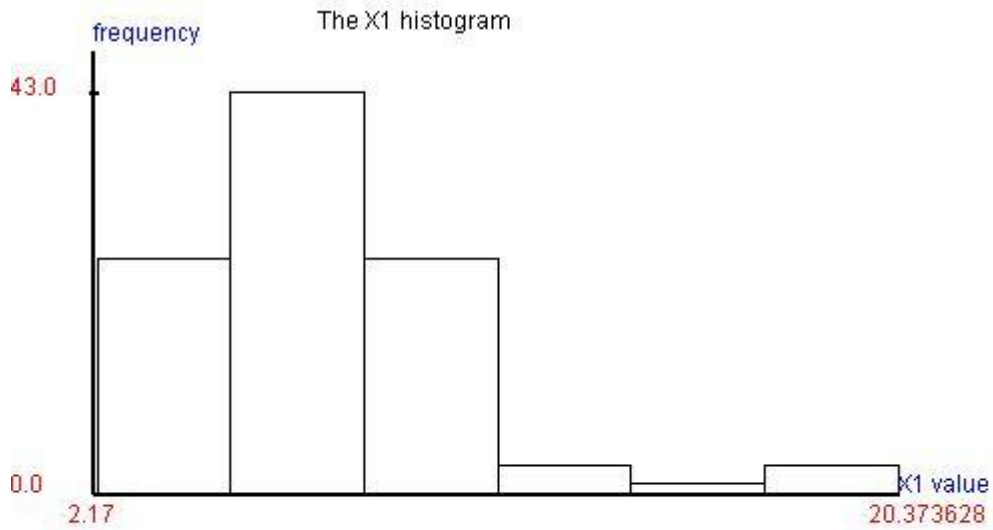
The mu value is setting to 6 and sigma value is 1.8.

X1 is Gumbel(mu=6.000000,sigma=2.000000),

| X1 |
|---------------|
| 5.4303968401 |
| 9.0670650275 |
| 8.6757364624 |
| 6.4023797338 |
| 4.2980412486 |
| 7.8250758910 |
| 4.3154618006 |
| 7.9955881539 |
| 5.8950714948 |
| 6.5271514675 |
| 7.2933205566 |
| 18.0779554457 |
| 8.2322936170 |
| 6.1330395400 |
| 5.6550516146 |
| 10.0464940065 |
| 5.1378365044 |
| 7.5994667461 |
| 5.9546404344 |
| 8.3314505025 |
| 4.1184690051 |
| 9.4460396616 |
| 5.0818138863 |
| 4.6792742522 |
| 5.9920398112 |
| 4.8830595028 |
| 6.7048091297 |
| 8.7306395911 |
| 4.6689721663 |
| 4.4178592673 |
| 7.2699136392 |
| 10.5891541344 |
| 5.5520896247 |
| 8.2659176112 |
| 7.9270892310 |
| 7.2938363818 |
| 11.4406945409 |
| 7.9903990477 |
| 7.1199781881 |
| 6.5249652101 |
| 7.0884570019 |
| 10.0028598916 |
| 15.4922450691 |
| 4.4602331659 |
| 7.6241383843 |
| 7.0323193262 |
| 7.9991626508 |
| 4.7603215757 |
| 2.1755608709 |
| 5.4126087712 |
| 19.9886757924 |
| 9.8133271118 |
| 8.3483594773 |
| 5.5165114386 |
| 4.3352841794 |
| 6.5360863931 |
| 8.7674454749 |
| 8.1495960481 |
| 10.9473717733 |
| 3.9653391831 |
| 4.3780480775 |
| 5.1329873342 |
| 7.4579231245 |
| 4.7279028378 |
| 6.9529604470 |
| 5.9046766607 |
| 8.2652844380 |
| 8.8609183880 |
| 4.9716694322 |
| 7.0689440480 |

10.3465034306
 6.0288683655
 6.8982291976
 8.4075004411
 5.4775995593
 4.0082543115
 4.1446535857
 6.7697851570
 6.8656359247
 9.1887431024
 8.3052081948
 20.3736282763
 6.3492289020
 3.2954146503
 7.8354167373
 6.0693071309
 4.9048052807
 9.1855428179
 13.1219177135
 11.0929564950
 5.0689199245
 14.0800520932
 8.6441881930
 8.4091129511
 8.2525288177
 6.6367461463
 4.0113177775
 4.5160664809
 9.6274293618
 7.4887923732

X1 is Gumbel($\mu=6.000000$, $\sigma=2.000000$),



H0: $X_1 \sim \text{Gumbel}(\mu=6.000000, \sigma=1.800000)$,

pearson goodness of fit

| class | [1] | [2] | [3] | [4] |
|-------------|----------|----------|----------|----------|
| [5] | [6] | [7] | | |
| lower limit | 2.17556 | 4.77528 | 7.37501 | 9.97473 |
| 12.57446 | 15.17418 | 17.77390 | | |
| upper limit | 4.77528 | 7.37501 | 9.97473 | 12.57446 |
| 15.17418 | 17.77390 | 20.37363 | | |
| observed no | 18.00000 | 38.00000 | 31.00000 | 7.00000 |
| 2.00000 | 1.00000 | 3.00000 | | |
| probability | 0.13881 | 0.48879 | 0.26832 | 0.07848 |
| 0.01950 | 0.00466 | 0.00144 | | |
| expected no | 13.88082 | 48.87938 | 26.83217 | 7.84826 |
| 1.94958 | 0.46559 | 0.14419 | | |
| chi square | 0.94265 | 3.11476 | 0.56035 | 0.10279 |
| 0.00127 | 0.28559 | 2.71855 | | |

Likelihood ratio chi square test statistic=7.725962

degree of freedom=6,p-value=0.357300

correction:

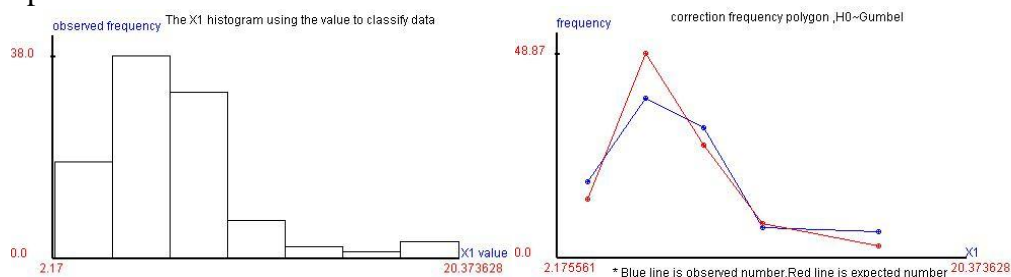
expected number ≥ 5 in each cell, the frequency table is adjusted

| class | [1] | [2] | [3] | [4] |
|-------------|----------|----------|----------|----------|
| [5] | | | | |
| lower limit | 2.17556 | 4.77528 | 7.37501 | 9.97473 |
| 12.57446 | | | | |
| upper limit | 4.77528 | 7.37501 | 9.97473 | 12.57446 |
| 20.37363 | | | | |
| observed no | 18.00000 | 38.00000 | 31.00000 | 7.00000 |
| 6.00000 | | | | |
| probability | 0.13881 | 0.48879 | 0.26832 | 0.07848 |
| 0.02559 | | | | |
| expected no | 13.88082 | 48.87938 | 26.83217 | 7.84826 |
| 2.55936 | | | | |
| chi square | 0.94265 | 3.11476 | 0.56035 | 0.10279 |
| 1.97300 | | | | |

degree of freedom=4

Likelihood ratio chi-square test statistic =6.693546

p-value=0.152900



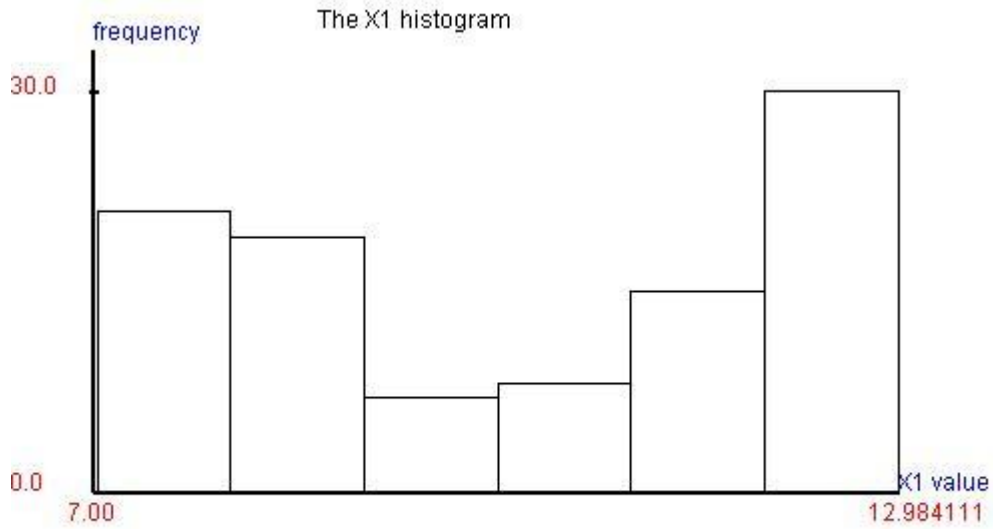
9.4.14)The population distribution is triangular 1 distribution.

X1 is Triangular1($\mu=10.000000,c=3.000000$),

| X1 |
|---------------|
| 8.6375799808 |
| 12.6822343668 |
| 9.0364902069 |
| 8.6587103492 |
| 8.0426949603 |
| 7.1144190271 |
| 9.0947254770 |
| 7.3801314446 |
| 11.4935673975 |
| 12.7052845217 |
| 12.8834822566 |
| 7.3639389700 |
| 12.4217283771 |
| 8.9508471029 |
| 12.4432204119 |
| 8.8169548636 |
| 12.4065598516 |
| 8.7208253463 |
| 12.1831500302 |
| 12.6632401934 |
| 7.3762745926 |
| 7.1018611788 |
| 11.8103473091 |
| 11.3686975362 |
| 9.5195845517 |
| 7.6429869313 |
| 12.0231651830 |
| 12.6373044503 |
| 8.0519938097 |
| 7.9838414104 |
| 12.6725294787 |
| 12.9841119623 |
| 8.7138603510 |
| 11.6802550026 |
| 8.0669731416 |
| 8.5762249346 |
| 12.4034575433 |
| 12.3085149739 |
| 7.0042068396 |
| 11.5741420666 |
| 12.2784589393 |
| 12.8553096640 |
| 11.3402290693 |
| 12.9050723874 |
| 11.7571561777 |
| 10.8530954551 |
| 8.0162551932 |
| 8.7581602556 |
| 8.4878466551 |
| 7.3041143011 |
| 11.4178810722 |
| 11.3614843255 |
| 8.6807129790 |
| 12.6166905924 |
| 12.7062325347 |
| 7.5589211193 |
| 11.1442217027 |
| 7.3088782914 |
| 7.8264162614 |
| 12.2742186247 |
| 8.1364228743 |
| 12.1025651315 |
| 7.1788368142 |
| 12.4509862251 |
| 11.2698774995 |
| 7.2733952605 |
| 9.1425104569 |
| 11.7489359992 |
| 10.6986463078 |
| 12.1107288707 |
| 7.1509773481 |

10.9829001949
 10.8649912190
 8.9466898226
 9.1042860054
 7.5778994748
 7.6237801001
 8.1219635481
 9.0999552623
 12.3688559611
 11.2555383094
 12.6661725751
 12.0240771856
 7.3972289243
 10.4439971640
 12.1834866715
 7.8155490683
 10.9835812976
 7.1622378497
 11.8687044542
 7.7721369075
 8.4569677045
 12.8648009795
 10.8081303724
 12.5378614818
 8.1675227176
 10.9741285830
 11.8597961869
 9.2198717926
 12.7028897175

X1 is Triangular1(mu=10.000000,c=3.000000),



H0: $X_1 \sim \text{Triangular 1}(\mu, c)$, μ, c are unknown

μ point estimated value=9.994159 (MLE)

c point estimated value=2.989953 (MLE)

pearson goodness of fit

| class | [1] | [2] | [3] | [4] |
|-------------|----------|----------|----------|----------|
| lower limit | 7.00421 | 7.85848 | 8.71275 | 9.56702 |
| 10.42130 | 11.27557 | 12.12984 | | |
| upper limit | 7.85848 | 8.71275 | 9.56702 | 10.42130 |
| 11.27557 | 12.12984 | 12.98411 | | |
| observed no | 20.00000 | 14.00000 | 13.00000 | 0.00000 |
| 11.00000 | 16.00000 | 26.00000 | | |
| probability | 0.24490 | 0.16327 | 0.08163 | 0.02041 |
| 0.08163 | 0.16327 | 0.24490 | | |
| expected no | 24.48980 | 16.32653 | 8.16327 | 2.04082 |
| 8.16327 | 16.32653 | 24.48980 | | |
| chi square | 1.00791 | 0.38662 | 1.79954 | |
| 0.73155 | 0.00666 | 0.08772 | | |

Likelihood ratio chi square test statistic=1.#INF00

degree of freedom=4,p-value=0.000000

correction:

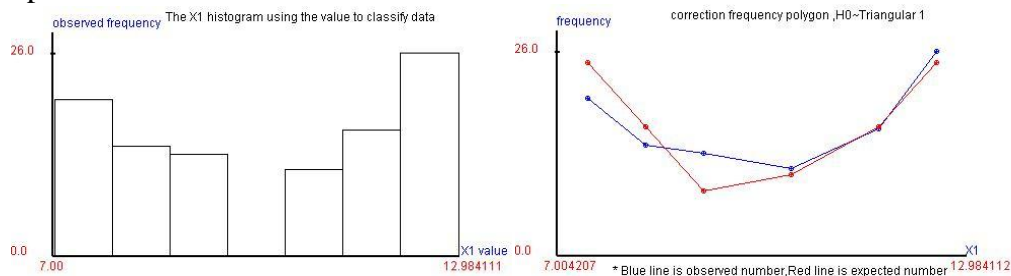
expected number ≥ 5 in each cell, the frequency table is adjusted

| class | [1] | [2] | [3] | [4] |
|-------------|----------|----------|----------|----------|
| lower limit | 7.00421 | 7.85848 | 8.71275 | 9.56702 |
| 11.27557 | 12.12984 | | | |
| upper limit | 7.85848 | 8.71275 | 9.56702 | 11.27557 |
| 12.12984 | 12.98411 | | | |
| observed no | 20.00000 | 14.00000 | 13.00000 | 11.00000 |
| 16.00000 | 26.00000 | | | |
| probability | 0.24490 | 0.16327 | 0.08163 | 0.10204 |
| 0.16327 | 0.24490 | | | |
| expected no | 24.48980 | 16.32653 | 8.16327 | 10.20408 |
| 16.32653 | 24.48980 | | | |
| chi square | 1.00791 | 0.38662 | 1.79954 | 0.05759 |
| 0.00666 | 0.08772 | | | |

degree of freedom=3

Likelihood ratio chi-square test statistic =3.346050

p-value=0.341200



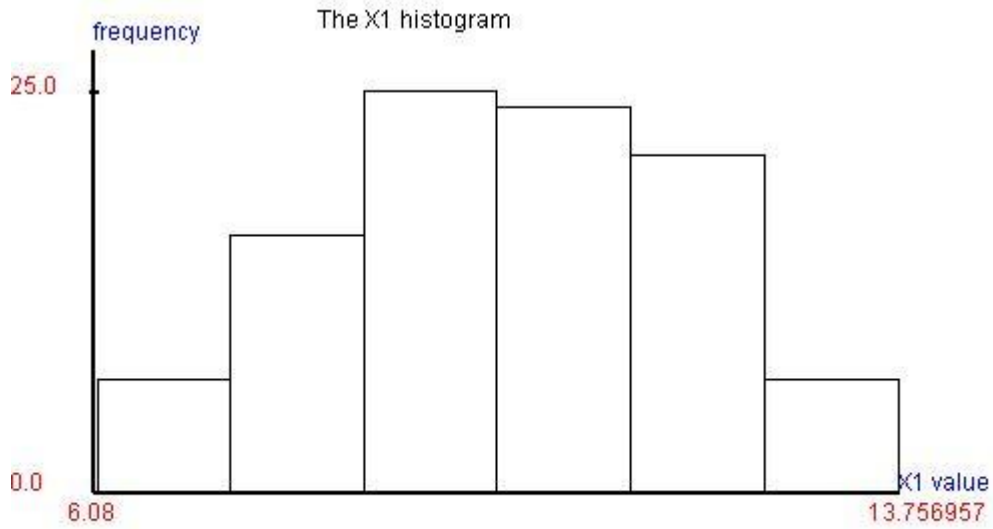
9.4.15) The population distribution is trapezoid distribution.

X_1 is Trapezoid($\mu=10.000000, c=3.000000$),

| X_1 |
|---------------|
| 6.0883314089 |
| 11.1506676711 |
| 9.4316785001 |
| 9.8210977732 |
| 7.3793084475 |
| 9.5728784889 |
| 7.2448754838 |
| 8.1855820333 |
| 6.7613696959 |
| 11.6137445010 |
| 10.8695298002 |
| 9.7975191860 |
| 11.6360439420 |
| 8.7210744755 |
| 10.5289618489 |
| 11.0073289595 |
| 12.0072331446 |
| 9.4017166481 |
| 8.4067966436 |
| 11.9660677316 |
| 10.3138018530 |
| 13.1389069505 |
| 13.3264731222 |
| 11.2280662286 |
| 9.3227787831 |
| 12.6143141927 |
| 10.3985071659 |
| 7.6184781023 |
| 8.7739422427 |
| 8.1154304091 |
| 11.5258785463 |
| 8.8845766166 |
| 8.8956257904 |
| 10.9947346607 |
| 11.5149949341 |
| 9.6995486626 |
| 8.4162058232 |
| 8.0473130579 |
| 6.8137857462 |
| 8.7891102110 |
| 9.3739807353 |
| 11.7736332974 |
| 8.5061438951 |
| 13.7265477610 |
| 10.6603663323 |
| 7.4501010934 |
| 7.2311111488 |
| 12.4479700692 |
| 8.3311491378 |
| 11.2841159045 |
| 9.9459431734 |
| 10.8836899676 |
| 8.6799327288 |
| 11.7848443950 |
| 12.4533341575 |
| 9.0602132809 |
| 12.0460655661 |
| 11.8984207658 |
| 9.9813170420 |
| 11.3515770646 |
| 9.1663694503 |
| 9.0728205557 |
| 12.4301685161 |
| 10.7673648337 |
| 8.9563249403 |
| 6.8698545996 |
| 10.0842807128 |
| 9.9230421815 |
| 12.1379074597 |
| 11.4743038486 |
| 10.2415217712 |

13.7431423703
 9.9546480094
 10.6258343151
 10.8000259853
 11.9148860046
 8.3503972123
 10.0153942125
 13.7569572842
 10.4990269101
 9.6159000481
 9.8107415065
 9.4992559410
 11.9856078641
 7.6116931868
 8.5924126289
 10.7235484708
 7.7869464831
 8.1852389968
 7.3925107170
 10.1257586671
 9.9766470199
 12.7226214554
 9.7068677345
 11.5770348264
 10.3866055451
 9.3607507085
 6.5812713024
 8.9483757511
 8.7690272305

X1 is Trapezoid($\mu=10.000000, c=3.000000$),



H0: $X_1 \sim \text{Trapezoid}(\mu, c)$, μ, c are unknown

μ point estimated value=9.922644 (MLE)

c point estimated value=2.556209 (MLE)

pearson goodness of fit

| class | [1] | [2] | [3] | [4] |
|-------------|----------|----------|----------|----------|
| lower limit | 6.08833 | 7.18385 | 8.27937 | 9.37489 |
| 10.47040 | 11.56592 | 12.66144 | | |
| upper limit | 7.18385 | 8.27937 | 9.37489 | 10.47040 |
| 11.56592 | 12.66144 | 13.75696 | | |
| observed no | 5.00000 | 12.00000 | 21.00000 | 22.00000 |
| 18.00000 | 16.00000 | 6.00000 | | |
| probability | 0.04592 | 0.13776 | 0.20918 | 0.21429 |
| 0.20918 | 0.13776 | 0.04592 | | |
| expected no | 4.59184 | 13.77551 | 20.91837 | 21.42857 |
| 20.91837 | 13.77551 | 4.59184 | | |
| chi square | 0.03332 | 0.26270 | 0.00032 | 0.01484 |
| 0.47316 | 0.30927 | 0.33049 | | |

Likelihood ratio chi square test statistic=1.424101

degree of freedom=4, p-value=0.984800

correction:

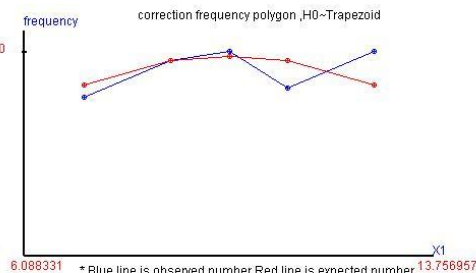
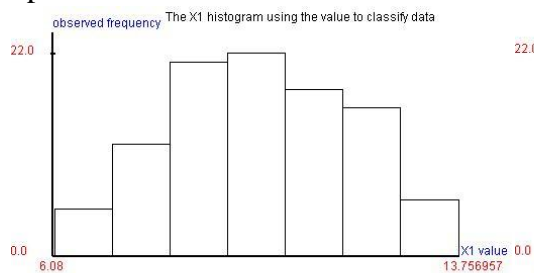
expected number ≥ 5 in each cell, the frequency table is adjusted

| class | [1] | [2] | [3] | [4] |
|-------------|----------|----------|----------|----------|
| lower limit | 6.08833 | 8.27937 | 9.37489 | 10.47040 |
| 11.56592 | | | | |
| upper limit | 8.27937 | 9.37489 | 10.47040 | 11.56592 |
| 13.75696 | | | | |
| observed no | 17.00000 | 21.00000 | 22.00000 | 18.00000 |
| 22.00000 | | | | |
| probability | 0.18367 | 0.20918 | 0.21429 | 0.20918 |
| 0.18367 | | | | |
| expected no | 18.36735 | 20.91837 | 21.42857 | 20.91837 |
| 18.36735 | | | | |
| chi square | 0.10998 | 0.00032 | 0.01484 | 0.47316 |
| 0.59983 | | | | |

degree of freedom=2

Likelihood ratio chi-square test statistic =1.198123

p-value=0.549300



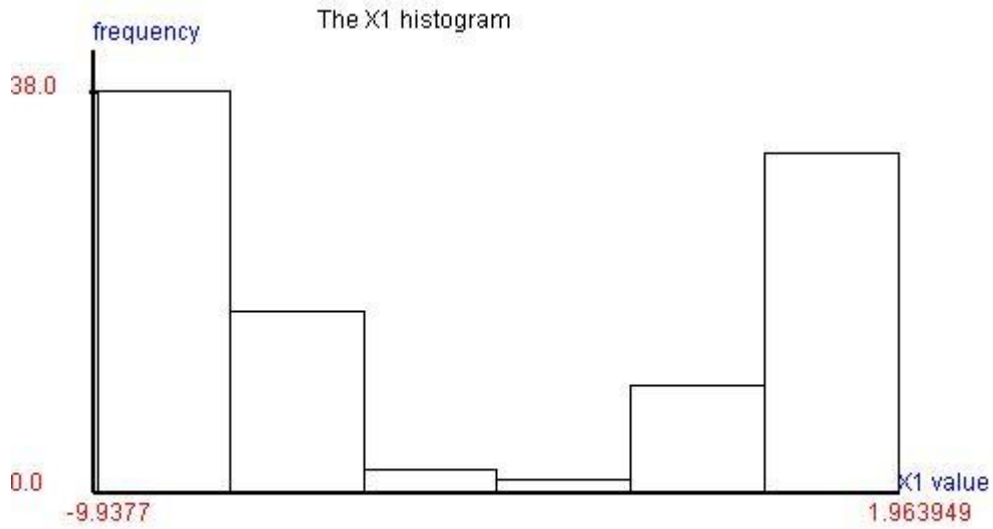
9.4.16)The population distribution is U quadratic distribution.

X1 is U-quadratic(a=-10.000000,b=2.000000),

| X1 |
|---------------|
| -9.5777311949 |
| 0.9067555874 |
| -9.5154715952 |
| -9.9288050056 |
| -8.7590594010 |
| 0.5878117932 |
| 1.0719797715 |
| 0.6947258407 |
| -9.6749596153 |
| 0.7071821191 |
| -9.7174159076 |
| 1.9639493327 |
| -9.0678246124 |
| -8.6893689278 |
| -9.9377901918 |
| -8.8791701008 |
| -7.1156425999 |
| -1.8064991012 |
| -6.7319001351 |
| -5.8665013071 |
| -8.1044218141 |
| -0.5634790804 |
| -9.9022765921 |
| 1.0720919961 |
| -9.0530245329 |
| -8.9636780522 |
| -9.7966430959 |
| -6.8651343718 |
| 0.7708875659 |
| 0.6727382594 |
| -7.7901963212 |
| 0.8879341880 |
| 1.7608245830 |
| -0.3941690874 |
| -8.8323464699 |
| -2.1640071059 |
| -8.1579585821 |
| -9.6007911285 |
| -7.1176044613 |
| 1.3334832843 |
| -7.2082510769 |
| -5.2794964301 |
| -8.8741775672 |
| -9.6100750899 |
| -7.2214499484 |
| 1.1737838460 |
| -8.3976441584 |
| -0.0826238759 |
| 0.8741077802 |
| 1.1740417653 |
| 1.0722173500 |
| -9.5791709849 |
| 1.0541194143 |
| -7.7920248231 |
| -8.2123014035 |
| -8.3381166310 |
| 1.7138407518 |
| -8.3197081280 |
| -7.1983444817 |
| 0.3335136979 |
| 1.0372874121 |
| 1.6118277790 |
| 0.8938784485 |
| -7.3886432287 |
| -7.6600983948 |
| -8.4527270651 |
| -0.5803648943 |
| 1.9491214563 |
| -7.7074830466 |
| 1.0152198191 |
| -9.0496903283 |

-8.4967323624
 -0.6593661263
 -7.9727403760
 0.7830172780
 -6.9542500015
 -9.5525155155
 -7.9259769002
 -0.6790053478
 -8.9170070456
 -8.9636048823
 1.7562880560
 1.5666242403
 1.9610082620
 -1.5160180293
 -6.6661551166
 -0.2655414044
 -9.5645023908
 -9.8119457131
 1.7411725263
 -6.0727731095
 -0.5194926184
 1.6028775799
 -8.8680925813
 1.3002668550
 0.7236759025
 -9.6325563864
 -9.4022225924
 -8.4299888718
 -7.9361821984

X1 is U-quadratic(a=-10.000000,b=2.000000),



H0: $X_1 \sim U_quadratic(a,b)$, a,b are unknown

a point estimated value=-9.937790 (MLE)

b point estimated value=1.963949 (MLE)

pearson goodness of fit

| class | [1] | [2] | [3] | [4] | [5] |
|-------------|----------|----------|----------|----------|----------|
| lower limit | -9.93779 | -8.23754 | -6.53729 | -4.83704 | -3.13680 |
| upper limit | -1.43655 | 0.26370 | | | |
| observed no | 34.00000 | 20.00000 | 3.00000 | 0.00000 | |
| 3.00000 | 8.00000 | 32.00000 | | | |
| probability | 0.31780 | 0.14290 | 0.03790 | 0.00290 | |
| 0.03790 | 0.14290 | 0.31770 | | | |
| expected no | 31.78000 | 14.29000 | 3.79000 | 0.29000 | |
| 3.79000 | 14.29000 | 31.77000 | | | |
| chi square | 0.14495 | 1.63021 | 0.20803 | | |
| 0.20803 | 4.94551 | 0.00165 | | | |

Likelihood ratio chi square test statistic=1.#INF00

degree of freedom=4,p-value=0.000000

correction:

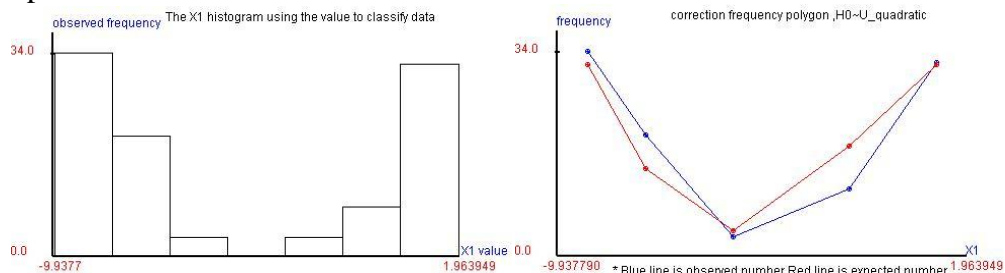
expected number ≥ 5 in each cell, the frequency table is adjusted

| class | [1] | [2] | [3] | [4] | [5] |
|-------------|----------|----------|----------|----------|---------|
| lower limit | -9.93779 | -8.23754 | -6.53729 | -3.13680 | 0.26370 |
| upper limit | -8.23754 | -6.53729 | -3.13680 | 0.26370 | 1.96395 |
| observed no | 34.00000 | 20.00000 | 3.00000 | 11.00000 | |
| 32.00000 | | | | | |
| probability | 0.31780 | 0.14290 | 0.04080 | 0.18080 | |
| 0.31770 | | | | | |
| expected no | 31.78000 | 14.29000 | 4.08000 | 18.08000 | |
| 31.77000 | | | | | |
| chi square | 0.14495 | 1.63021 | 0.38880 | 4.55695 | |
| 0.00165 | | | | | |

degree of freedom=2

Likelihood ratio chi-square test statistic =6.722557

p-value=0.034600



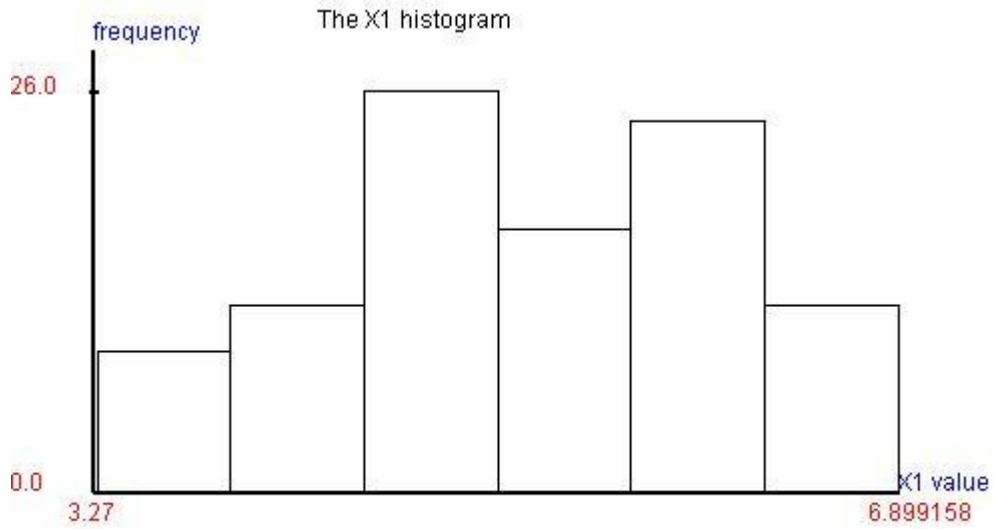
9.4.17) The population distribution is semi circle distribution.

X1 is Semi-circle($\mu=5.000000, R=2.000000$),

| X1 |
|--------------|
| 6.3528366932 |
| 4.5864481187 |
| 5.2389891954 |
| 6.2777630972 |
| 4.6387667365 |
| 4.4331501991 |
| 4.7986294350 |
| 5.7340343807 |
| 5.5183432146 |
| 4.8936975583 |
| 4.4534506858 |
| 4.6280486256 |
| 5.0003157274 |
| 3.3392756418 |
| 5.3989019100 |
| 6.4416208471 |
| 3.3984266076 |
| 6.5202355290 |
| 6.4184617634 |
| 6.2978638509 |
| 6.0701435185 |
| 4.9012684428 |
| 5.1151655419 |
| 4.5517877717 |
| 4.8688092003 |
| 6.0349015511 |
| 4.1962521976 |
| 5.0470226898 |
| 3.9602319491 |
| 3.9646611958 |
| 5.8084003335 |
| 3.2777303146 |
| 6.2940040728 |
| 5.7315640811 |
| 5.6767632650 |
| 4.9876726765 |
| 4.6775816262 |
| 6.4469502191 |
| 5.0182900312 |
| 4.2230174260 |
| 4.9118748115 |
| 6.0191538442 |
| 5.1886694867 |
| 4.9282213272 |
| 4.6974678971 |
| 4.5936880176 |
| 5.4200211780 |
| 4.1069215866 |
| 4.2126406219 |
| 6.6780257351 |
| 6.0853714516 |
| 5.8273138579 |
| 4.6185729386 |
| 5.4447261167 |
| 6.0062458919 |
| 3.3039683210 |
| 5.6949835400 |
| 5.4893620467 |
| 4.2506246923 |
| 5.7116693206 |
| 4.2912813191 |
| 4.8519660634 |
| 6.2382089020 |
| 6.8065629897 |
| 5.8360041590 |
| 5.0458726511 |
| 5.8858218511 |
| 4.2906259010 |
| 4.8581832195 |
| 3.4113197358 |
| 6.6068955575 |

5.5897874871
 5.9816398185
 3.4066559696
 3.8527471177
 5.7581293589
 3.5433233948
 5.2931069909
 6.8991584424
 5.3530126786
 6.0601723370
 3.9734653701
 5.6482524897
 4.5963664747
 4.5191667880
 6.2121791635
 5.2734587564
 5.4077023789
 6.1495242617
 6.2754809628
 6.3012786897
 4.6377189022
 5.0871202104
 6.2017803275
 5.7460121034
 5.4598922854
 3.6437647752
 6.5901386794
 5.5664237491
 5.0248589728

X1 is Semi-circle($\mu=5.000000, R=2.000000$),



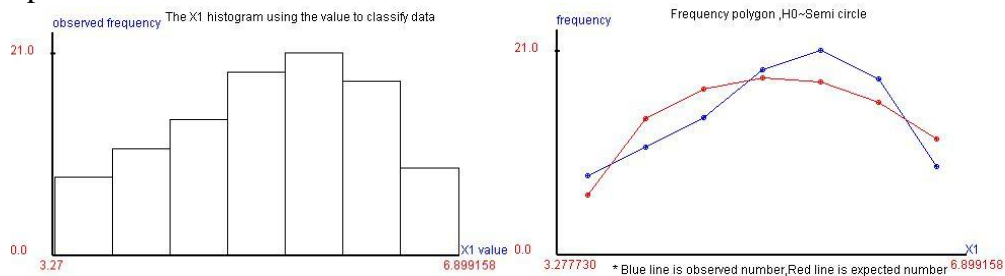
H0: $X_1 \sim \text{Semi-circle}(\mu, R)$, μ, R are unknown
 μ point estimated value=5.205861 (MLE)
 R point estimated value=1.810714 (MLE)

pearson goodness of fit

| class | [1] | [2] | [3] | [4] |
|-------------|----------|----------|----------|----------|
| lower limit | 3.27773 | 3.79508 | 4.31242 | 4.82977 |
| 5.34712 | 5.86446 | 6.38181 | | |
| upper limit | 3.79508 | 4.31242 | 4.82977 | 5.34712 |
| 5.86446 | 6.38181 | 6.89916 | | |
| observed no | 8.00000 | 11.00000 | 14.00000 | 19.00000 |
| 21.00000 | 18.00000 | 9.00000 | | |
| probability | 0.06020 | 0.13900 | 0.16960 | 0.18080 |
| 0.17680 | 0.15580 | 0.11780 | | |
| expected no | 6.02000 | 13.90000 | 16.96000 | 18.08000 |
| 17.68000 | 15.58000 | 11.78000 | | |
| chi square | 0.49005 | 0.76455 | 0.62583 | 0.04455 |
| 0.52488 | 0.32536 | 0.85871 | | |

degree of freedom=4

Likelihood ratio chi-square test statistic =3.633914
p-value=0.457800



9.4.18)The population distribution is logistic distribution.

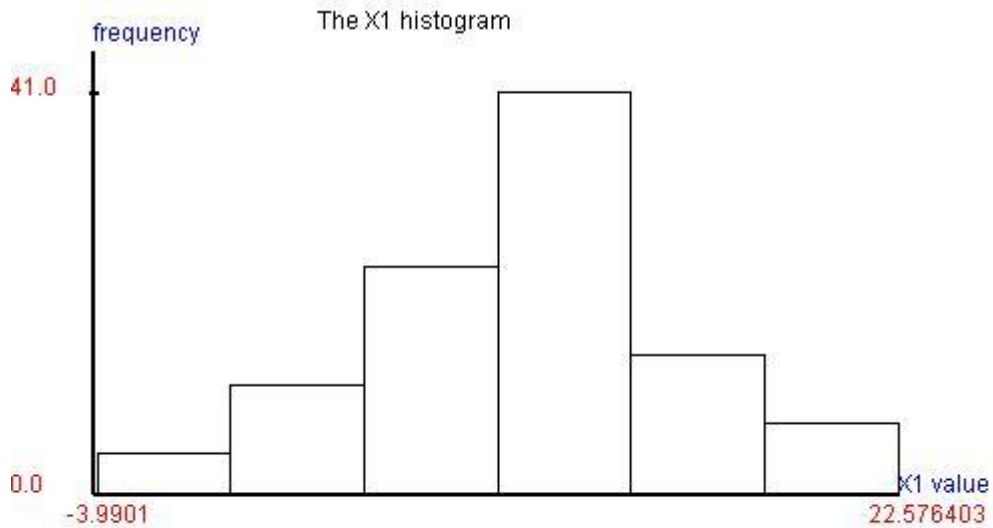
The mu value is setting to 10.5 and sigma value is 3.2.

X1 is Logistic(mu=10.000000,sigma=3.000000),

| X1 |
|---------------|
| 7.5031168679 |
| 12.7140516439 |
| 8.0792426936 |
| 17.3069793276 |
| 10.3628497030 |
| 3.4165369494 |
| 13.6477279207 |
| 11.0876065833 |
| 4.8338725424 |
| 12.6400903769 |
| 12.3246668889 |
| 15.1533606517 |
| 8.8415348295 |
| 10.6005571156 |
| 5.7239033980 |
| 5.4926207692 |
| 6.7237529627 |
| 9.0325719834 |
| 11.3568443125 |
| 5.0136134972 |
| 11.4231319243 |
| -0.9438318613 |
| 13.3421906503 |
| 4.3908483785 |
| 13.3208726984 |
| 6.8663983215 |
| 8.3370019949 |
| 2.5057254261 |
| 12.8701008759 |
| 11.2224012433 |
| 12.1755293222 |
| 10.4074088239 |
| -1.2670291452 |
| 11.4984832720 |
| 1.8738892469 |
| 12.7098421043 |
| 7.6368083575 |
| 11.8248741299 |
| 15.7341941160 |
| 12.2326512883 |
| 7.1834630724 |
| 10.1506194939 |
| 7.7774778445 |
| 12.5133267784 |
| 11.6646936649 |
| 9.5550845017 |
| 10.8977955901 |
| 12.4075586145 |
| 6.0792714835 |
| 5.3156273086 |
| 16.3241586164 |
| 14.3413098341 |
| 6.2315065057 |
| 16.3299876475 |
| 18.9385900722 |
| 4.8348404084 |
| 11.2281051423 |
| 12.6716118932 |
| 6.9658011982 |
| 11.0680857555 |
| 10.1376420828 |
| 19.0375640050 |
| 13.0819221282 |
| 12.8550780353 |
| 18.7056844350 |
| 12.6775665893 |
| 7.7752289281 |
| 20.2413200344 |
| 9.7295716375 |
| 9.5738344353 |

15.2285008422
 13.5409069385
 3.5014931490
 15.8721533080
 18.0533805509
 16.6495821665
 6.7157285715
 12.0940734636
 -0.1805908839
 21.3201556000
 16.1256739609
 11.8235062214
 11.3493973646
 22.2624126581
 13.2228392535
 15.2803512212
 9.4858186973
 -3.9901959159
 12.6845733196
 14.5934679553
 3.4633870326
 2.4923907220
 7.2248028463
 22.5764035431
 8.3537573502
 3.0901658420
 6.0825333692
 5.5713264286
 0.6031055328
 17.9058503825

X1 is Logistic(mu=10.000000,sigma=3.000000),



H0: $X1 \sim \text{Logistic}(\mu=10.500000, \sigma=3.200000)$,

pearson goodness of fit

| class | [1] | [2] | [3] | [4] |
|-------------|----------|----------|----------|----------|
| [5] | [6] | [7] | | |
| lower limit | -3.99020 | -0.19497 | 3.60026 | 7.39549 |
| 11.19072 | 14.98595 | 18.78118 | | |
| upper limit | -0.19497 | 3.60026 | 7.39549 | 11.19072 |
| 14.98595 | 18.78118 | 22.57640 | | |
| observed no | 3.00000 | 9.00000 | 17.00000 | 21.00000 |
| 31.00000 | 13.00000 | 6.00000 | | |
| probability | 0.03415 | 0.06960 | 0.17109 | 0.27891 |
| 0.24872 | 0.12760 | 0.06992 | | |
| expected no | 3.41523 | 6.96039 | 17.10925 | 27.89051 |
| 24.87248 | 12.75980 | 6.99234 | | |
| chi square | 0.05747 | 0.46222 | 0.00070 | 2.26091 |
| 1.21118 | 0.00444 | 0.16412 | | |

Likelihood ratio chi square test statistic=4.161042

degree of freedom=6,p-value=0.761000

correction:

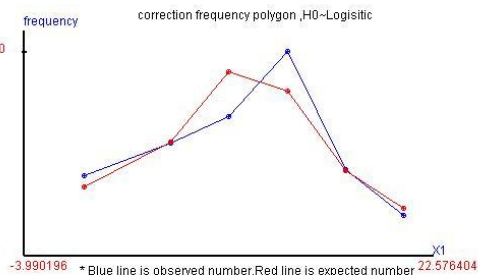
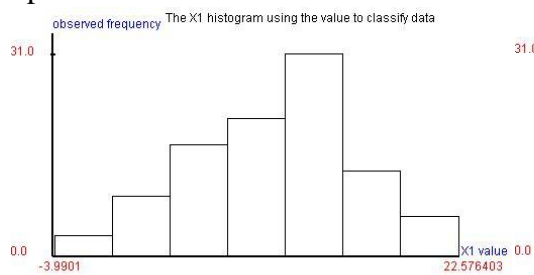
expected number ≥ 5 in each cell, the frequency table is adjusted

| class | [1] | [2] | [3] | [4] |
|-------------|----------|----------|----------|----------|
| [5] | [6] | | | |
| lower limit | -3.99020 | 3.60026 | 7.39549 | 11.19072 |
| 14.98595 | 18.78118 | | | |
| upper limit | 3.60026 | 7.39549 | 11.19072 | 14.98595 |
| 18.78118 | 22.57640 | | | |
| observed no | 12.00000 | 17.00000 | 21.00000 | 31.00000 |
| 13.00000 | 6.00000 | | | |
| probability | 0.10376 | 0.17109 | 0.27891 | 0.24872 |
| 0.12760 | 0.06992 | | | |
| expected no | 10.37562 | 17.10925 | 27.89051 | 24.87248 |
| 12.75980 | 6.99234 | | | |
| chi square | 0.21988 | 0.00070 | 2.26091 | 1.21118 |
| 0.00444 | 0.16412 | | | |

degree of freedom=5

Likelihood ratio chi-square test statistic =3.861234

p-value=0.569500



9.4.19) The population distribution is Weibull distribution.

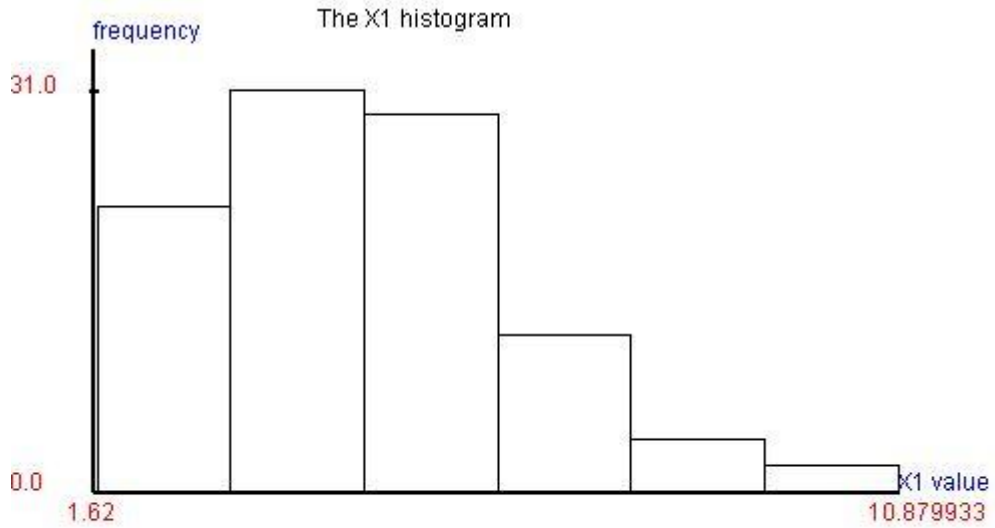
The alpha value is supposed to be 1 and the beta value is 3.8 and gamma value is 2.

X1 is Weibull(alpha=1.000000,beta=4.000000,gamma=2.000000),

| X1 |
|---------------|
| 4.7555584608 |
| 6.7539452859 |
| 4.2333052024 |
| 2.4862689156 |
| 3.1675588583 |
| 5.2231901478 |
| 5.6367851034 |
| 4.6152261790 |
| 5.8274053710 |
| 3.3908052637 |
| 2.1954996590 |
| 6.1853785148 |
| 4.8769693049 |
| 4.0283218600 |
| 3.6584938594 |
| 2.4309454502 |
| 7.3603610441 |
| 3.7921899361 |
| 6.0856544882 |
| 2.5151657448 |
| 4.2000726944 |
| 7.1773923071 |
| 4.2955810175 |
| 5.4114382748 |
| 4.4687536969 |
| 7.2683133558 |
| 4.9649547546 |
| 4.3762135554 |
| 4.6712663262 |
| 1.8317013737 |
| 5.2837902804 |
| 4.6866283492 |
| 7.0879188263 |
| 2.3375755664 |
| 4.2168418046 |
| 5.0509968567 |
| 2.5342539466 |
| 3.9427711710 |
| 2.0736097476 |
| 4.1232267792 |
| 4.5682857886 |
| 9.7124824098 |
| 4.6556396946 |
| 6.8669958835 |
| 3.3975463348 |
| 4.8888840726 |
| 5.9244196264 |
| 7.4900242507 |
| 4.6112742827 |
| 5.9719882755 |
| 5.9110297850 |
| 4.9339316004 |
| 4.1139510163 |
| 4.6854371425 |
| 3.3804731678 |
| 5.8943232317 |
| 8.3224731331 |
| 3.2377740967 |
| 1.8165149212 |
| 5.0320040566 |
| 4.9551606303 |
| 10.8799338918 |
| 6.4080474795 |
| 3.7723946603 |
| 2.5243518227 |
| 2.6107937277 |
| 5.3952507453 |
| 4.3552087490 |
| 6.5323007716 |
| 5.4437420978 |

5.5549831647
 2.2384995620
 2.5125568969
 4.1041729478
 7.0926993221
 7.7719196328
 4.6808811090
 1.6280155188
 5.8208846313
 4.8047115498
 2.3000755373
 6.8851421721
 8.0582070736
 2.9024294514
 8.7033652673
 2.5280423704
 4.9390323440
 3.3857775184
 4.9813259707
 2.9778283975
 5.5127664952
 4.0384409700
 3.8526250878
 4.2488472302
 2.8172323689
 5.3970013399
 2.5351381733
 5.1092800188
 8.5796522108
 2.1950884769

X1 is Weibull(alpha=1.000000,beta=4.000000,gamma=2.000000),



H0: $X_1 \sim \text{Weibull}(\alpha=1.000000, \beta=3.800000, \gamma=2.000000)$,

pearson goodness of fit

| class | [1] | [2] | [3] | [4] |
|-------------|----------|----------|----------|----------|
| [5] | [6] | [7] | | |
| lower limit | 1.62802 | 2.94972 | 4.27142 | 5.59312 |
| 6.91483 | 8.23653 | 9.55823 | | |
| upper limit | 2.94972 | 4.27142 | 5.59312 | 6.91483 |
| 8.23653 | 9.55823 | 10.87993 | | |
| observed no | 20.00000 | 21.00000 | 32.00000 | 14.00000 |
| 8.00000 | 3.00000 | 2.00000 | | |
| probability | 0.23145 | 0.29198 | 0.24456 | 0.14333 |
| 0.06207 | 0.02034 | 0.00627 | | |
| expected no | 23.14540 | 29.19800 | 24.45609 | 14.33317 |
| 6.20655 | 2.03396 | 0.62683 | | |
| chi square | 0.49468 | 3.20034 | 1.77846 | 0.00793 |
| 0.40206 | 0.31108 | 0.94280 | | |

Likelihood ratio chi square test statistic=7.137343

degree of freedom=7,p-value=0.414700

correction:

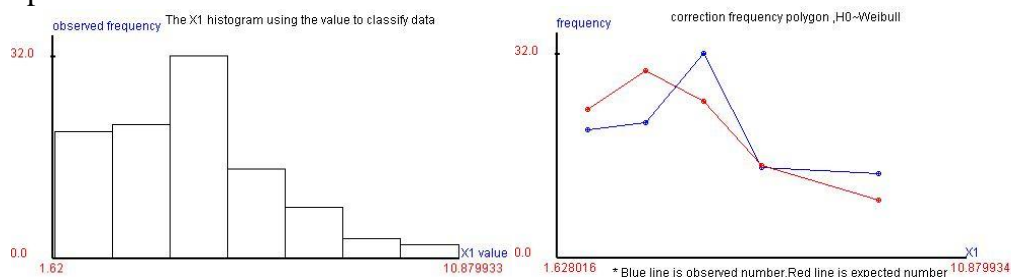
expected number ≥ 5 in each cell, the frequency table is adjusted

| class | [1] | [2] | [3] | [4] |
|-------------|----------|----------|----------|----------|
| [5] | | | | |
| lower limit | 1.62802 | 2.94972 | 4.27142 | 5.59312 |
| 6.91483 | | | | |
| upper limit | 2.94972 | 4.27142 | 5.59312 | 6.91483 |
| 10.87993 | | | | |
| observed no | 20.00000 | 21.00000 | 32.00000 | 14.00000 |
| 13.00000 | | | | |
| probability | 0.23145 | 0.29198 | 0.24456 | 0.14333 |
| 0.08867 | | | | |
| expected no | 23.14540 | 29.19800 | 24.45609 | 14.33317 |
| 8.86734 | | | | |
| chi square | 0.49468 | 3.20034 | 1.77846 | 0.00793 |
| 1.31376 | | | | |

degree of freedom=5

Likelihood ratio chi-square test statistic =6.795169

p-value=0.236300



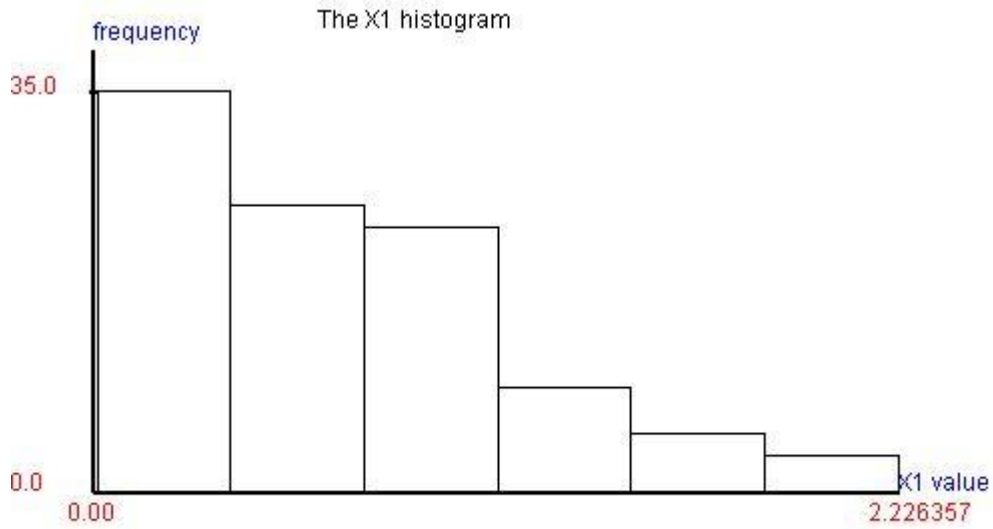
9.4.20)The population distribution is pareto 3 distribution.

X1 is Pareto3(lamda=5.000000,c=4.000000),

| X1 |
|--------------|
| 1.5983418516 |
| 0.2100529889 |
| 0.0843554830 |
| 1.0998783957 |
| 1.0571027027 |
| 2.0164323515 |
| 1.6512135671 |
| 0.7824749298 |
| 0.3987885329 |
| 0.3446071707 |
| 0.4269488196 |
| 0.9333337712 |
| 1.2507872630 |
| 0.0277081823 |
| 0.5429672552 |
| 0.2329862303 |
| 0.5247162181 |
| 0.5742099887 |
| 0.2415577922 |
| 0.8759587963 |
| 0.6840262124 |
| 0.5917243297 |
| 0.6295935244 |
| 0.7768511416 |
| 0.0331388620 |
| 0.4123012155 |
| 0.1550902858 |
| 0.0273252877 |
| 0.6038226631 |
| 0.0888068843 |
| 0.0536572933 |
| 0.8981690956 |
| 0.3678883329 |
| 0.5056181718 |
| 0.2154752562 |
| 0.6244401062 |
| 0.1378728992 |
| 0.0473075400 |
| 1.4476275093 |
| 1.0466392735 |
| 0.2855528305 |
| 1.8089346743 |
| 1.5339378559 |
| 0.6970002278 |
| 0.9162170380 |
| 0.4002729552 |
| 1.0628104278 |
| 0.5935122123 |
| 1.4140899586 |
| 0.6346750631 |
| 1.4688532845 |
| 0.9941381173 |
| 0.2719851652 |
| 0.8306120791 |
| 0.1371996548 |
| 2.2263578599 |
| 0.3227058392 |
| 0.6934984934 |
| 0.0386702750 |
| 1.0371501377 |
| 0.5252932337 |
| 1.1418098470 |
| 0.0011918661 |
| 0.1163604056 |
| 1.7487634233 |
| 0.7228241749 |
| 0.1805080748 |
| 0.5052672488 |
| 0.4706802892 |
| 2.1971661507 |
| 0.9450348298 |

0.0572778306
1.3190320576
0.4022869111
0.0871338840
0.7665564739
0.7351613058
0.5617307140
0.1046233965
0.8550525984
0.0587659001
1.1676401049
0.2008007540
1.174222576
0.8749244866
1.1721989147
0.0788575750
0.8292236703
0.1648937207
0.8314687011
0.8366000779
0.2053971362
0.4695273226
0.0985832036
0.7798925022
0.2829724505
0.3690098275
0.7938131412
0.0984265092
0.7834361761

X1 is Pareto3(lamda=5.000000,c=4.000000),



H0: $X_1 \sim \text{Pareto } 3(\lambda, c)$, λ, c are unknown

λ point estimated value=2.311893 (MLE)

c point estimated value=2.226358 (MLE)

pearson goodness of fit

| class | [1] | [2] | [3] | [4] |
|-------------|----------|----------|----------|----------|
| lower limit | 0.00119 | 0.31907 | 0.63695 | 0.95483 |
| 1.27272 | 1.59060 | 1.90848 | | |
| upper limit | 0.31907 | 0.63695 | 0.95483 | 1.27272 |
| 1.59060 | 1.90848 | 2.22636 | | |
| observed no | 31.00000 | 24.00000 | 22.00000 | 11.00000 |
| 5.00000 | 4.00000 | 3.00000 | | |
| probability | 0.30066 | 0.24053 | 0.18491 | 0.13305 |
| 0.08568 | 0.04405 | 0.01111 | | |
| expected no | 30.06598 | 24.05333 | 18.49117 | 13.30508 |
| 8.56829 | 4.40521 | 1.11093 | | |
| chi square | 0.02814 | 0.00012 | 0.55963 | 0.48304 |
| 2.54654 | 0.04105 | 1.18953 | | |

Likelihood ratio chi square test statistic=4.848050

degree of freedom=4, p-value=0.678500

correction:

expected number ≥ 5 in each cell, the frequency table is adjusted

| class | [1] | [2] | [3] | [4] |
|-------------|----------|----------|----------|----------|
| lower limit | 0.00119 | 0.31907 | 0.63695 | 0.95483 |
| 1.27272 | 1.59060 | | | |
| upper limit | 0.31907 | 0.63695 | 0.95483 | 1.27272 |
| 1.59060 | 2.22636 | | | |
| observed no | 31.00000 | 24.00000 | 22.00000 | 11.00000 |
| 5.00000 | 7.00000 | | | |
| probability | 0.30066 | 0.24053 | 0.18491 | 0.13305 |
| 0.08568 | 0.05516 | | | |
| expected no | 30.06598 | 24.05333 | 18.49117 | 13.30508 |
| 8.56829 | 5.51614 | | | |
| chi square | 0.02814 | 0.00012 | 0.55963 | 0.48304 |
| 2.54654 | 0.31455 | | | |

degree of freedom=3

Likelihood ratio chi-square test statistic =3.932020

p-value=0.268800

