

## Chapter six The goodness of fit test

8)MLR test 1:

8.1)The multi-nominal distribution is the sampling distribution, each category has a probability and the summation of probability is 1.

|             |       |       |       |       |
|-------------|-------|-------|-------|-------|
| category    | 1     | 2     | ..... | k     |
| probability | $p_1$ | $p_2$ | ..... | $p_k$ |

$$p_1 + \dots + p_k = 1,$$

$$f(x_1, \dots, x_k) = \frac{n!}{x_1! \times \dots \times x_k!} \times p_1^{x_1} \times \dots \times p_k^{x_k}, x_1 + \dots + x_k = n \quad \text{that is trial number.}$$

8.2)

$$X_i = np_i + \varepsilon_i, E(\varepsilon_i) = 0, E[(\varepsilon_i)^2] = np_i(1 - p_i), i = 1, 2, \dots, k$$

$$X_i - np_i = \varepsilon_i, E\left[\frac{(\varepsilon_i)^2}{np_i}\right] = E\left[\frac{(X_i - np_i)^2}{np_i}\right] = 1 - p_i,$$

$$\sum_{i=1}^k E\left[\frac{(X_i - np_i)^2}{np_i}\right] = E\left[\sum_{i=1}^k \left(\frac{(X_i - np_i)^2}{np_i}\right)\right] = k - \sum_{i=1}^k p_i = k - 1,$$

$$\sum_{i=1}^k \left(\frac{(X_i - np_i)^2}{np_i}\right) \rightarrow \chi_{k-1}^2$$

$$H_0 : p_1 = p_{01}, p_2 = p_{02}, \dots, p_k = p_{0k} \quad H_1 : \text{against } H_0$$

$p_{01}, p_{02}, \dots, p_{0k}$  are known value and  $p_{01} + p_{02} + \dots + p_{0k} = 1,$

$$E_i = np_{i0}, i = 1, 2, \dots, k, O_i = x_i,$$

$O_i$  : The observed sample number of cell i,

$E_i$  : The expected sample number of cell i,

$$\chi_v^2 = \sum_{i=1}^k \frac{(O_i - E_i)^2}{O_i}, \quad v = k - 1 - (\text{number of point estimator}) \text{ is degree of freedom of}$$

pearsson chi square test.

$$\chi_v^2 > \chi_{\alpha, v}^2 \Rightarrow \text{reject } H_0.$$

8.3)The process of test

$H_0$  : Population distribution is a continuous probability distribution,

$H_1$  : against  $H_0$

The sample size is n.

The process: i)The class number of frequency distribution:  $k = \log_2(n) + 1.$

ii)The class limit of frequency distribution: Let each class probability are equally and the class limit will be gotten.

iii)The class sample number of frequency distribution( $O_i$ ):

The frequency distribution is done and getting the  $O_i$  of each class.

iv) The class expected number of frequency distribution( $E_i$ ):

$$E_i = n \times \text{the probability of each class.}$$

Note: There are 20 kinds of continuous probability distribution that is can be assigned to null hypothesis.

#### 8.4) Example (The simulated sample data and computing the result by the P\_S\_CCC)

The Likelihood ratio chi square test (goodness of fit), the traditional frequency distribution, please select the population distribution

|                                       |                                    |
|---------------------------------------|------------------------------------|
| 1.H0:Uniform distribution             | 13.H0:Gumbel distribution          |
| 2.H0:Normal distribution              | 14.H0:Triangular 1 distribution    |
| 3.H0:Shifted exponential distribution | 15.H0:Trapezoid distribution       |
| 4.H0:Pareto 1 distribution            | 16.H0:U-quadratic distribution     |
| 5.H0:Pareto 2 distribution            | 17.H0:Semi-circle distribution     |
| 6.H0:Rayleigh distribution            | 18.H0:Logistic distribution        |
| 7.H0:Double exponential distribution  | 19.H0:Weibull distribution         |
| 8.H0:Log normal distribution          | 20.H0:Pareto 3 distribution        |
| 9.H0:Gamma distribution               | ** Above Ho population all do once |
| 10.H0:Beta distribution               |                                    |
| 11.H0:Cauchy distribution             |                                    |
| 12.H0:Arcsin distribution             |                                    |

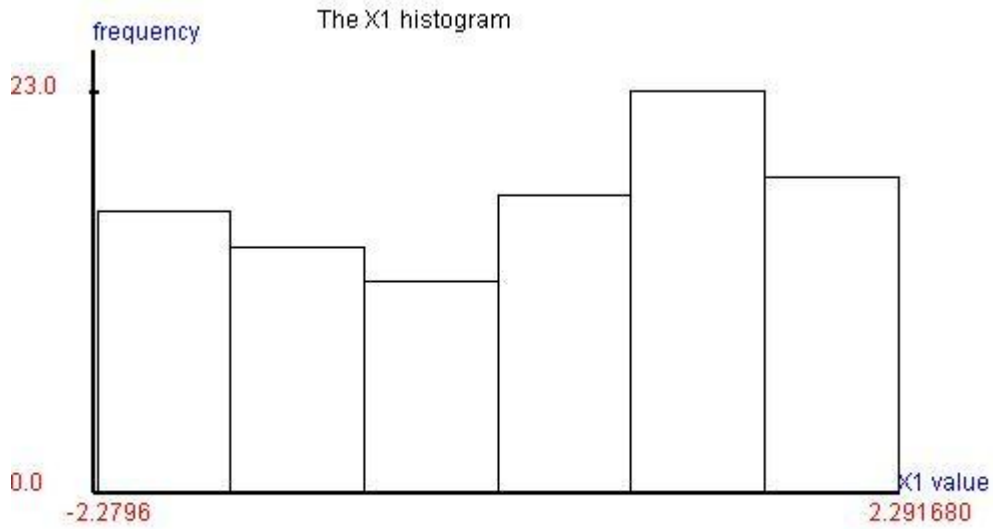
8.4.1)The population distribution is uniform distribution.

X1 is Uniform( $\alpha=-2.300000$ , $\beta=2.300000$ ),

| X1            |
|---------------|
| 1.0260895304  |
| 1.8135209041  |
| -1.7994523225 |
| 0.3125696451  |
| -0.1909980756 |
| 0.9309985906  |
| -1.0732491522 |
| -2.0742064745 |
| 2.2916807655  |
| -2.2796890372 |
| 0.9505830273  |
| 0.8201458402  |
| 2.1896672636  |
| 1.6621781431  |
| 0.3457197860  |
| 0.3399795399  |
| 0.5764683353  |
| 1.6497989861  |
| 0.7366301557  |
| 1.4535824434  |
| 1.5426662496  |
| -1.3546054764 |
| 1.5755073379  |
| 1.6844716369  |
| 1.9723064462  |
| -0.5761324885 |
| 2.0290196507  |
| -0.5118371837 |
| 0.5887511025  |
| 0.5858546754  |
| 0.7746127025  |
| -2.0001177090 |
| -1.4706541936 |
| -1.0758671528 |
| 1.9173244565  |
| -0.0130275794 |
| -1.7958247254 |
| 2.1207420506  |
| -1.8713310514 |
| 1.3365120763  |
| 0.2941843312  |
| 2.2580848171  |
| 1.0186908654  |
| 1.9106394178  |
| 1.0466074263  |
| 1.1252955848  |
| -2.1562705617 |
| -0.3004629138 |
| -1.0775787138 |
| -0.7996682112 |
| 1.1587476718  |
| 0.9031632088  |
| -1.0455339536 |
| 0.4904842670  |
| 1.5170460843  |
| -1.9260655782 |
| 0.8680012429  |
| 0.3095125178  |
| -0.9708349222 |
| -0.4484151739 |
| 0.7250093739  |
| -1.0225283691 |
| 2.0805663742  |
| -2.0795583237 |
| 1.3454172532  |
| -0.6810905382 |
| -1.1803832533 |
| 2.1214645646  |
| 1.4627575215  |
| -1.2540779006 |
| 1.3207991266  |

-2.0507107922  
 0.2691250504  
 -1.7967095872  
 -2.1745911227  
 -1.6330967344  
 1.1073718428  
 1.9590972068  
 0.3739259080  
 -2.2326471094  
 0.3659567308  
 -1.6791445102  
 0.2281086267  
 0.8158094146  
 0.5327087488  
 1.0609686368  
 -0.0053860584  
 -0.1991665582  
 1.2637628733  
 -0.9715356567  
 -0.7460547636  
 2.1295694691  
 -1.7763537989  
 -0.4680537718  
 1.4010573117  
 0.2930409809  
 -1.2455420398  
 -0.8262063273  
 1.3050702927  
 -0.2611937529

X1 is Uniform(alpha=-2.300000,beta=2.300000),



likelihood ratio goodness of fit

| class       | [ 1 ]    | [ 2 ]    | [ 3 ]    | [ 4 ]    |
|-------------|----------|----------|----------|----------|
| lower limit | -2.27969 | -1.62664 | -0.97358 | -0.32053 |
| 0.33252     | 0.98558  | 1.63863  |          |          |
| upper limit | -1.62664 | -0.97358 | -0.32053 | 0.33252  |
| 0.98558     | 1.63863  | 2.29168  |          |          |
| observed no | 16.00000 | 10.00000 | 10.00000 | 12.00000 |
| 18.00000    | 18.00000 | 16.00000 |          |          |
| probability | 0.14286  | 0.14286  | 0.14286  | 0.14286  |
| 0.14286     | 0.14286  | 0.14286  |          |          |
| expected no | 14.28571 | 14.28571 | 14.28571 | 14.28571 |
| 14.28571    | 14.28571 | 14.28571 |          |          |
| chi square  | 0.18367  | 1.83673  | 1.83673  | 0.43537  |
| 0.76644     | 0.76644  | 0.18367  |          |          |

degree of freedom=4

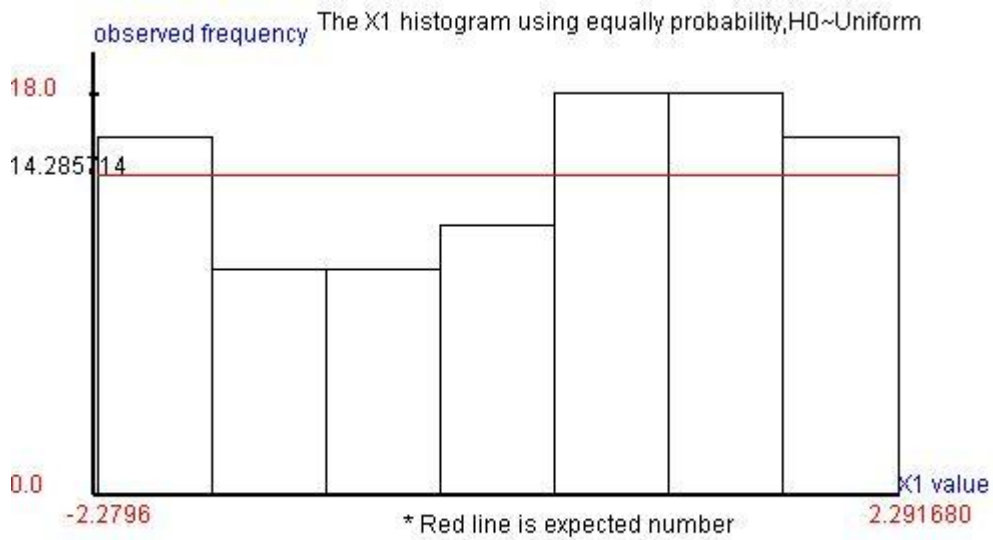
H0:  $X_1 \sim \text{Uniform}(\alpha, \beta)$ ,  $\alpha, \beta$  are unknown

$\alpha$  point estimated value = -2.279689 (MLE)

$\beta$  point estimated value = 2.291681 (MLE)

Likelihood ratio chi-square test statistic = 6.009070

p-value = 0.198400



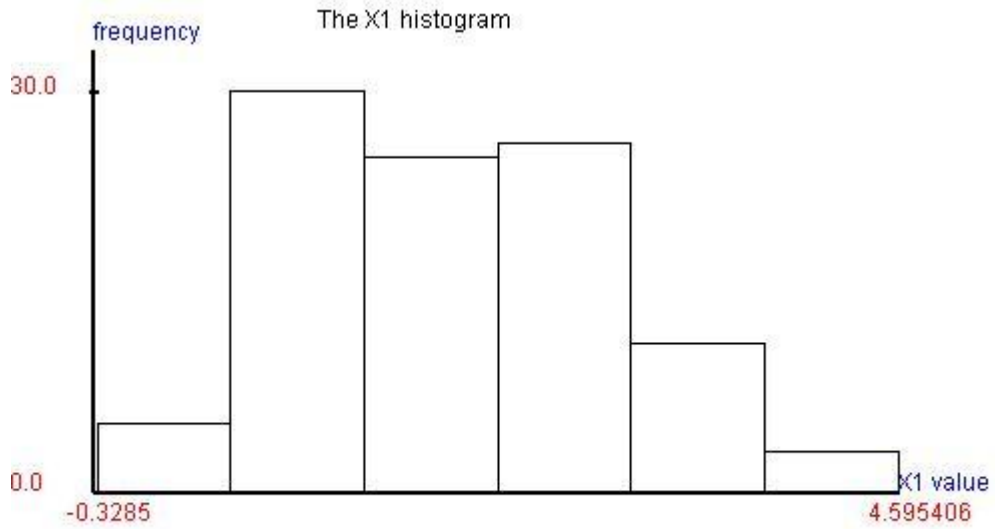
8.4.2)The population distribution is normal distribution.

X1 is Normal( $\mu=2.000000$ , $\sigma^2=1.000000$ ),

| X1            |
|---------------|
| 2.2068669569  |
| 2.7169878333  |
| 2.6464333915  |
| 2.3835645127  |
| 1.0102370826  |
| 2.1256933754  |
| 0.8224976900  |
| 2.2854060958  |
| 3.8254815208  |
| 3.1001641439  |
| 2.0081231002  |
| 2.7824166217  |
| 0.6328531896  |
| 1.7402674481  |
| 1.0359437238  |
| 1.3913588820  |
| 1.7471881560  |
| 4.0794790553  |
| 1.0260269432  |
| 1.1198314607  |
| 3.7379806606  |
| 2.0861976339  |
| 2.3348913805  |
| 1.7992574931  |
| 2.3490848240  |
| 0.5404750924  |
| 2.8313065854  |
| 2.3804158695  |
| 0.7188384806  |
| 1.9084473235  |
| 2.3342623634  |
| 0.9564239183  |
| 2.4317246835  |
| 3.1381011405  |
| 2.9536731897  |
| -0.3285625994 |
| 1.7187811335  |
| 1.5171710474  |
| 1.1188561413  |
| 0.3223699263  |
| 1.2737747346  |
| 1.0718196951  |
| 2.3896864065  |
| 3.1111452511  |
| 1.1681015357  |
| 0.3129838812  |
| 3.3243283948  |
| 1.5126628738  |
| 2.5318079220  |
| 1.4316597735  |
| 2.2774777113  |
| 2.9923428529  |
| 0.8862287211  |
| 1.7663298515  |
| 2.2251068455  |
| 0.7449474847  |
| 1.5240573082  |
| 1.3093146787  |
| 1.9066552749  |
| 2.7659612478  |
| 0.6916345741  |
| 0.8024335586  |
| 2.3410700410  |
| 3.4245324054  |
| 2.6226981869  |
| 2.6481455746  |
| 2.8638684863  |
| 0.9460017551  |
| 1.7313502002  |
| 0.2858512335  |
| 1.7436194460  |

1.9298739673  
 4.5954060957  
 2.8156803416  
 2.0135101081  
 2.6591871797  
 0.9285406943  
 2.6947828008  
 1.6982656580  
 3.6444790891  
 0.5646977972  
 1.5664523617  
 1.9824229927  
 3.5219637527  
 0.7025838916  
 0.8672887216  
 1.7121713114  
 0.7079115634  
 0.8413974089  
 3.4884255998  
 0.7413133455  
 0.3092128291  
 1.3906599847  
 3.3010024806  
 0.6230939061  
 2.2188544059  
 1.4354220148  
 1.0617633082  
 1.2131274854  
 1.0139596252

X1 is Normal(mu=2.000000,sigma\*sigma=1.000000)



likelihood ratio goodness of fit

| class       | [ 1 ]    | [ 2 ]    | [ 3 ]    | [ 4 ]    |
|-------------|----------|----------|----------|----------|
| [ 5 ]       | [ 6 ]    | [ 7 ]    |          |          |
| lower limit |          | 0.81067  | 1.30575  | 1.68637  |
| 2.04158     | 2.42211  | 2.91699  |          |          |
| upper limit | 0.81067  | 1.30575  | 1.68637  | 2.04158  |
| 2.42211     | 2.91699  |          |          |          |
| observed no | 16.00000 | 18.00000 | 9.00000  | 15.00000 |
| 14.00000    | 13.00000 | 15.00000 |          |          |
| probability | 0.14286  | 0.14286  | 0.14286  | 0.14286  |
| 0.14286     | 0.14286  | 0.14286  |          |          |
| expected no | 14.28571 | 14.28571 | 14.28571 | 14.28571 |
| 14.28571    | 14.28571 | 14.28571 |          |          |
| chi square  | 0.18367  | 0.76644  | 3.10431  | 0.03401  |
| 0.00583     | 0.12716  | 0.03401  |          |          |

degree of freedom=4

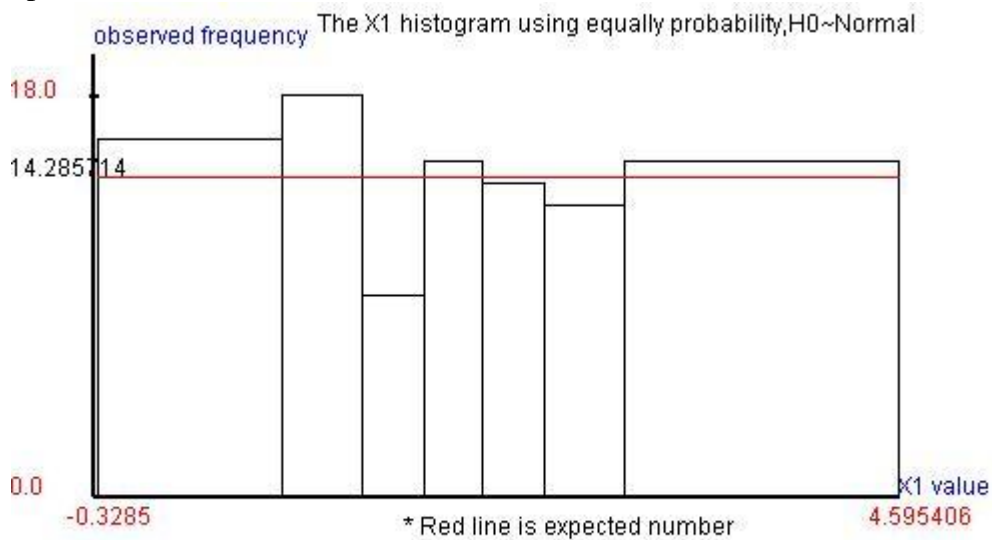
H0:  $X_1 \sim \text{Normal}(\mu, \sigma^2)$ ,  $\mu, \sigma$  are unknown

population mean( $\mu$ ) point estimated value=1.864076 (MLE)

population variance( $\sigma^2$ ) which point estimated value=0.973194 (MLE)

Likelihood ratio chi-square test statistic =4.255438

p-value=0.372500





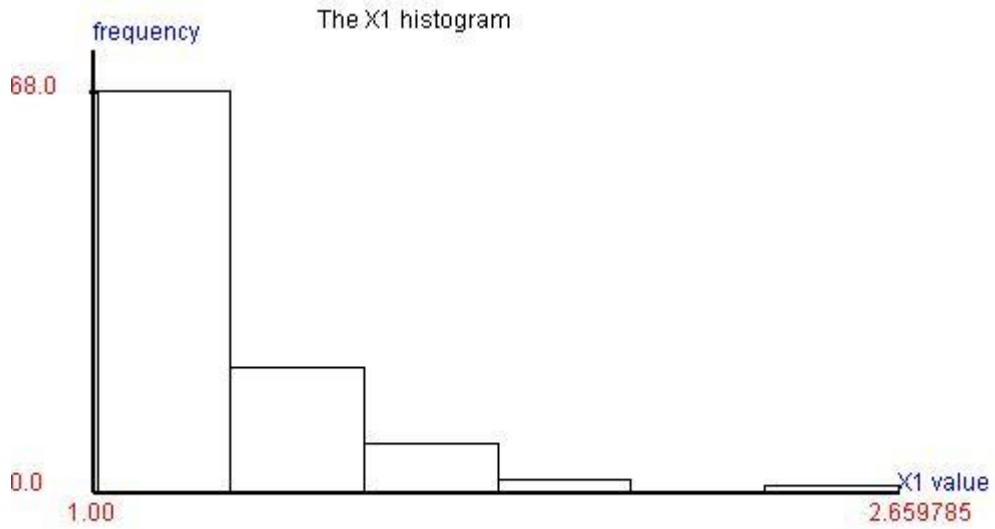
8.4.3)The population distribution is shifted exponential distribution.

X1 is Exponential( $\lambda=4.000000, c=1.000000$ ),

| X1           |
|--------------|
| 1.0636386859 |
| 1.2208838720 |
| 1.4135372635 |
| 1.1907903489 |
| 2.0299791450 |
| 1.4475361920 |
| 1.0002861062 |
| 1.0402698301 |
| 1.1504041042 |
| 1.0931955102 |
| 1.1301019225 |
| 1.0940401022 |
| 1.0354008585 |
| 1.2079443607 |
| 1.0339378063 |
| 1.5042278899 |
| 1.2690110882 |
| 1.0224952563 |
| 1.2034109256 |
| 2.0510981309 |
| 1.1513242379 |
| 1.0518249628 |
| 1.0603475142 |
| 1.3366861938 |
| 1.3475280240 |
| 1.0705956984 |
| 1.8146222455 |
| 1.0083777377 |
| 2.6597850959 |
| 1.0990797468 |
| 1.2237512777 |
| 1.5851139183 |
| 1.2546145199 |
| 1.0491401241 |
| 1.3953906115 |
| 1.0599882764 |
| 1.0745872777 |
| 1.2300869478 |
| 1.1719743030 |
| 1.3037420671 |
| 1.0304671156 |
| 1.0553347694 |
| 1.2162993373 |
| 1.1399880345 |
| 1.0108385050 |
| 1.1696056892 |
| 1.8001920856 |
| 1.2095416478 |
| 1.3240350366 |
| 1.1380091958 |
| 1.0165772303 |
| 1.2311109714 |
| 1.1100344873 |
| 1.0629193593 |
| 1.2741680184 |
| 1.5393292179 |
| 1.2642757596 |
| 1.2631975296 |
| 1.0257348989 |
| 1.2037425495 |
| 1.0790049315 |
| 1.2478782965 |
| 1.4639108000 |
| 1.5211910095 |
| 1.2155178446 |
| 1.0644685574 |
| 1.1746786926 |
| 1.5364547659 |
| 1.4058893883 |
| 1.0101616792 |
| 1.0348923504 |

1.1598052377  
 1.2094020018  
 1.6622216005  
 1.7556301509  
 1.4270684970  
 1.3848457921  
 1.0401143536  
 1.1500216858  
 1.0524172399  
 1.7140361534  
 1.4514109126  
 1.4587276306  
 1.0332926161  
 1.3429888624  
 1.5461249443  
 1.5607444627  
 1.0000214373  
 1.0422586801  
 1.0567266708  
 1.0455239671  
 1.7873458643  
 1.3798223039  
 1.0913165467  
 1.2184495724  
 1.0598548155  
 1.2729156124  
 1.1041342718  
 1.1280664307  
 1.3406307466

X1 is Exponential(lamda=4.000000,c=1.000000),



likelihood ratio goodness of fit

| class       | [ 1 ]    | [ 2 ]    | [ 3 ]    | [ 4 ]    |
|-------------|----------|----------|----------|----------|
| [ 5 ]       | [ 6 ]    | [ 7 ]    |          |          |
| lower limit | 1.00002  | 1.04082  | 1.08907  | 1.14812  |
| 1.22425     | 1.33155  | 1.51499  |          |          |
| upper limit | 1.04082  | 1.08907  | 1.14812  | 1.22425  |
| 1.33155     | 1.51499  |          |          |          |
| observed no | 15.00000 | 16.00000 | 10.00000 | 18.00000 |
| 11.00000    | 15.00000 | 15.00000 |          |          |
| probability | 0.14286  | 0.14286  | 0.14286  | 0.14286  |
| 0.14286     | 0.14286  | 0.14286  |          |          |
| expected no | 14.28571 | 14.28571 | 14.28571 | 14.28571 |
| 14.28571    | 14.28571 | 14.28571 |          |          |
| chi square  | 0.03401  | 0.18367  | 1.83673  | 0.76644  |
| 0.98145     | 0.03401  | 0.03401  |          |          |

degree of freedom=4

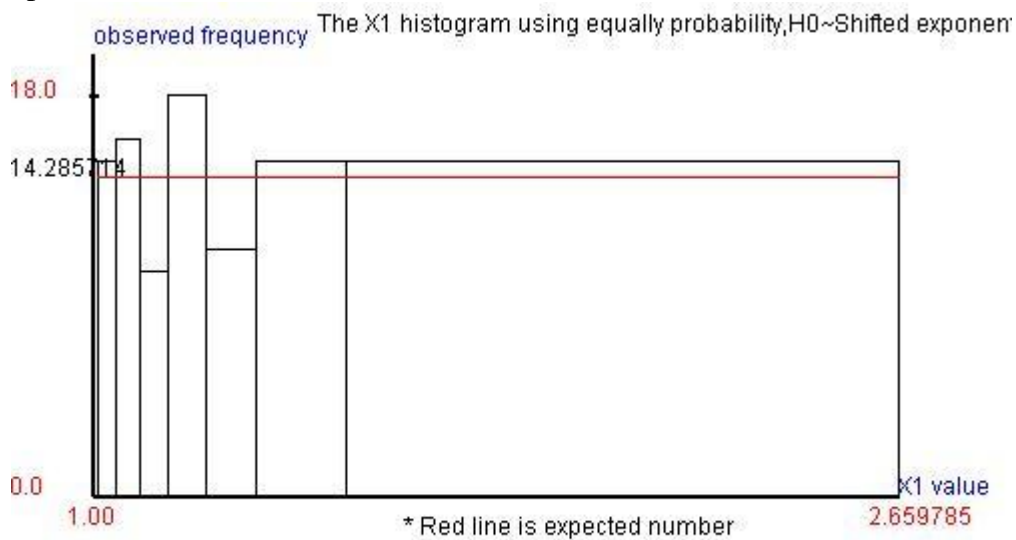
H0:  $X_1 \sim \text{Shifted exponential}(\lambda, c)$ ,  $\lambda, c$  are unknown

$\lambda$  point estimated value=3.778722 (MLE)

$c$  point estimated value=1.000021 (MLE)

Likelihood ratio chi-square test statistic =3.870336

p-value=0.423800



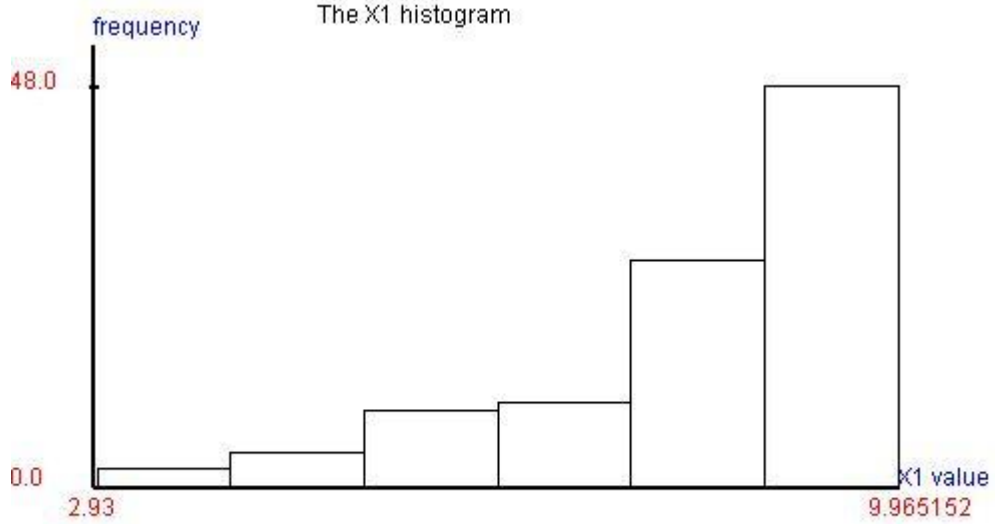
8.4.4)The population distribution is pareto1 distribution.

X1 is Pareto1(lamda=5.000000,c=10.000000),

| X1           |
|--------------|
| 8.0695171083 |
| 5.8601217639 |
| 9.0169907362 |
| 7.9942554119 |
| 9.8996664487 |
| 8.5824348724 |
| 6.7122121207 |
| 7.6367436895 |
| 9.8939030504 |
| 8.3329357581 |
| 8.3799560434 |
| 7.5507526722 |
| 9.3323116289 |
| 7.9293063314 |
| 9.9651522282 |
| 9.9593842480 |
| 9.4501550734 |
| 7.4680574235 |
| 4.1741179556 |
| 9.8365471430 |
| 9.3526922021 |
| 8.3221060168 |
| 9.5713655998 |
| 9.9275868699 |
| 8.3760449985 |
| 2.9321683163 |
| 8.4507829357 |
| 7.5422639818 |
| 5.6574849538 |
| 5.2612841830 |
| 8.7684216690 |
| 5.6673439359 |
| 9.0127135348 |
| 7.9182131520 |
| 6.6333379085 |
| 9.8362023549 |
| 6.8326676220 |
| 8.7913284456 |
| 9.5262510760 |
| 9.4021896865 |
| 9.0710834424 |
| 8.8485096540 |
| 9.0714355862 |
| 6.2582665235 |
| 8.1406347071 |
| 9.4094073248 |
| 9.0875186452 |
| 9.3570972690 |
| 5.5319402858 |
| 8.6595393533 |
| 5.9006068974 |
| 9.9587678430 |
| 9.1255733534 |
| 9.5207848990 |
| 4.9049938831 |
| 9.0511432222 |
| 8.2454909761 |
| 9.9374402015 |
| 9.9637144854 |
| 8.6001193766 |
| 6.6406749032 |
| 9.0576187583 |
| 9.1059212258 |
| 8.7344267374 |
| 9.0083848482 |
| 8.6036432216 |
| 9.4441950427 |
| 9.5206864447 |
| 9.6358470934 |
| 9.1478023414 |
| 7.6654383542 |

9.1748041491  
 9.6151598267  
 9.4656164838  
 8.0308718605  
 5.8381041219  
 7.0716237863  
 8.2919481726  
 9.1958452646  
 4.3067627703  
 9.9354882339  
 8.7340461957  
 9.6994709739  
 5.3721513746  
 9.2768166199  
 6.3255528113  
 3.3710609123  
 9.9218822555  
 7.9921415458  
 9.8427726117  
 7.5894059427  
 9.0878767637  
 8.9856819987  
 8.1135172779  
 9.0763882451  
 9.6830101332  
 7.2982989511  
 8.2053133641  
 9.7141215798  
 8.7554892817

X1 is Pareto1(lamda=5.000000,c=10.000000),



likelihood ratio goodness of fit

| class       | [ 1 ]    | [ 2 ]    | [ 3 ]    | [ 4 ]    |
|-------------|----------|----------|----------|----------|
| [ 5 ]       | [ 6 ]    | [ 7 ]    |          |          |
| lower limit | 0.00000  | 6.61748  | 7.65638  | 8.33814  |
| 8.85837     | 9.28414  | 9.64717  |          |          |
| upper limit | 6.61748  | 7.65638  | 8.33814  | 8.85837  |
| 9.28414     | 9.64717  | 9.96515  |          |          |
| observed no | 15.00000 | 11.00000 | 14.00000 | 13.00000 |
| 17.00000    | 14.00000 | 16.00000 |          |          |
| probability | 0.14286  | 0.14286  | 0.14286  | 0.14286  |
| 0.14286     | 0.14286  | 0.14286  |          |          |
| expected no | 14.28571 | 14.28571 | 14.28571 | 14.28571 |
| 14.28571    | 14.28571 | 14.28571 |          |          |
| chi square  | 0.03401  | 0.98145  | 0.00583  | 0.12716  |
| 0.43337     | 0.00583  | 0.18367  |          |          |

degree of freedom=4

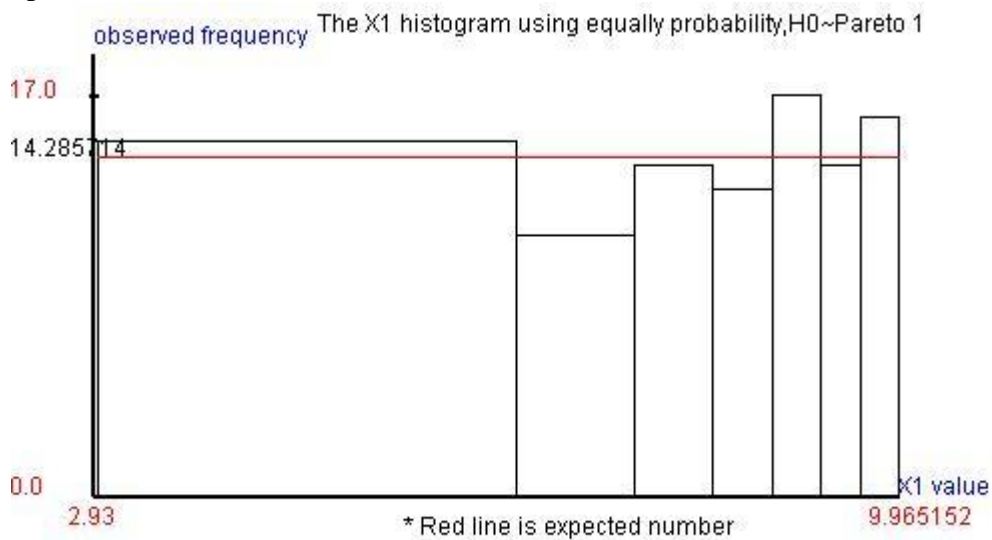
H0:  $X_1 \sim \text{Pareto 1}(\lambda, c)$ ,  $\lambda, c$  are unknown

$\lambda$  point estimated value=4.753317 (MLE)

$c$  point estimated value=9.965152 (MLE)

Likelihood ratio chi-square test statistic =1.771328

p-value=0.777700



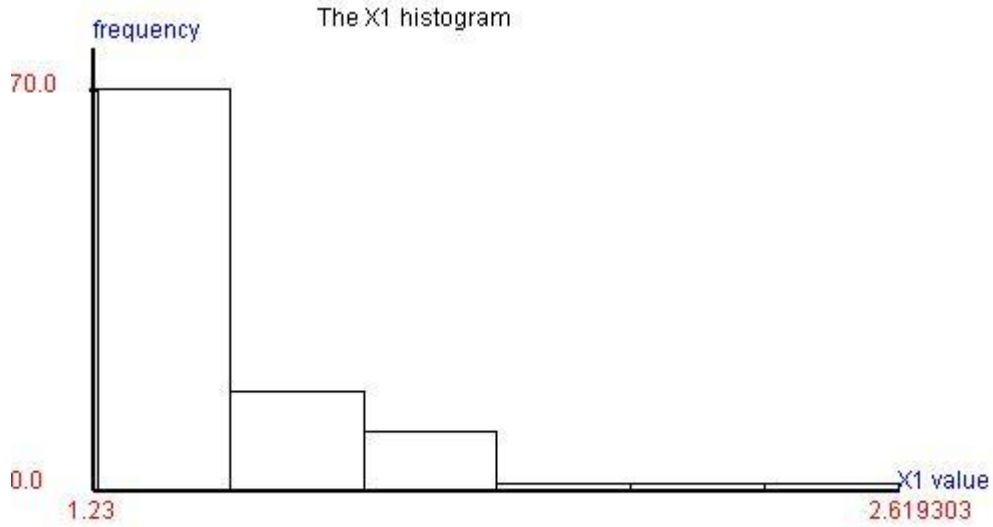
8.4.5)The population distribution is pareto2 distribution.

X1 is Pareto2(lamda=6.400000,c=1.230000),

| X1           |
|--------------|
| 1.2949557532 |
| 1.4594178288 |
| 1.3486978807 |
| 1.3071560468 |
| 1.7187259044 |
| 1.4222329494 |
| 1.3349006618 |
| 1.4734967191 |
| 1.2323347850 |
| 1.3618862482 |
| 1.8793905719 |
| 1.2400586172 |
| 1.2381065828 |
| 1.5973072748 |
| 1.3225638927 |
| 1.2522632844 |
| 1.2650744963 |
| 1.2972574421 |
| 1.2844486958 |
| 1.6577129812 |
| 1.2437341750 |
| 1.5394119169 |
| 1.3140325227 |
| 1.3833946166 |
| 1.3594346892 |
| 1.2598908674 |
| 1.5160268828 |
| 1.3560116048 |
| 1.3933280501 |
| 1.2634927849 |
| 1.4151366717 |
| 1.5389428870 |
| 1.2776868842 |
| 2.6193035173 |
| 1.8462935615 |
| 1.7018558837 |
| 1.3010970152 |
| 1.5750789424 |
| 1.2747022159 |
| 1.2416788169 |
| 1.4158144882 |
| 1.2437827015 |
| 1.6155559161 |
| 1.4792905917 |
| 1.5279984194 |
| 1.8779743741 |
| 1.3591391561 |
| 1.3630179698 |
| 1.2503319982 |
| 1.2888126318 |
| 1.3138931812 |
| 1.3825361981 |
| 1.3011922867 |
| 1.4681175621 |
| 1.2903134001 |
| 1.2942301475 |
| 1.7579442046 |
| 1.3605316054 |
| 1.4522027090 |
| 1.3178204301 |
| 1.5027457113 |
| 1.3150024095 |
| 1.3316212845 |
| 1.2880696991 |
| 1.2733247365 |
| 1.2378038090 |
| 1.4011859284 |
| 1.7984924712 |
| 1.3740580349 |
| 1.6648866550 |
| 1.2340122386 |

1.2635184348  
 1.4366755198  
 1.2397054343  
 1.7673931166  
 1.2387431369  
 1.2900817161  
 1.2462042828  
 2.2185982833  
 1.2377331192  
 1.4282873176  
 1.5338597347  
 1.7025508985  
 1.2512006385  
 1.2451250181  
 1.2843607336  
 1.2711843078  
 1.4422955879  
 1.5276283094  
 1.9309821210  
 1.3551593531  
 1.3042787136  
 1.7335739947  
 1.2941230540  
 1.2365218945  
 1.5345819855  
 1.2667695189  
 1.3000358080  
 1.5556391316  
 1.2600652848

X1 is Pareto2(lamda=6.400000,c=1.230000),





likelihood ratio goodness of fit

| class       | [ 1 ]    | [ 2 ]    | [ 3 ]    | [ 4 ]    |
|-------------|----------|----------|----------|----------|
| [ 5 ]       | [ 6 ]    | [ 7 ]    |          |          |
| lower limit | 1.23233  | 1.25831  | 1.28974  | 1.32928  |
| 1.38205     | 1.46000  | 1.60357  |          |          |
| upper limit | 1.25831  | 1.28974  | 1.32928  | 1.38205  |
| 1.46000     | 1.60357  |          |          |          |
| observed no | 17.00000 | 14.00000 | 16.00000 | 11.00000 |
| 12.00000    | 14.00000 | 16.00000 |          |          |
| probability | 0.14286  | 0.14286  | 0.14286  | 0.14286  |
| 0.14286     | 0.14286  | 0.14286  |          |          |
| expected no | 14.28571 | 14.28571 | 14.28571 | 14.28571 |
| 14.28571    | 14.28571 | 14.28571 |          |          |
| chi square  | 0.43337  | 0.00583  | 0.18367  | 0.98145  |
| 0.43537     | 0.00583  | 0.18367  |          |          |

degree of freedom=4

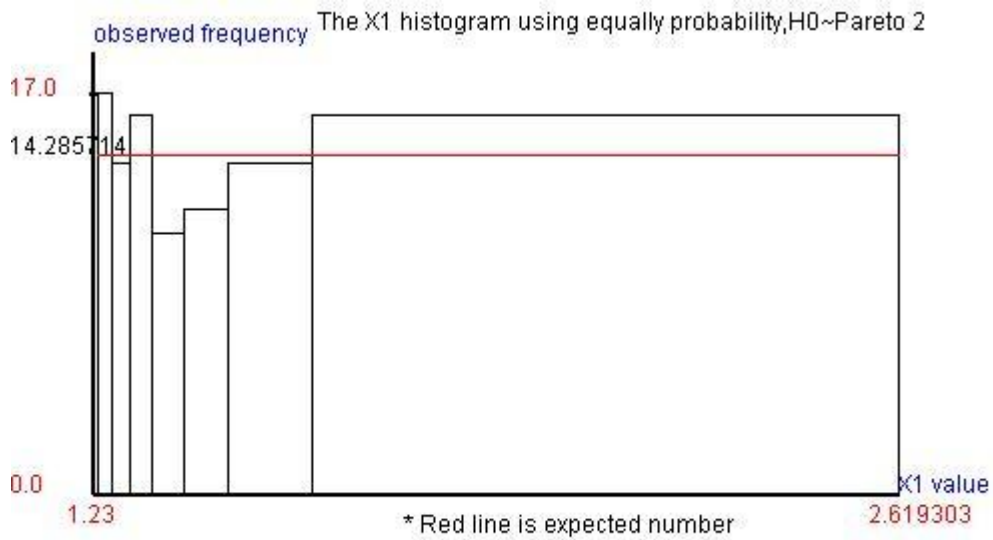
H0:  $X_1 \sim \text{Pareto } 2(\lambda, c)$ ,  $\lambda, c$  are unknown

$\lambda$  point estimated value=7.389930 (MLE)

$c$  point estimated value=1.232335 (MLE)

Likelihood ratio chi-square test statistic =2.229203

p-value=0.693600



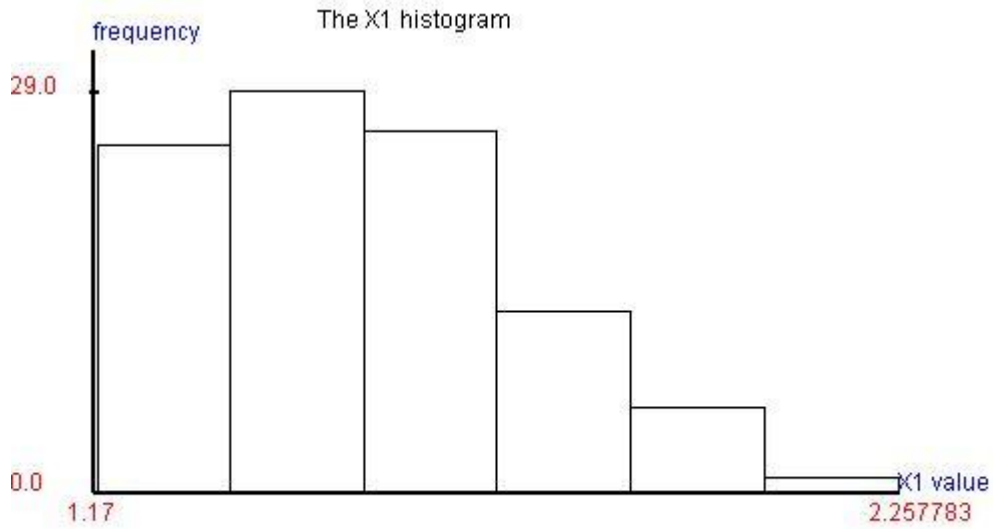
8.4.6)The population distribution is rayleigh distribution.

X1 is Rayleigh( $\lambda=4.000000, c=1.100000$ ),

| X1           |
|--------------|
| 1.5302659988 |
| 1.3017837189 |
| 1.2124944911 |
| 1.5690588713 |
| 1.7118864154 |
| 1.3026124236 |
| 1.5066758252 |
| 1.5851672945 |
| 1.6428095844 |
| 1.6326709834 |
| 1.3384940815 |
| 1.6213446843 |
| 1.5365168452 |
| 1.2496319604 |
| 1.4130702357 |
| 1.3075422301 |
| 1.4195634418 |
| 1.4788125419 |
| 1.2634824987 |
| 1.5210217937 |
| 1.7967995831 |
| 1.2113785880 |
| 1.7494171628 |
| 1.2627519562 |
| 1.4585383377 |
| 1.8728124144 |
| 1.5357156297 |
| 1.3073904934 |
| 1.5300166401 |
| 2.0256038826 |
| 1.6833718746 |
| 1.4958344888 |
| 1.5720383837 |
| 1.4040480579 |
| 1.8225481939 |
| 2.0726271651 |
| 1.3644879139 |
| 1.3299887022 |
| 1.6320820280 |
| 1.3963296941 |
| 1.6637446899 |
| 2.0063078105 |
| 1.5049490933 |
| 1.1879801273 |
| 1.2052355564 |
| 1.8018931086 |
| 1.4215925286 |
| 1.1769456936 |
| 1.2704010578 |
| 1.6296420768 |
| 1.1960523629 |
| 1.4918645798 |
| 1.5978392620 |
| 1.3925579144 |
| 2.0745062043 |
| 1.7794424446 |
| 1.9715197495 |
| 1.2688550394 |
| 1.9591139480 |
| 1.7948185454 |
| 1.7059036826 |
| 1.4199926065 |
| 1.6880317807 |
| 1.2501312810 |
| 1.6384919810 |
| 1.3075651552 |
| 1.6590006699 |
| 1.6034940827 |
| 1.6265523930 |
| 1.7120863131 |
| 1.2517949145 |

1.5803467412  
 1.4432218000  
 1.4158732829  
 1.3974604684  
 1.4721605947  
 1.4253171107  
 1.6364187481  
 1.2471165666  
 1.2845681287  
 1.2451226959  
 1.2668644117  
 1.5211703550  
 1.4662522647  
 2.2577830278  
 1.7340563551  
 1.7909219590  
 1.6194601371  
 1.3591027229  
 1.4165754305  
 1.4379986748  
 1.6921064675  
 1.4466915207  
 1.8793818516  
 1.8888953265  
 1.7759716227  
 1.1743600215  
 1.5649610121  
 1.5098362072  
 1.8846370057

X1 is Rayleigh(lamda=4.000000,c=1.100000),



likelihood ratio goodness of fit

| class       | [ 1 ]    | [ 2 ]    | [ 3 ]    | [ 4 ]    |
|-------------|----------|----------|----------|----------|
| [ 5 ]       | [ 6 ]    | [ 7 ]    |          |          |
| lower limit | 1.17436  | 1.34429  | 1.42541  | 1.49813  |
| 1.57275     | 1.65878  | 1.77810  |          |          |
| upper limit | 1.34429  | 1.42541  | 1.49813  | 1.57275  |
| 1.65878     | 1.77810  |          |          |          |
| observed no | 25.00000 | 13.00000 | 9.00000  | 12.00000 |
| 13.00000    | 11.00000 | 17.00000 |          |          |
| probability | 0.14286  | 0.14286  | 0.14286  | 0.14286  |
| 0.14286     | 0.14286  | 0.14286  |          |          |
| expected no | 14.28571 | 14.28571 | 14.28571 | 14.28571 |
| 14.28571    | 14.28571 | 14.28571 |          |          |
| chi square  | 4.59184  | 0.12716  | 3.10431  | 0.43537  |
| 0.12716     | 0.98145  | 0.43337  |          |          |

degree of freedom=4

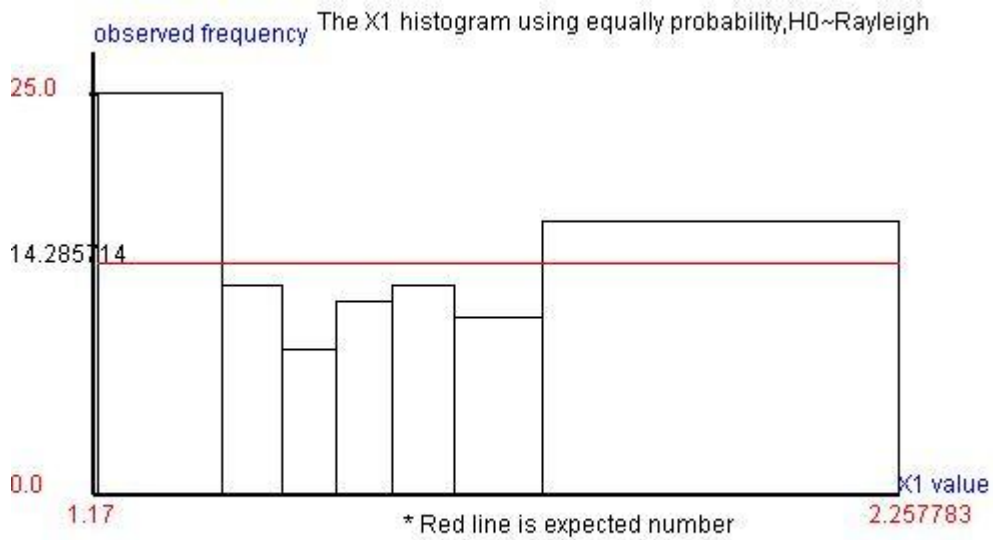
H0:  $X_1 \sim \text{Rayleigh}(\lambda, c)$ ,  $\lambda, c$  are unknown

$\lambda$  point estimated value=5.338539 (MLE)

$c$  point estimated value=1.174360 (MLE)

Likelihood ratio chi-square test statistic =9.800657

p-value=0.043900



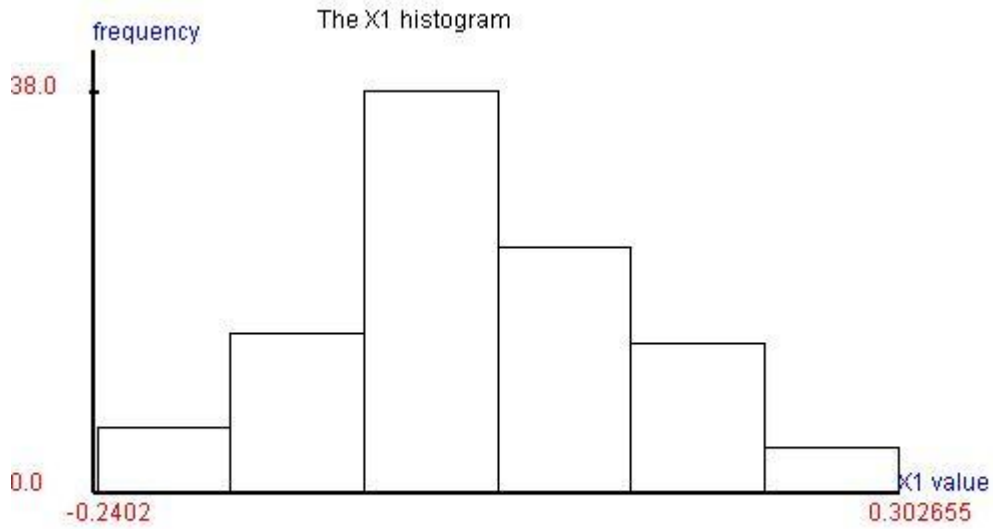
8.4.7)The population distribution is double exponential distribution.

X1 is Double Exponential( $\lambda=10.000000,\mu=0.000000$ ),

| X1            |
|---------------|
| -0.0824176500 |
| 0.0561704070  |
| -0.1050007481 |
| -0.0431537800 |
| 0.0690953039  |
| 0.0617634706  |
| 0.0020911520  |
| 0.1654532177  |
| 0.0137378890  |
| 0.0030081281  |
| 0.1766191950  |
| -0.0647276376 |
| -0.0209613670 |
| 0.0024931457  |
| 0.0139922664  |
| 0.0266311440  |
| 0.0905194321  |
| 0.0895917844  |
| 0.1326861696  |
| -0.1621304818 |
| 0.1717520938  |
| -0.0153129375 |
| -0.1345593098 |
| 0.1543563976  |
| -0.0362267415 |
| -0.0844083701 |
| -0.0349125307 |
| 0.0367783101  |
| -0.1956897031 |
| 0.0410870563  |
| -0.0691630148 |
| -0.0189381222 |
| -0.0332251066 |
| -0.1096580464 |
| -0.0433183457 |
| 0.0324389888  |
| 0.2278200388  |
| 0.0055697802  |
| 0.0209882802  |
| -0.0349748440 |
| 0.0685089771  |
| -0.1354074917 |
| 0.3026552307  |
| -0.0417048256 |
| 0.0134850113  |
| -0.0156556704 |
| 0.2302771098  |
| -0.1681455498 |
| -0.2335174062 |
| 0.0257558570  |
| -0.2402816163 |
| -0.1282305047 |
| 0.1638028771  |
| 0.0433476628  |
| 0.1320196327  |
| -0.0028324986 |
| -0.0361794014 |
| 0.0695929657  |
| -0.1184038962 |
| -0.0433488544 |
| 0.0161467694  |
| 0.0809308052  |
| -0.0559789144 |
| -0.0511917590 |
| 0.1009238864  |
| 0.0469650519  |
| 0.1286425311  |
| 0.0742556452  |
| -0.0814845562 |
| 0.2595246992  |
| 0.1777048661  |

0.0417872569  
 -0.0850230615  
 -0.0400009698  
 0.0005024377  
 -0.0001811121  
 0.0331964985  
 -0.0573323627  
 -0.1561814655  
 0.0075360734  
 0.0668160916  
 -0.0983543271  
 -0.0191357351  
 0.1656697531  
 0.1681197858  
 -0.0267250946  
 0.0385544963  
 -0.0488624339  
 0.0549817961  
 0.0519185495  
 -0.0673982593  
 -0.0325112211  
 -0.0519620866  
 0.1004492124  
 0.1739405168  
 0.1375415636  
 0.0818071159  
 0.0194662997  
 -0.0679170318  
 0.1419594110

X1 is Double Exponential(lamda=10.000000,mu=0.000000),



likelihood ratio goodness of fit

| class       | [ 1 ]    | [ 2 ]    | [ 3 ]    | [ 4 ]    |
|-------------|----------|----------|----------|----------|
| lower limit |          | -0.09591 | -0.03922 | -0.00605 |
| 0.01916     | 0.05232  | 0.10901  |          |          |
| upper limit | -0.09591 | -0.03922 | -0.00605 | 0.01916  |
| 0.05232     | 0.10901  |          |          |          |
| observed no | 13.00000 | 18.00000 | 12.00000 | 12.00000 |
| 13.00000    | 14.00000 | 18.00000 |          |          |
| probability | 0.14286  | 0.14286  | 0.14286  | 0.14286  |
| 0.14286     | 0.14286  | 0.14286  |          |          |
| expected no | 14.28571 | 14.28571 | 14.28571 | 14.28571 |
| 14.28571    | 14.28571 | 14.28571 |          |          |
| chi square  | 0.12716  | 0.76644  | 0.43537  | 0.43537  |
| 0.12716     | 0.00583  | 0.76644  |          |          |

degree of freedom=4

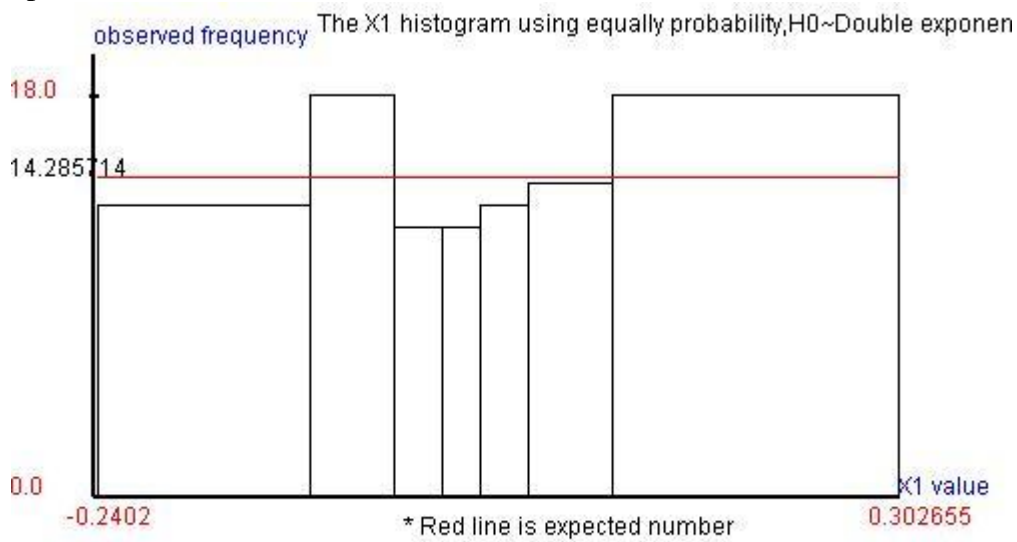
H0:  $X_1 \sim \text{Double exponential}(\lambda, \mu)$ ,  $\lambda, \mu$  are unknown

$\lambda$  point estimated value=12.226691 (MLE)

$\mu$  point estimated value=0.006553 (MLE)

Likelihood ratio chi-square test statistic =2.663776

p-value=0.615500



8.4.8)The population distribution is lognormal distribution.

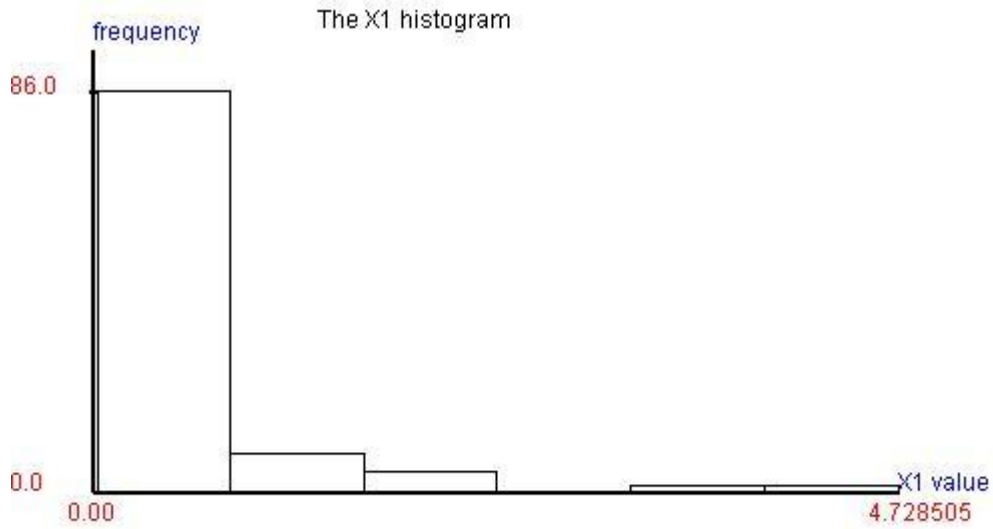
X1 is Log normal( $\mu=-2.000000$ , $\sigma=2.000000$ ),

| X1           |
|--------------|
| 0.5470313675 |
| 0.2723363631 |
| 0.3707976395 |
| 0.0344700212 |
| 0.4017229330 |
| 0.0072703934 |
| 0.0297803416 |
| 0.2074382579 |
| 0.2027586186 |
| 0.2081148525 |
| 0.1306828155 |
| 0.0113355735 |
| 1.9111239952 |
| 0.1634152694 |
| 1.5325008538 |
| 1.4725180316 |
| 0.5373947652 |
| 0.7341173929 |
| 0.0183538448 |
| 0.8497916672 |
| 0.0059454990 |
| 0.0023892354 |
| 1.0173166040 |
| 0.3348981152 |
| 0.1316768430 |
| 0.5048985846 |
| 0.3222856733 |
| 0.0578832849 |
| 0.1625931072 |
| 0.5523896750 |
| 0.1380309757 |
| 0.4281384234 |
| 0.0765404885 |
| 0.2187079852 |
| 0.7229742909 |
| 0.1021432777 |
| 0.0912268586 |
| 0.0821125024 |
| 0.0283797718 |
| 0.1394034020 |
| 0.1975096161 |
| 0.1296280923 |
| 0.3846328893 |
| 0.1198308081 |
| 0.0841397882 |
| 0.0031264406 |
| 1.1461692857 |
| 0.0755650440 |
| 1.8067296861 |
| 0.0660423817 |
| 0.0303830617 |
| 0.5497862814 |
| 0.1497334307 |
| 0.1484226161 |
| 0.0092478469 |
| 2.0068984019 |
| 0.0384228871 |
| 0.0403761656 |
| 0.0458620081 |
| 0.0436019433 |
| 0.0177932334 |
| 0.6116688094 |
| 3.3679346422 |
| 0.2640984497 |
| 0.1791894741 |
| 0.0140634921 |
| 0.1357297814 |
| 1.2372680704 |
| 0.7560723128 |
| 1.3006317798 |
| 0.2746344755 |



0.0164811404  
 0.2555754718  
 0.0117858831  
 0.0435365177  
 0.3450941006  
 4.7285050629  
 0.2049800201  
 0.0443021795  
 0.2474951255  
 0.5662841907  
 0.2120734593  
 0.0442407867  
 0.0172573343  
 0.0177802611  
 1.7541226616  
 0.0647326303  
 0.3366469347  
 1.0145307811  
 0.1255955218  
 0.1659808593  
 0.0921057420  
 0.3935601194  
 0.0386324036  
 0.1847879007  
 0.0025836954  
 0.0057471305  
 0.2577752748  
 0.2201760222  
 0.0628417012

X1 is Log normal(mu=-2.000000,sigma=2.000000),



likelihood ratio goodness of fit

| class       | [ 1 ]    | [ 2 ]    | [ 3 ]    | [ 4 ]    |
|-------------|----------|----------|----------|----------|
| lower limit |          | 0.02469  | 0.05608  | 0.10539  |
| 0.18989     | 0.35679  | 0.81029  |          |          |
| upper limit | 0.02469  | 0.05608  | 0.10539  | 0.18989  |
| 0.35679     | 0.81029  |          |          |          |
| observed no | 15.00000 | 12.00000 | 11.00000 | 15.00000 |
| 18.00000    | 15.00000 | 14.00000 |          |          |
| probability | 0.14286  | 0.14286  | 0.14286  | 0.14286  |
| 0.14286     | 0.14286  | 0.14286  |          |          |
| expected no | 14.28571 | 14.28571 | 14.28571 | 14.28571 |
| 14.28571    | 14.28571 | 14.28571 |          |          |
| chi square  | 0.03401  | 0.43537  | 0.98145  | 0.03401  |
| 0.76644     | 0.03401  | 0.00583  |          |          |

degree of freedom=4

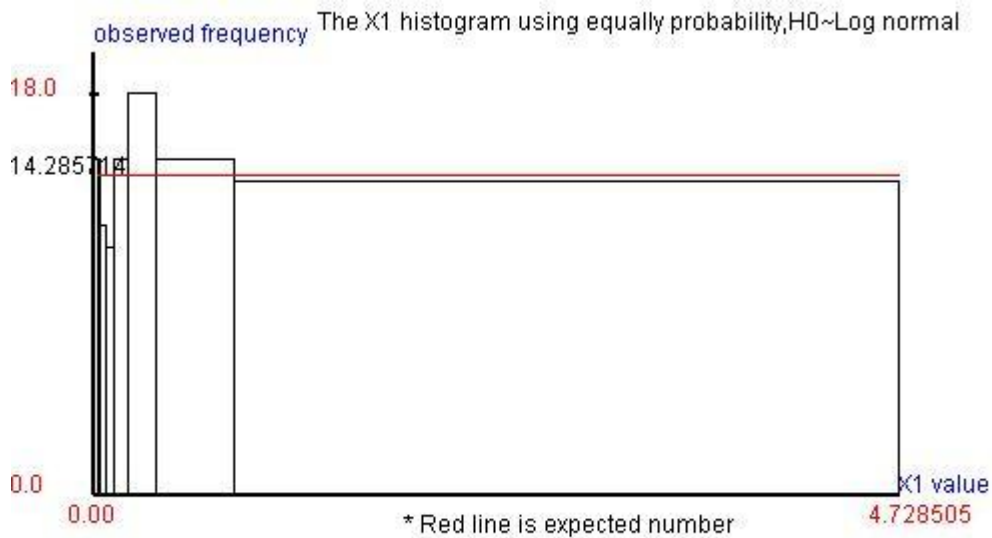
H0:  $X_1 \sim \text{Log\_Normal}(\mu, \sigma^2)$ ,  $\mu, \sigma$  are unknown

population mean( $\mu$ ) point estimated value=-1.955536 (MLE)

population variance( $\sigma^2$ ) which point estimated value=2.673564 (MLE)

Likelihood ratio chi-square test statistic =2.291133

p-value=0.682300

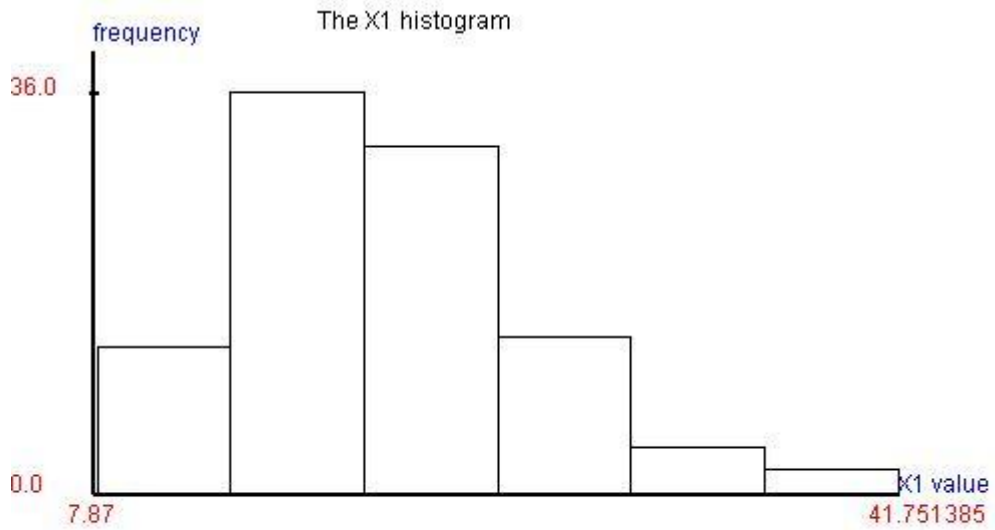


8.4.9)The population distribution is gamma distribution.  
The alpha value is setting to 10 and beta value is setting to 2.  
X1 is Gamma(alpha=10.500000,beta=2.000000),

| X1            |
|---------------|
| 27.5780152894 |
| 19.1377939960 |
| 16.4234485535 |
| 15.0990590568 |
| 20.3378279650 |
| 17.5654367138 |
| 18.7913330374 |
| 27.5388166967 |
| 22.0631065931 |
| 13.3953503242 |
| 21.6674419139 |
| 15.3211837678 |
| 26.2373532124 |
| 20.5843375701 |
| 14.1113948841 |
| 20.0373508548 |
| 23.1019342711 |
| 30.1868248756 |
| 26.0322488318 |
| 26.0805833632 |
| 10.8075404594 |
| 16.6160855660 |
| 21.9027006250 |
| 26.2866773058 |
| 13.8104580905 |
| 17.0200863565 |
| 27.5136058679 |
| 12.9490888071 |
| 13.0111586082 |
| 23.8952504371 |
| 16.2845467840 |
| 24.0877938553 |
| 8.7131289032  |
| 23.0168258384 |
| 16.4150758415 |
| 29.7840839932 |
| 17.3156664721 |
| 11.1078082526 |
| 21.8337462255 |
| 41.7513859173 |
| 27.2810038854 |
| 23.9040217934 |
| 17.7224249036 |
| 16.8255769881 |
| 14.6226488419 |
| 33.9045382273 |
| 16.0488024221 |
| 12.9034183067 |
| 40.3835393614 |
| 10.4866787744 |
| 10.9186355890 |
| 25.7477826259 |
| 21.2923557948 |
| 24.0195477733 |
| 7.8709877995  |
| 18.6438930127 |
| 23.8416103241 |
| 17.3254104894 |
| 18.7560833706 |
| 19.2375391316 |
| 16.4557789911 |
| 11.7253520164 |
| 18.4992294519 |
| 35.3037023718 |
| 23.5330551900 |
| 17.0014005081 |
| 24.2293492401 |
| 13.2686657059 |
| 18.9675576880 |
| 35.6499294360 |

19.9684721553  
 24.1274305789  
 23.2318859123  
 25.2339922926  
 19.1296688550  
 14.4645879948  
 18.2556841233  
 18.0180236727  
 18.7757087256  
 21.0657959731  
 19.5931861897  
 20.0215885196  
 28.8559696243  
 15.7674262344  
 20.4983132349  
 22.6702371034  
 15.8929007126  
 27.8301028665  
 21.7423940090  
 17.6865289118  
 14.8643749716  
 19.4420365553  
 21.9266815423  
 23.4418868279  
 31.2783478877  
 17.1768232558  
 13.3509764060  
 19.2404680700  
 18.2652622762  
 15.3699069390

X1 is Gamma(alpha=10.500000,beta=2.000000),



likelihood ratio goodness of fit

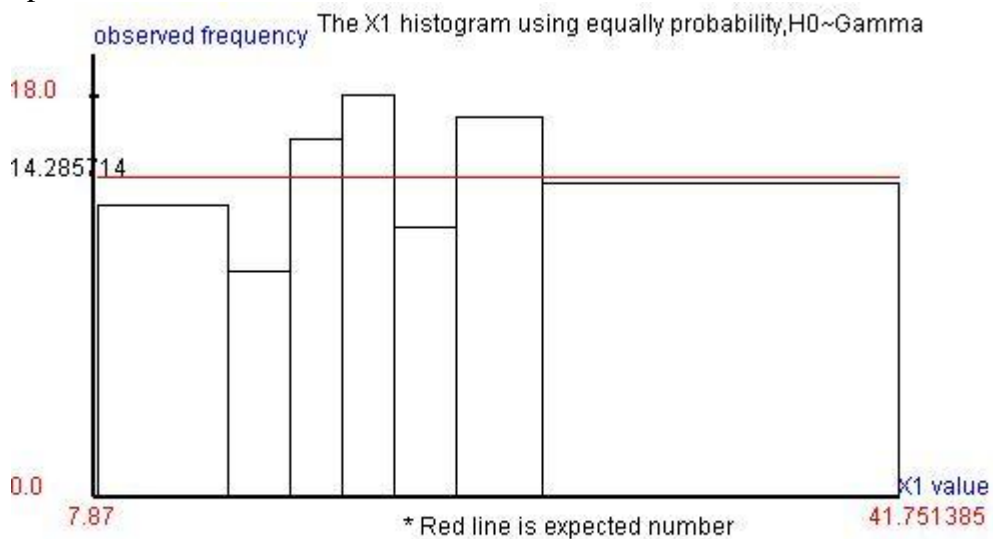
| class       | [ 1 ]    | [ 2 ]    | [ 3 ]    | [ 4 ]    |
|-------------|----------|----------|----------|----------|
| [ 5 ]       | [ 6 ]    | [ 7 ]    |          |          |
| lower limit |          | 13.45114 | 16.03716 | 18.24161 |
| 20.47489    | 23.06128 | 26.73453 |          |          |
| upper limit | 13.45114 | 16.03716 | 18.24161 | 20.47489 |
| 23.06128    | 26.73453 |          |          |          |
| observed no | 13.00000 | 10.00000 | 16.00000 | 18.00000 |
| 12.00000    | 17.00000 | 14.00000 |          |          |
| probability | 0.14286  | 0.14286  | 0.14286  | 0.14286  |
| 0.14286     | 0.14286  | 0.14286  |          |          |
| expected no | 14.28571 | 14.28571 | 14.28571 | 14.28571 |
| 14.28571    | 14.28571 | 14.28571 |          |          |
| chi square  | 0.12716  | 1.83673  | 0.18367  | 0.76644  |
| 0.43537     | 0.43337  | 0.00583  |          |          |

degree of freedom=6

H0:  $X_1 \sim \text{Gamma}(\alpha=10.000000, \beta=2.000000)$ ,

Likelihood ratio chi-square test statistic =3.788585

p-value=0.705200

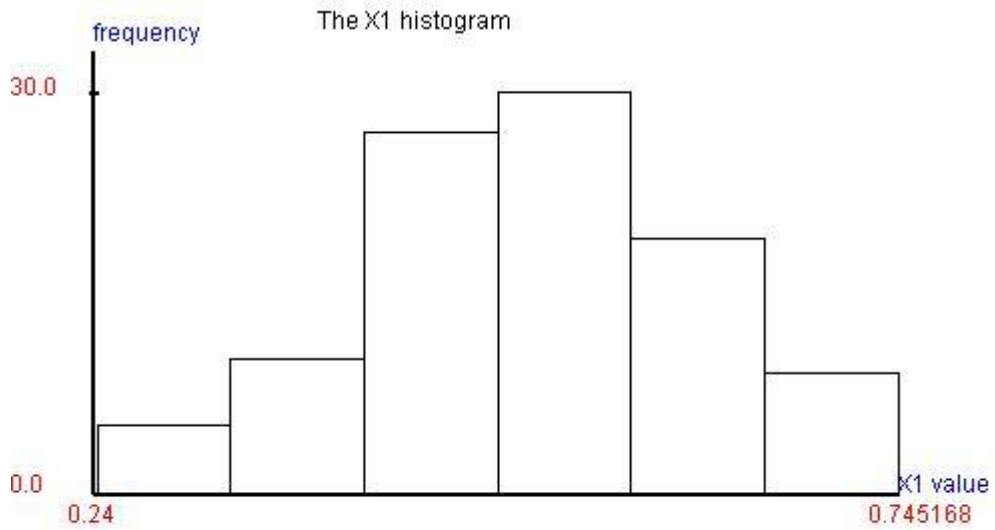


8.4.10)The population distribution is beta distribution.  
The alpha value is setting to 10 and beta value is setting to 12.  
X1 is Beta(alpha=10.500000,beta=10.500000),

| X1           |
|--------------|
| 0.4973915553 |
| 0.6108862764 |
| 0.4155234080 |
| 0.5940686415 |
| 0.5032133246 |
| 0.4237821217 |
| 0.6894221524 |
| 0.2829373714 |
| 0.5116860819 |
| 0.4593126881 |
| 0.4750888956 |
| 0.7003945825 |
| 0.3525427179 |
| 0.3422104550 |
| 0.5322930610 |
| 0.5938664400 |
| 0.4656678538 |
| 0.5383455069 |
| 0.6179943600 |
| 0.4366098292 |
| 0.6032496739 |
| 0.6483125842 |
| 0.4550781932 |
| 0.5741471102 |
| 0.3701051375 |
| 0.5209746653 |
| 0.5252623622 |
| 0.4307789606 |
| 0.6615852489 |
| 0.5995111929 |
| 0.4287362647 |
| 0.6840185680 |
| 0.7451688763 |
| 0.2493419485 |
| 0.2958429677 |
| 0.5228415558 |
| 0.5545192300 |
| 0.4759000361 |
| 0.5662437256 |
| 0.5389044129 |
| 0.4440898161 |
| 0.4035865958 |
| 0.4214949605 |
| 0.6796819720 |
| 0.5213870742 |
| 0.6790540433 |
| 0.4974152102 |
| 0.5301844225 |
| 0.3893809781 |
| 0.4671114703 |
| 0.5738736386 |
| 0.5762740469 |
| 0.5357388493 |
| 0.6032668296 |
| 0.4407211743 |
| 0.4282135539 |
| 0.5216233886 |
| 0.4287998892 |
| 0.6236680552 |
| 0.6373076465 |
| 0.3474765133 |
| 0.7228252141 |
| 0.5410867432 |
| 0.5137020761 |
| 0.2743659195 |
| 0.7233114054 |
| 0.6603745977 |
| 0.4984057585 |
| 0.5595400588 |
| 0.4404843811 |

0.4762779734  
0.4798604504  
0.4607567189  
0.6310630932  
0.5358638909  
0.4245232999  
0.3462414200  
0.5449705326  
0.4650948181  
0.4932805113  
0.5174218368  
0.5301739245  
0.3992909829  
0.6098263758  
0.4707789032  
0.5210282159  
0.6866188342  
0.6025131119  
0.3962523421  
0.4924858007  
0.4199394079  
0.4847861548  
0.5809744786  
0.3862977516  
0.3292091425  
0.5800819058  
0.5862261015  
0.5171296573  
0.5679994989  
0.5920971417

X1 is Beta(alpha=10.500000,beta=10.500000),



likelihood ratio goodness of fit

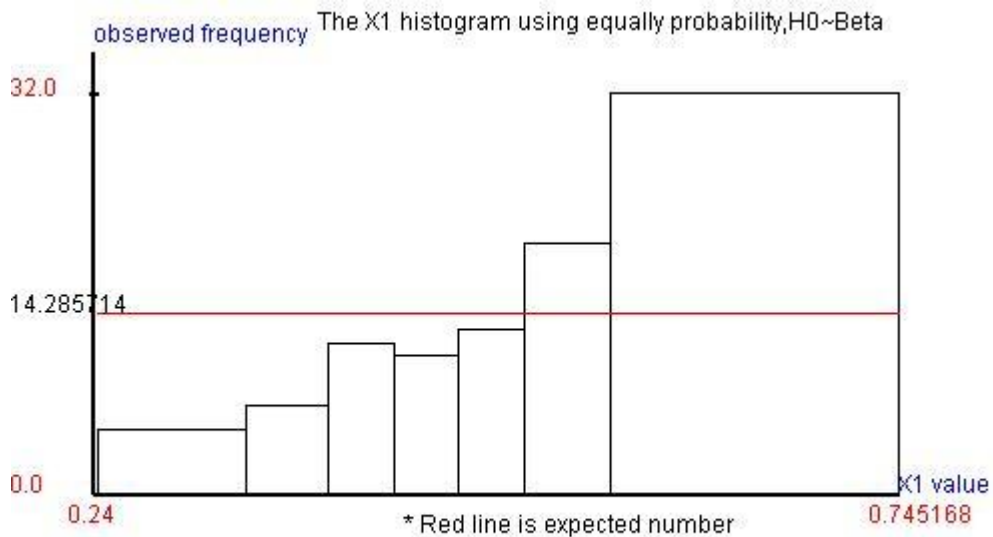
| class       | [ 1 ]    | [ 2 ]    | [ 3 ]    | [ 4 ]    |
|-------------|----------|----------|----------|----------|
| lower limit | 0.05346  | 0.34165  | 0.39306  | 0.43386  |
| 0.47250     | 0.51411  | 0.56785  |          |          |
| upper limit | 0.34165  | 0.39306  | 0.43386  | 0.47250  |
| 0.51411     | 0.56785  | 0.91621  |          |          |
| observed no | 5.00000  | 7.00000  | 12.00000 | 11.00000 |
| 13.00000    | 20.00000 | 32.00000 |          |          |
| probability | 0.14286  | 0.14286  | 0.14286  | 0.14286  |
| 0.14286     | 0.14286  | 0.14286  |          |          |
| expected no | 14.28571 | 14.28571 | 14.28571 | 14.28571 |
| 14.28571    | 14.28571 | 14.28571 |          |          |
| chi square  | 17.24490 | 7.58309  | 0.43537  | 0.98145  |
| 0.12716     | 1.63265  | 9.80612  |          |          |

degree of freedom=6

H0:  $X_1 \sim \text{Beta}(\alpha=10.000000, \beta=12.000000)$ ,

Likelihood ratio chi-square test statistic =37.810744

p-value=0.000000





8.4.11)The population distribution is cauchy distribution.

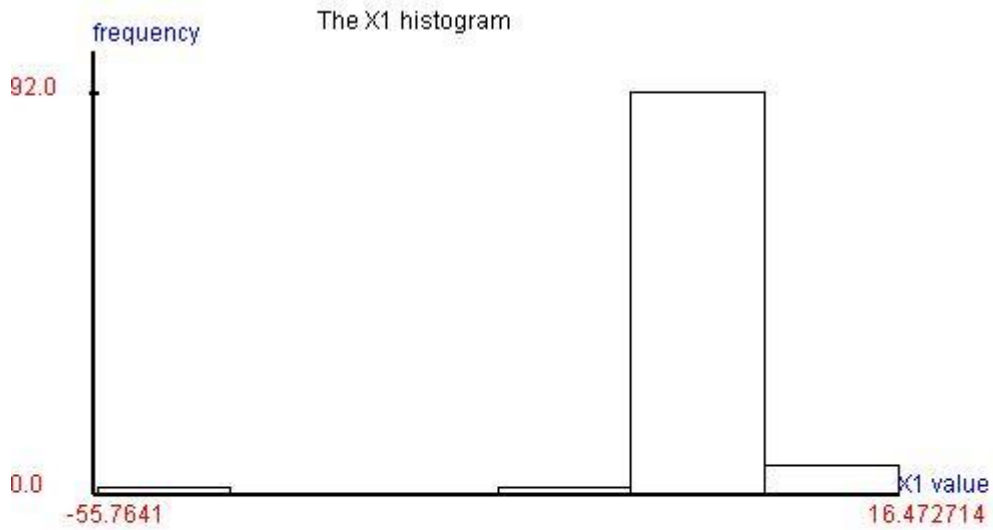
The mua value is setting to 0.01 and sigma value is setting to 1.01.

X1 is Cauchy( $\mu=0.000000$ , $\sigma=1.000000$ ),

| X1             |
|----------------|
| -0.3847857563  |
| -0.2151695723  |
| 2.2525406400   |
| 0.1895820442   |
| 0.2095138774   |
| -1.0524269864  |
| -2.0937257078  |
| 0.8794478384   |
| 9.7410249162   |
| -0.5369634532  |
| -5.6726714985  |
| -6.9829132141  |
| 0.8615103869   |
| 0.6089271039   |
| 2.6845867140   |
| 0.0388740827   |
| 0.2364453344   |
| 1.5450555167   |
| -1.2415434002  |
| 0.5637639339   |
| -0.2699324061  |
| -5.5402382005  |
| -2.5483890756  |
| 0.2778425815   |
| -1.0295102827  |
| -55.7641412726 |
| 0.9484540580   |
| 16.4727145410  |
| 4.0754165279   |
| -0.9426284604  |
| -1.4008242628  |
| 0.5297487916   |
| 2.0874840661   |
| -0.0673117281  |
| -1.8766448219  |
| -0.4122327632  |
| 1.3158799797   |
| 0.1890467450   |
| 5.0988506794   |
| 0.3061648855   |
| -0.0652612901  |
| 4.6496096184   |
| 0.6365720978   |
| -0.8951311121  |
| 0.7326922865   |
| -0.0067667150  |
| -1.2621558086  |
| 9.9984692284   |
| -0.5953137989  |
| 0.2060910758   |
| 0.1136392469   |
| 9.9142322254   |
| -1.3854784581  |
| 0.3022305415   |
| -0.3335425858  |
| 3.1003219327   |
| 4.1535284531   |
| -0.6512914556  |
| 1.2721312902   |
| -0.0194077097  |
| 2.9380072506   |
| -1.3794801935  |
| -0.5727913013  |
| 2.8204774094   |
| -0.9744781610  |
| 0.2435660910   |
| 0.0664738087   |
| -1.8620603569  |
| 1.6406948503   |
| 1.2708149865   |

1.7474854464  
 1.6940049057  
 0.3816584866  
 -1.6978960572  
 -0.4133994061  
 -0.4346984588  
 -3.1504038832  
 -1.1077265391  
 0.6228999939  
 -0.5644973383  
 -0.1233089603  
 -5.2216991042  
 -0.2891811227  
 -2.2986810556  
 4.1868986304  
 0.1650246324  
 -1.0168123297  
 1.4785552363  
 -0.7221887395  
 0.2926511630  
 0.5562107413  
 -0.5208519066  
 -3.5569963935  
 0.8055064384  
 -1.3912280616  
 0.7352131794  
 -8.2209042232  
 1.1629423583  
 -0.8451166432  
 -0.5915333820

X1 is Cauchy(mu=0.000000,sigma=1.000000),



likelihood ratio goodness of fit

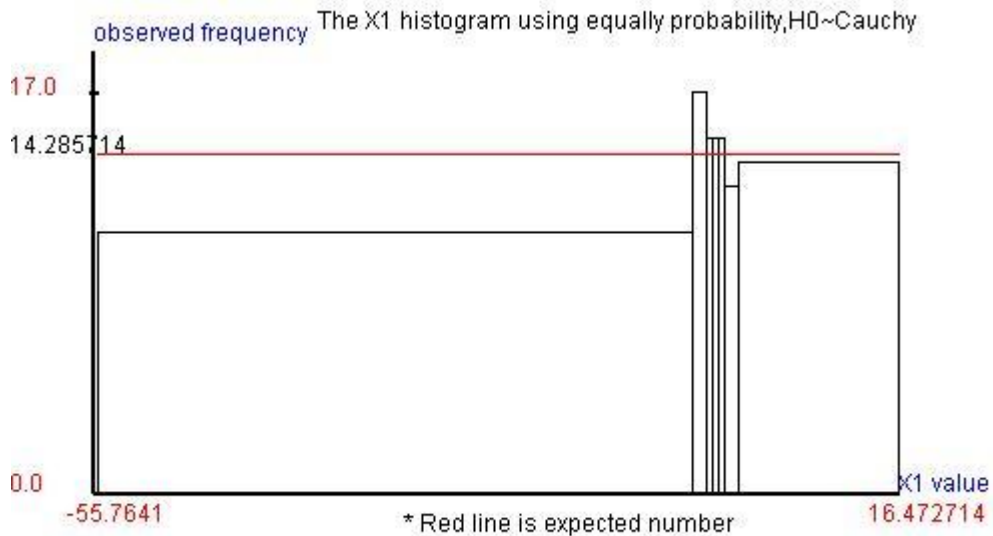
| class       | [ 1 ]    | [ 2 ]    | [ 3 ]    | [ 4 ]    |
|-------------|----------|----------|----------|----------|
| [ 5 ]       | [ 6 ]    | [ 7 ]    |          |          |
| lower limit |          | -2.08729 | -0.79545 | -0.22053 |
| 0.24053     | 0.81545  | 2.10729  |          |          |
| upper limit | -2.08729 | -0.79545 | -0.22053 | 0.24053  |
| 0.81545     | 2.10729  |          |          |          |
| observed no | 11.00000 | 17.00000 | 15.00000 | 15.00000 |
| 15.00000    | 13.00000 | 14.00000 |          |          |
| probability | 0.14286  | 0.14286  | 0.14286  | 0.14286  |
| 0.14286     | 0.14286  | 0.14286  |          |          |
| expected no | 14.28571 | 14.28571 | 14.28571 | 14.28571 |
| 14.28571    | 14.28571 | 14.28571 |          |          |
| chi square  | 0.98145  | 0.43337  | 0.03401  | 0.03401  |
| 0.03401     | 0.12716  | 0.00583  |          |          |

degree of freedom=6

H0:  $X_1 \sim \text{Cauchy}(\mu=0.010000, \sigma=1.010000)$ ,

Likelihood ratio chi-square test statistic = 1.649851

p-value=0.948900



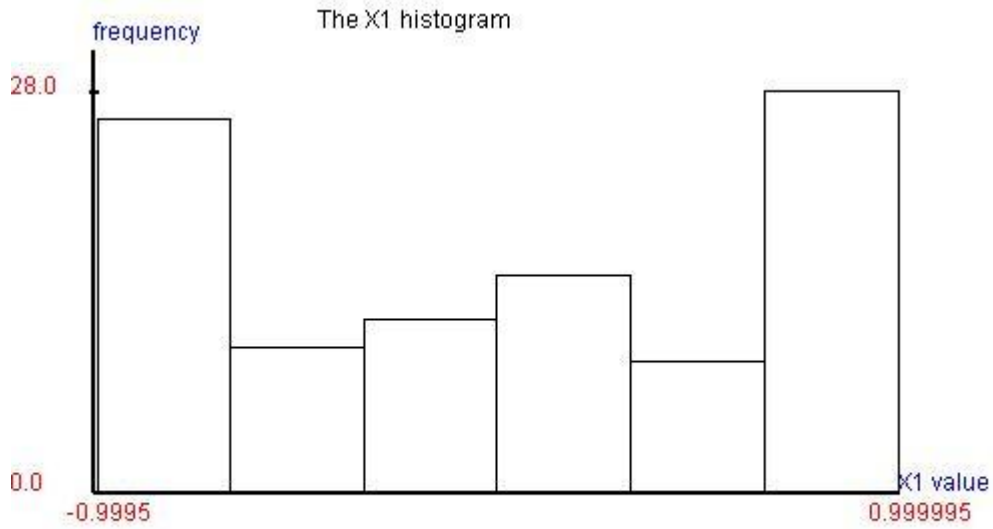
8.4.12)The population distribution is arcsin distribution.

X1 is Arcsin( $\mu=0.000000,c=1.000000$ ),

| X1            |
|---------------|
| 0.9907769214  |
| 0.9837387541  |
| -0.0513295262 |
| -0.9645162065 |
| -0.1881504762 |
| -0.3637881664 |
| -0.9967614251 |
| -0.8559346921 |
| -0.9089340053 |
| 0.9860394588  |
| 0.9999953952  |
| -0.5879676977 |
| -0.9572246502 |
| 0.0720243468  |
| 0.4694057073  |
| -0.5013381396 |
| 0.0825424295  |
| 0.7872479600  |
| 0.4230373729  |
| 0.9675168750  |
| 0.9697715232  |
| -0.9565933613 |
| 0.9469920332  |
| 0.5237852250  |
| 0.7221796086  |
| 0.9946553482  |
| -0.7904908407 |
| -0.9995738201 |
| 0.8906559542  |
| -0.7294104790 |
| -0.4188600322 |
| 0.1723480065  |
| -0.5102388485 |
| 0.8164476568  |
| 0.9670216165  |
| -0.7212022405 |
| -0.7383562945 |
| 0.9994626243  |
| 0.4083601991  |
| 0.9996973327  |
| 0.6834697526  |
| 0.2126921766  |
| -0.0139627856 |
| -0.9976843744 |
| 0.9997734461  |
| -0.3601173982 |
| -0.3776370359 |
| 0.8049111138  |
| 0.3123207672  |
| 0.3111817305  |
| -0.5232537538 |
| 0.3008470133  |
| 0.6332331865  |
| 0.2894071053  |
| -0.0927428473 |
| 0.5862681902  |
| 0.7556506692  |
| -0.1877339829 |
| 0.1978698076  |
| -0.8043563151 |
| 0.8326447679  |
| -0.9848364327 |
| -0.9149357352 |
| -0.1088292541 |
| -0.0569864719 |
| 0.9858212011  |
| -0.6621357936 |
| -0.8555929952 |
| 0.3443767579  |
| 0.2718383955  |
| -0.9911255879 |

-0.9955894666  
 -0.1555871810  
 -0.2244667492  
 0.0846714487  
 -0.9734155700  
 0.0162009670  
 0.0569121461  
 -0.0969892645  
 0.3638271712  
 -0.9982464757  
 0.7395716503  
 0.2161142690  
 0.8466478183  
 -0.9790795367  
 0.8274988313  
 -0.3294030657  
 0.9209785015  
 -0.6553226966  
 0.1087999670  
 0.9567594735  
 0.8320860598  
 -0.8591206414  
 -0.8326089494  
 0.4074448836  
 0.9967884173  
 -0.8678153432  
 -0.9088875394  
 -0.1665478411  
 -0.9887612312

X1 is Arcsin(mu=0.000000,c=1.000000).



likelihood ratio goodness of fit

| class       | [ 1 ]    | [ 2 ]    | [ 3 ]    | [ 4 ]    |
|-------------|----------|----------|----------|----------|
| lower limit | -0.99957 | -0.90056 | -0.62314 | -0.22226 |
| 0.22268     | 0.62357  | 0.90099  |          |          |
| upper limit | -0.90056 | -0.62314 | -0.22226 | 0.22268  |
| 0.62357     | 0.90099  | 1.00000  |          |          |
| observed no | 16.00000 | 12.00000 | 10.00000 | 20.00000 |
| 13.00000    | 13.00000 | 16.00000 |          |          |
| probability | 0.14286  | 0.14286  | 0.14286  | 0.14286  |
| 0.14286     | 0.14286  | 0.14286  |          |          |
| expected no | 14.28571 | 14.28571 | 14.28571 | 14.28571 |
| 14.28571    | 14.28571 | 14.28571 |          |          |
| chi square  | 0.18367  | 0.43537  | 1.83673  | 1.63265  |
| 0.12716     | 0.12716  | 0.18367  |          |          |

degree of freedom=4

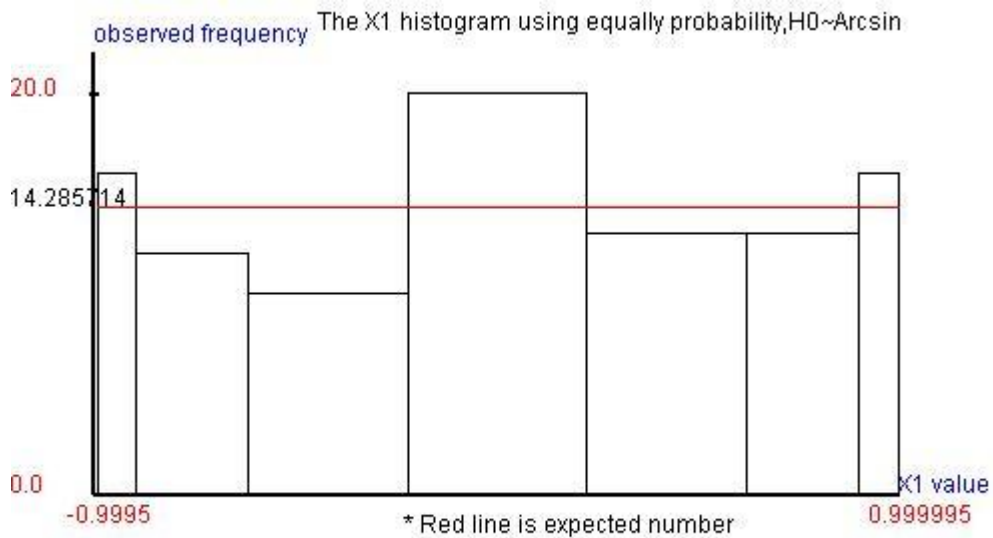
H0:  $X_1 \sim \text{Arcsin}(\mu, c)$ ,  $\mu, c$  are unknown

$\mu$  point estimated value=0.000211

$c$  point estimated value=0.999785 (MLE)

Likelihood ratio chi-square test statistic =4.526426

p-value=0.339400

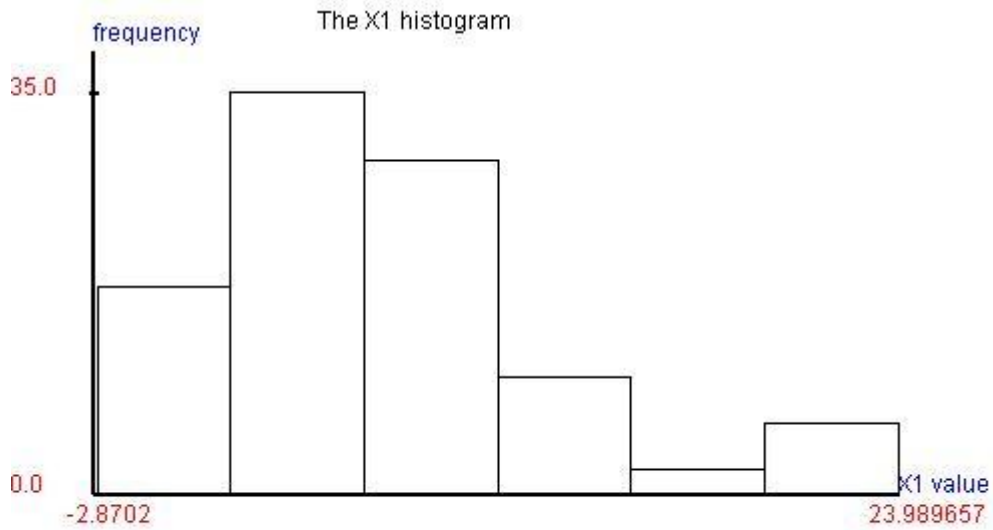


8.4.13)The population distribution is gumbel distribution.  
The  $\mu$  value is setting to 4.2 and  $\sigma$  value is setting to 5.5.  
 $X_1$  is Gumbel( $\mu=4.000000,\sigma=5.000000$ ),

| $X_1$         |
|---------------|
| 1.7598929176  |
| 4.7737386275  |
| 1.2120057337  |
| 6.0206000180  |
| 7.0260269498  |
| 9.3791455469  |
| 23.7809089754 |
| 7.2628145493  |
| 7.2539237702  |
| 4.6141412509  |
| 7.3763907125  |
| 1.0785999387  |
| -1.1846955484 |
| 20.5787255991 |
| 4.6742211770  |
| 20.5180120179 |
| 1.4028739819  |
| 8.3588358827  |
| 11.3273634671 |
| 11.5152729372 |
| 0.7893450288  |
| 10.0976195231 |
| 19.2877354175 |
| 5.5469856186  |
| 2.8570458416  |
| 0.8256341138  |
| 5.6327928818  |
| 3.4995989717  |
| 15.3365522922 |
| 12.6013205878 |
| 5.4097859357  |
| 3.9975842505  |
| 9.7232204601  |
| 10.3101717089 |
| 2.1341733782  |
| -0.7843071150 |
| 21.7816658177 |
| 5.2168789849  |
| 9.7822994749  |
| 11.9299126422 |
| -2.8702124003 |
| 3.3243799387  |
| 6.4330200599  |
| 13.8830810104 |
| 10.1105626245 |
| 4.1540579349  |
| 5.9335711386  |
| -0.2266279467 |
| 5.2074126530  |
| 12.0837873477 |
| 23.9896575118 |
| 1.8134139845  |
| 5.2447185273  |
| -1.1998175573 |
| 9.3492540652  |
| 9.2404242981  |
| 2.3528175160  |
| 5.4894109337  |
| 3.4817339070  |
| 3.9886981974  |
| 3.4809867633  |
| 10.4177696525 |
| 5.9207754672  |
| 5.2924264319  |
| 8.2340128554  |
| 14.0029160503 |
| 8.7624263448  |
| 5.8548778219  |
| 8.7335948754  |
| 8.5620062409  |

5.1187170948  
 8.3947193510  
 -0.5957676453  
 -0.3144686995  
 6.9463231240  
 11.7578048153  
 -1.3828194899  
 -1.7315820440  
 7.4027935829  
 1.2443712300  
 2.3581265407  
 12.6070236322  
 0.3843540965  
 7.9068489962  
 3.8406114483  
 3.2211530241  
 8.4476734835  
 7.3725663548  
 2.8030460761  
 6.5134615418  
 0.8177554261  
 7.2168736116  
 21.1351809719  
 1.7903224597  
 -0.4401135074  
 3.9190126024  
 12.8092807400  
 6.3703319754  
 2.7867861533  
 9.2309882033

X1 is Gumbel( $\mu=4.000000$ , $\sigma=5.000000$ ),





likelihood ratio goodness of fit

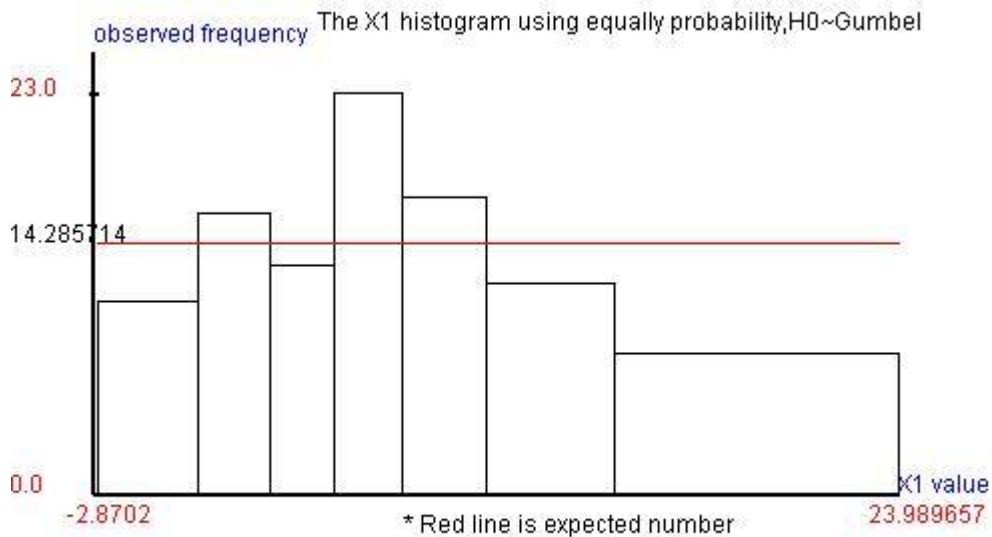
| class       | [ 1 ]    | [ 2 ]    | [ 3 ]    | [ 4 ]    |
|-------------|----------|----------|----------|----------|
| lower limit | 7.39278  | 10.19082 | 14.48404 | 17.00000 |
| upper limit | 10.19082 | 14.48404 | 17.00000 | 23.00000 |
| observed no | 11.00000 | 16.00000 | 13.00000 | 23.00000 |
| probability | 0.14286  | 0.14286  | 0.14286  | 0.14286  |
| expected no | 14.28571 | 14.28571 | 14.28571 | 14.28571 |
| chi square  | 0.98145  | 0.18367  | 0.12716  | 3.30169  |

degree of freedom=6

H0: X1~Gumbel(mu=4.200000,sigma=5.500000),

Likelihood ratio chi-square test statistic =10.401488

p-value=0.108700



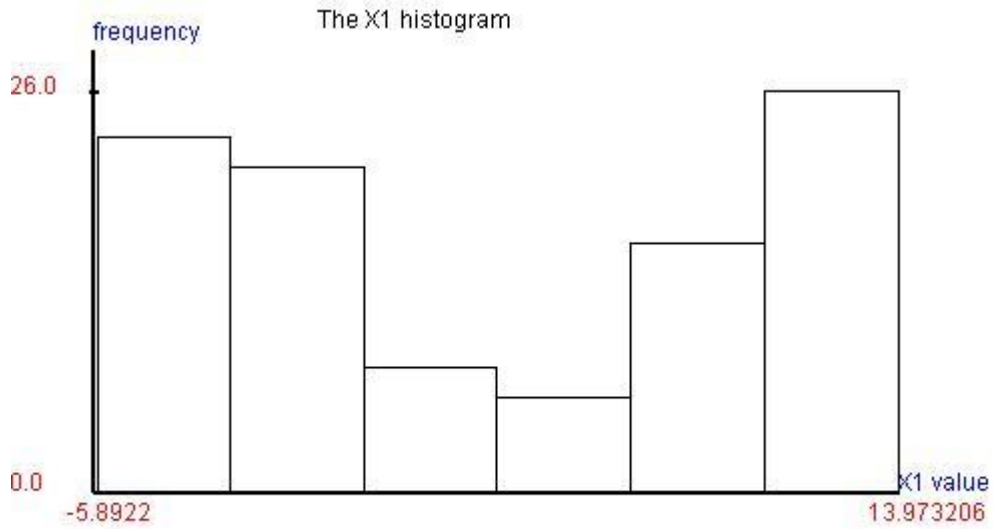
8.4.14) The population distribution is triangular 1 distribution.

X1 is  $\text{Triangular1}(\mu=4.000000, c=10.000000)$ ,

| X1            |
|---------------|
| 9.8633649697  |
| -4.4345203308 |
| -2.2804504826 |
| 2.2190209223  |
| -2.4874824653 |
| 5.4203630049  |
| 13.2743261032 |
| 12.3589977006 |
| -0.7930036180 |
| 8.4635230266  |
| -0.9267534292 |
| 12.6890387125 |
| -0.8298207323 |
| 10.9865228149 |
| -3.7472900529 |
| 9.7172829513  |
| -1.7583134109 |
| -4.3574610893 |
| -5.1639241848 |
| 11.2957147707 |
| 10.5351772893 |
| -4.8293365698 |
| 10.3830121526 |
| 13.7791067082 |
| -1.4983092660 |
| -1.8652381308 |
| 10.9498171625 |
| -3.4845294073 |
| 1.5581022230  |
| -0.8588151663 |
| 13.6335448109 |
| 7.4511279011  |
| 12.7833942783 |
| -2.2598863556 |
| 1.9968425679  |
| -1.3422447555 |
| -2.5908039797 |
| -2.5168169184 |
| -4.9539762505 |
| 6.6456264220  |
| 9.5409499011  |
| 5.9652223267  |
| 12.0138286840 |
| 13.7027317673 |
| 12.3121529627 |
| -4.5335908967 |
| 0.7794951473  |
| -5.8170450686 |
| -3.2421545527 |
| -2.2188843680 |
| -1.9377961724 |
| -4.3507372876 |
| 6.8847769074  |
| 0.5405447870  |
| -3.8009341700 |
| 10.8418901881 |
| 10.7244560027 |
| 5.8168753442  |
| -3.6845238710 |
| 13.1027282875 |
| -5.2836964373 |
| 6.9725928575  |
| 13.9732068890 |
| 9.3659176121  |
| -1.7800347547 |
| -4.4609724657 |
| 13.6364056079 |
| 12.0420968187 |
| -5.3241283051 |
| -3.1186949442 |
| 8.5615022980  |

7.7580356135  
 -5.7486683363  
 13.4223948657  
 -5.8922547243  
 8.8229846381  
 10.8132095864  
 0.1626698865  
 2.4307548909  
 12.0370699127  
 -4.4241886747  
 -5.2164994629  
 -2.5478441283  
 12.1196203502  
 1.1953269123  
 9.7851943898  
 1.3886547262  
 12.0169929575  
 -5.5993828300  
 10.4345673877  
 12.8179766342  
 0.0938284352  
 1.0590695711  
 12.7566098409  
 -1.8854652761  
 10.6690808485  
 9.9924699539  
 9.5215517185  
 10.3735755000  
 -0.2959834799

X1 is Triangular1(mu=4.000000,c=10.000000),



likelihood ratio goodness of fit

| class       | [ 1 ]    | [ 2 ]    | [ 3 ]    | [ 4 ]    |
|-------------|----------|----------|----------|----------|
| [ 5 ]       | [ 6 ]    | [ 7 ]    |          |          |
| lower limit | -5.89225 | -4.35421 | -2.46202 | 0.28626  |
| 7.79470     | 10.54297 | 12.43517 |          |          |
| upper limit | -4.35421 | -2.46202 | 0.28626  | 7.79470  |
| 10.54297    | 12.43517 | 13.97321 |          |          |
| observed no | 15.00000 | 11.00000 | 17.00000 | 17.00000 |
| 14.00000    | 14.00000 | 12.00000 |          |          |
| probability | 0.14286  | 0.14286  | 0.14286  | 0.14286  |
| 0.14286     | 0.14286  | 0.14286  |          |          |
| expected no | 14.28571 | 14.28571 | 14.28571 | 14.28571 |
| 14.28571    | 14.28571 | 14.28571 |          |          |
| chi square  | 0.03401  | 0.98145  | 0.43337  | 0.43337  |
| 0.00583     | 0.00583  | 0.43537  |          |          |

degree of freedom=4

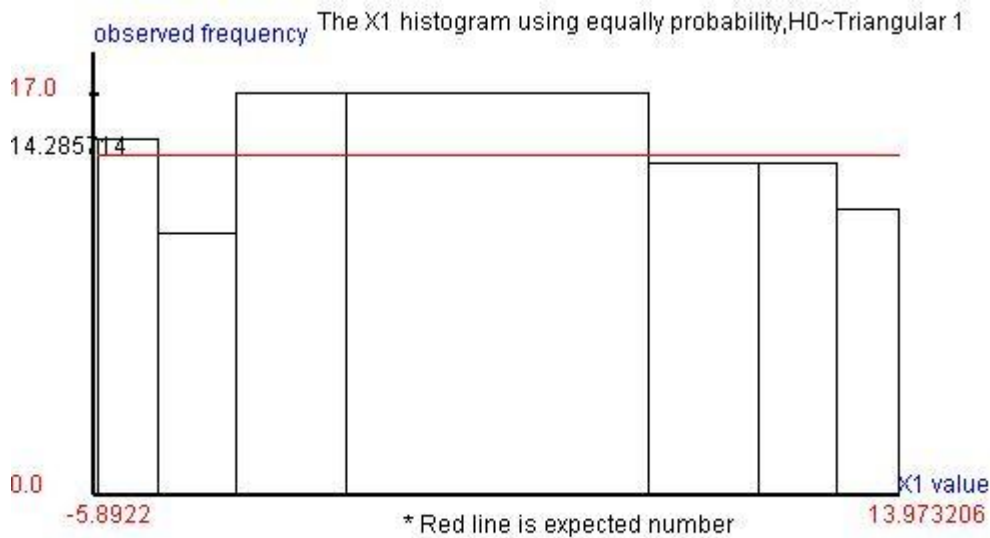
H0:  $X_1 \sim \text{Triangular 1}(\mu, c)$ ,  $\mu, c$  are unknown

$\mu$  point estimated value=4.040476 (MLE)

$c$  point estimated value=9.932731 (MLE)

Likelihood ratio chi-square test statistic =2.329243

p-value=0.675400



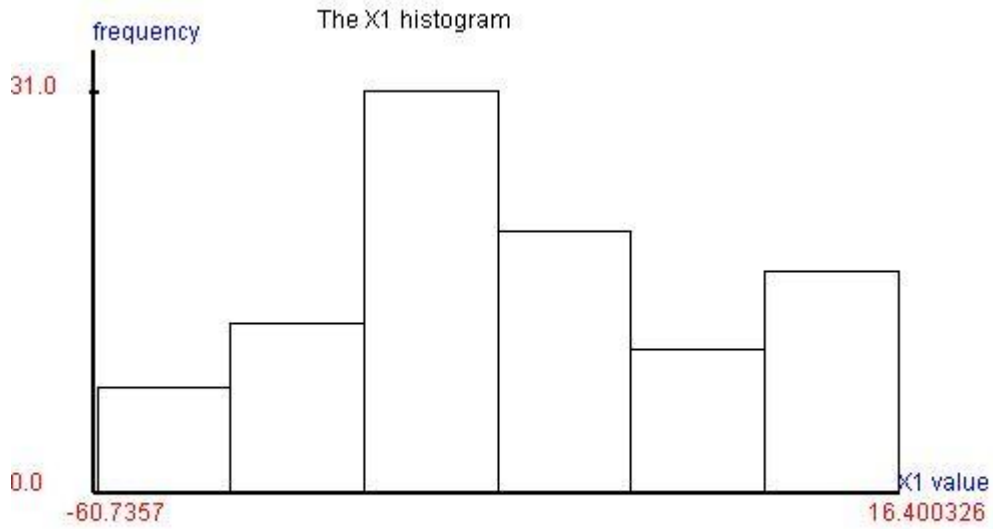
8.4.15)The population distribution is trapezoid distribution.

X1 is Trapezoid( $\mu=-20.000000,c=30.000000$ ),

X1  
4.8604415285  
-21.0919520548  
-22.9687188985  
-17.9399404526  
-2.8980742862  
-28.9508742518  
-44.2056377336  
13.4803763006  
-41.2405157171  
-29.1730611087  
16.1883945694  
16.4003261154  
-30.9436797575  
-23.8737025606  
-39.1859857988  
-31.4553054734  
-25.9858077671  
-35.8182210847  
-51.2176068546  
-31.7294040103  
-24.3185845282  
-12.5257755133  
-24.4504357216  
-28.7097676141  
-60.7357920490  
11.5788089148  
-8.9245493527  
8.1519337476  
-12.0522705610  
4.8292200301  
-27.5699204809  
-38.4002500703  
-11.0133203193  
6.6394912179  
-22.5728588014  
-26.5148635968  
-1.7188326861  
-27.6173661222  
-38.4775470575  
-11.3956651137  
-24.2621657914  
-33.8723581238  
-10.9420711907  
-16.8363349132  
-32.8899310079  
-17.6081883932  
-0.6967982233  
-29.4598819719  
16.1160536722  
-49.1706438068  
-27.8641402781  
-11.4231119339  
-53.6471832895  
-33.4317800791  
-51.8412624589  
6.3429431828  
-14.0791548299  
-29.8937260875  
-10.3348894669  
-24.4869168849  
-5.8968497441  
-55.1052081142  
-7.3556280332  
-55.7029124607  
-27.0014714370  
-37.4452113670  
-11.0144689895  
-31.2037674234  
7.8017050757  
8.0394911002  
-4.4142655765

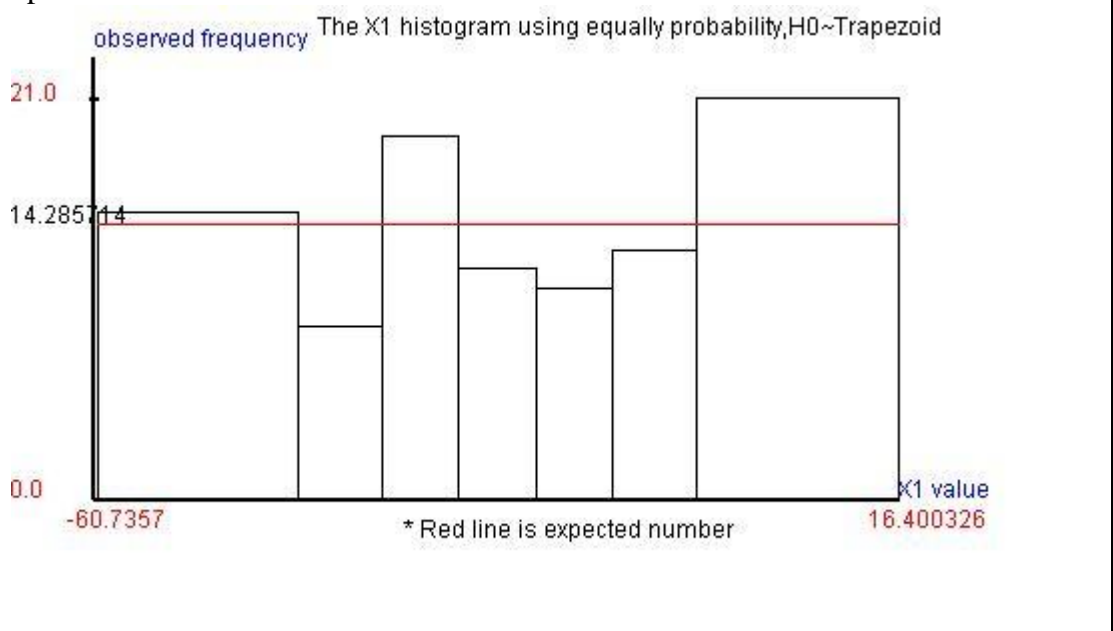
-13.0874210541  
 -29.5655171631  
 -47.3380224652  
 -10.7259830994  
 -20.2437325968  
 -46.2587814711  
 4.9519759960  
 -25.3939532887  
 8.6154433373  
 -18.7954946353  
 -6.7305725981  
 -30.9559475333  
 -49.9768774462  
 5.2152524603  
 -2.0136048625  
 -34.2870925130  
 -10.0313324978  
 -42.7585214014  
 -27.9568919771  
 -6.3565785888  
 -43.5055278755  
 -42.1980850760  
 -15.8747969985  
 -24.5157304857  
 -42.8199933568  
 -8.0488662538  
 5.7897756518  
 -17.2425885899  
 12.7068159119

X1 is Trapezoid(mu=-20.000000,c=30.000000),



| likelihood ratio goodness of fit |           |           |           |           |           |
|----------------------------------|-----------|-----------|-----------|-----------|-----------|
| class                            | [ 1 ]     | [ 2 ]     | [ 3 ]     | [ 4 ]     | [ 5 ]     |
|                                  | [ 6 ]     | [ 7 ]     |           |           |           |
| lower limit                      | -60.73579 | -41.29932 | -33.18718 | -25.84088 | -18.49458 |
|                                  | -11.14829 | -3.03615  |           |           |           |
| upper limit                      | -41.29932 | -33.18718 | -25.84088 | -18.49458 | -11.14829 |
|                                  | -3.03615  | 16.40033  |           |           |           |
| observed no                      | 15.00000  | 9.00000   | 19.00000  | 12.00000  |           |
|                                  | 11.00000  | 13.00000  | 21.00000  |           |           |
| probability                      | 0.14286   | 0.14286   | 0.14286   | 0.14286   |           |
|                                  | 0.14286   | 0.14286   |           |           |           |
| expected no                      | 14.28571  | 14.28571  | 14.28571  | 14.28571  |           |
|                                  | 14.28571  | 14.28571  |           |           |           |
| chi square                       | 0.03401   | 3.10431   | 1.16971   | 0.43537   |           |
|                                  | 0.98145   | 0.12716   | 2.14674   |           |           |

degree of freedom=4  
 H0:  $X_1 \sim \text{Trapezoid}(\mu, c)$ ,  $\mu, c$  are unknown  
 $\mu$  point estimated value = -22.167733 (MLE)  
 $c$  point estimated value = 25.712039 (MLE)  
 Likelihood ratio chi-square test statistic = 7.998756  
 p-value = 0.091600



8.4.16) The population distribution is U quadratic distribution.

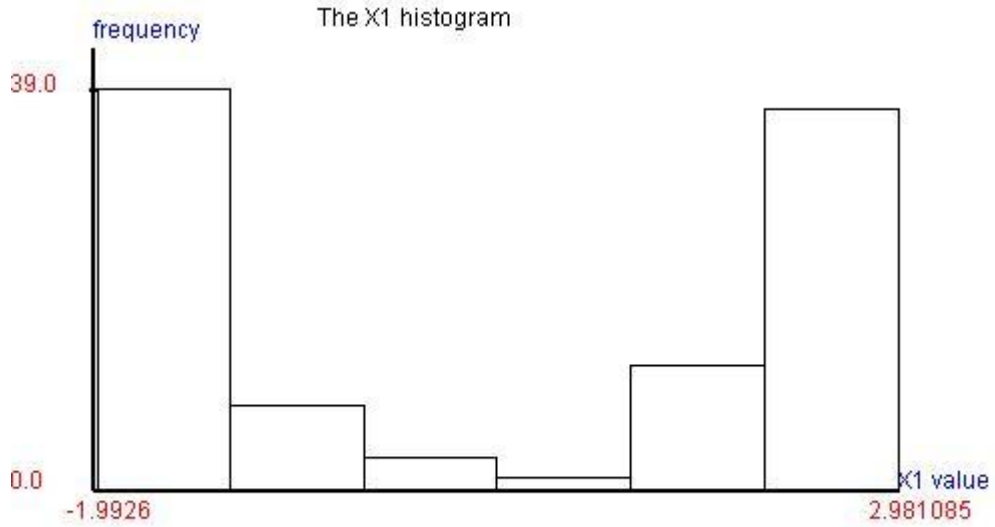
X1 is U-quadratic(a=-2.000000,b=3.000000),

X1  
-1.9589726698  
-0.1688615639  
2.8536248154  
-1.5250936601  
-1.9862612500  
1.3619526418  
2.7959734668  
-1.4757236301  
2.9796521044  
-1.7061326733  
2.3387256505  
-1.3119353194  
0.0577646132  
2.3837523405  
-1.2155490937  
-0.3930671487  
-1.2288782111  
-1.4090192552  
2.9175871144  
2.6455828456  
2.7290039672  
-1.9926838297  
2.6343051612  
-1.9358325119  
2.8012720409  
-1.7151730393  
1.7098666252  
-0.1574297786  
1.4248766600  
2.6382901852  
2.1415894887  
2.6925325187  
-1.1479164759  
1.8945798888  
1.7732763864  
-1.8673586159  
2.6546811824  
2.8659679944  
-1.6314622618  
-0.9528435863  
2.3109157919  
2.6599089950  
-1.7737642906  
2.5709126689  
-1.8720296581  
2.8018643820  
2.1795843423  
2.9351111286  
-1.2790052455  
2.4293188572  
2.9810855132  
-1.5036074800  
-1.9038175170  
-1.5999299313  
2.5421865139  
-1.2853464185  
-1.8101908350  
-1.3265248045  
2.8002848890  
2.5421443909  
-1.6644424182  
2.5948427384  
1.9718505919  
-1.8129356420  
-0.7628268346  
2.0274461197  
-1.6197618760  
-1.1261327409  
2.8321996541  
-1.7517324080  
-1.8427446894



-1.9752205327  
 -1.8311707723  
 1.5548031879  
 1.8966185607  
 -1.6368957571  
 2.9650329126  
 2.8420431497  
 -1.3914008873  
 -1.9777745598  
 -0.9461366888  
 2.6166193395  
 1.8044699984  
 -1.0391671463  
 2.4673148279  
 -1.4343596737  
 2.2541533399  
 1.1860007534  
 2.7652986306  
 2.5969338757  
 -1.3534078219  
 2.6452315469  
 -1.2700762936  
 2.2030305710  
 1.5548743820  
 2.4928393967  
 -1.4430059834  
 -1.7035595137  
 -0.7680308593  
 -1.5036744012

X1 is U-quadratic(a=-2.000000,b=3.000000),



likelihood ratio goodness of fit

| class       | [ 1 ]    | [ 2 ]    | [ 3 ]    | [ 4 ]    |
|-------------|----------|----------|----------|----------|
| lower limit | 1.79427  | -1.72896 | -1.38074 | -0.80602 |
| upper limit | 2.36895  | 2.71713  | -0.80602 | 1.79427  |
| observed no | 15.00000 | 16.00000 | 13.00000 | 13.00000 |
| expected no | 14.28571 | 14.28571 | 14.28571 | 14.28571 |
| chi square  | 0.03401  | 0.18367  | 0.12716  | 0.12716  |

degree of freedom=4

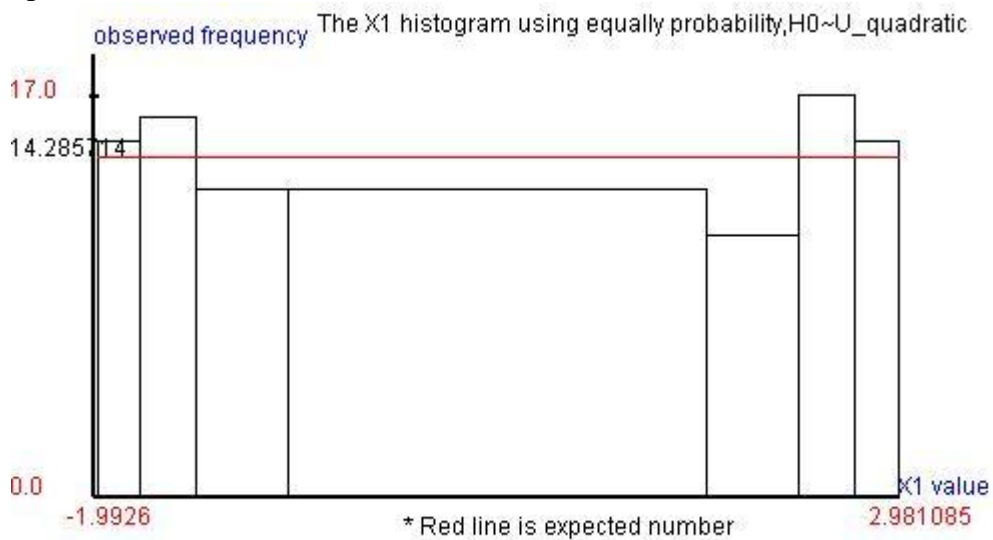
H0:  $X_1 \sim U\_quadratic(a,b)$ , a,b are unknown

a point estimated value=-1.992684 (MLE)

b point estimated value=2.981086 (MLE)

Likelihood ratio chi-square test statistic =1.920838

p-value=0.750200



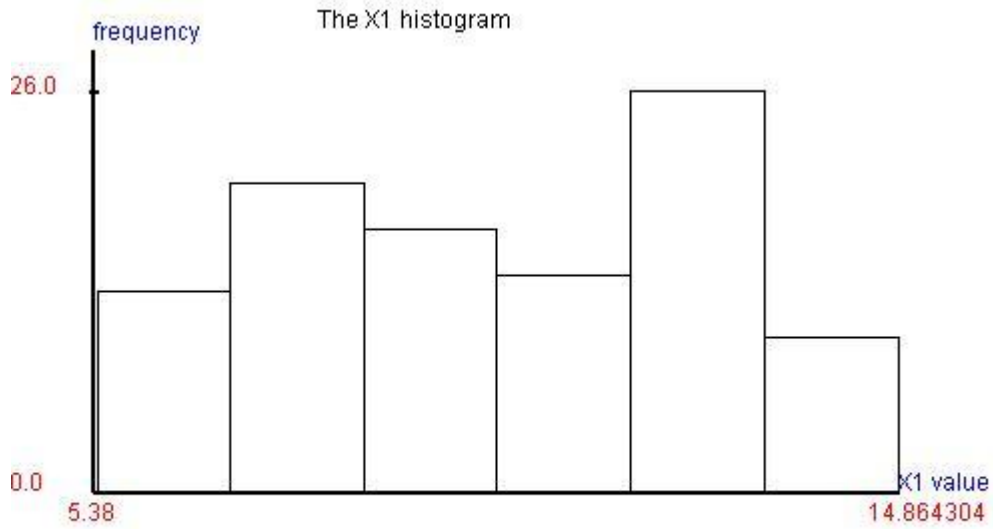
8.4.17)The population distribution is semi circle distribution.

X1 is Semi-circle( $\mu=10.000000,R=5.000000$ ),

| X1            |
|---------------|
| 9.0485339606  |
| 11.3485875004 |
| 10.0991592216 |
| 11.0084848712 |
| 5.8169187792  |
| 10.5116623180 |
| 12.2165961291 |
| 12.0716170291 |
| 14.5753794592 |
| 12.5163350377 |
| 12.6298321419 |
| 8.9043418555  |
| 6.6176369061  |
| 10.8867504261 |
| 7.2482691982  |
| 9.2282993186  |
| 12.6435597720 |
| 5.3871093852  |
| 12.1232688384 |
| 8.5097245054  |
| 10.0352329042 |
| 10.7266339191 |
| 11.6571365724 |
| 12.8145047428 |
| 5.9255379067  |
| 9.6940316246  |
| 13.6144632915 |
| 13.2848632232 |
| 11.9403983115 |
| 10.1056135948 |
| 8.2062811145  |
| 10.1206572553 |
| 6.4614270694  |
| 13.3612901096 |
| 10.6117699650 |
| 6.4288837953  |
| 7.0960479213  |
| 9.4901132339  |
| 6.9806718464  |
| 10.7878054085 |
| 12.1403835883 |
| 11.5390801244 |
| 13.0602509935 |
| 11.8998660628 |
| 8.4631859960  |
| 8.4868336927  |
| 12.2142392036 |
| 12.7830640707 |
| 10.9514743622 |
| 10.8791863701 |
| 13.8153687897 |
| 9.5104361643  |
| 11.1471736172 |
| 9.8711373531  |
| 8.5574665210  |
| 7.2136416052  |
| 13.6570405229 |
| 12.3100337700 |
| 8.7362780831  |
| 12.8754095305 |
| 8.4212847075  |
| 8.6364433896  |
| 14.8097220590 |
| 7.4929990896  |
| 6.6903757112  |
| 9.7445404666  |
| 9.8788134069  |
| 7.5466563358  |
| 13.1921655327 |
| 14.1731334554 |
| 7.2201849473  |

11.8444661454  
 10.5067479932  
 11.7807030495  
 11.7400631492  
 12.6498371536  
 12.9807380021  
 12.5141973773  
 5.4955772839  
 13.2758819635  
 8.2301850438  
 7.7732422464  
 8.4990611995  
 8.7290291677  
 6.6711931796  
 5.6322277788  
 8.0358252191  
 12.4763005208  
 5.8516858680  
 14.0267810775  
 6.4436875729  
 12.4905631442  
 6.3494259393  
 8.1441680524  
 11.7423538721  
 11.5010064587  
 7.0365532221  
 8.2850271488  
 14.8643047953  
 7.6094033111

X1 is Semi-circle(mu=10.000000,R=5.000000),



likelihood ratio goodness of fit

| class       | [ 1 ]    | [ 2 ]    | [ 3 ]    | [ 4 ]    |
|-------------|----------|----------|----------|----------|
| lower limit | 7.28652  | 8.49814  | 9.59250  |          |
| 10.65844    | 11.75280 | 12.96413 |          |          |
| upper limit | 7.28652  | 8.49814  | 9.59250  | 10.65844 |
| 11.75280    | 12.96413 |          |          |          |
| observed no | 19.00000 | 12.00000 | 11.00000 | 11.00000 |
| 13.00000    | 20.00000 | 14.00000 |          |          |
| probability | 0.14286  | 0.14286  | 0.14286  | 0.14286  |
| 0.14286     | 0.14286  | 0.14286  |          |          |
| expected no | 14.28571 | 14.28571 | 14.28571 | 14.28571 |
| 14.28571    | 14.28571 | 14.28571 |          |          |
| chi square  | 1.16971  | 0.43537  | 0.98145  | 0.98145  |
| 0.12716     | 1.63265  | 0.00583  |          |          |

degree of freedom=4

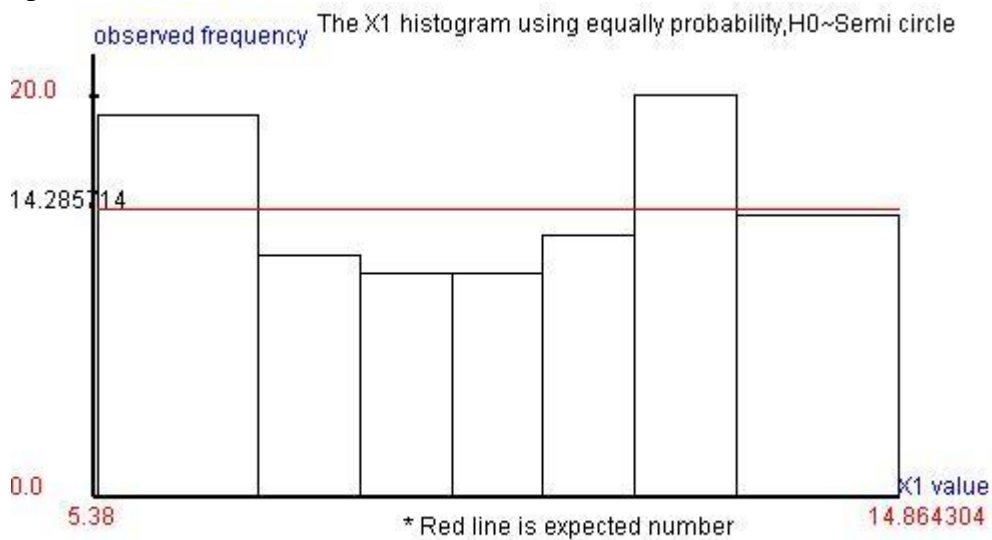
H0:  $X_1 \sim \text{Semi-circle}(\mu, R)$ ,  $\mu, R$  are unknown

$\mu$  point estimated value=10.125707(MLE)

$R$  point estimated value=4.738598 (MLE)

Likelihood ratio chi-square test statistic =5.333621

p-value=0.254700



8.4.18)The population distribution is logistic distribution.

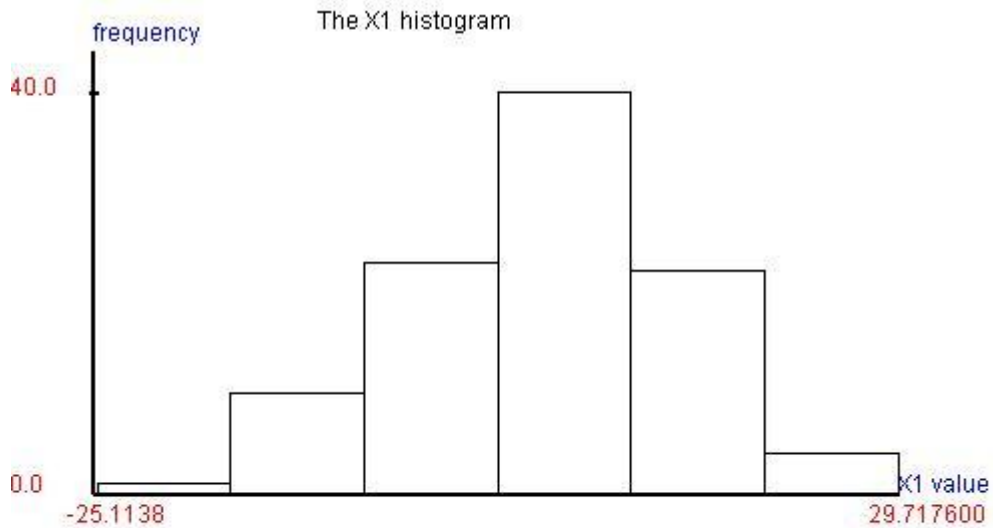
The  $\mu$  value is setting to 4.2 and  $\sigma$  value is setting to 5.5.

$X_1$  is Logistic( $\mu=4.000000,\sigma=6.000000$ ),

| $X_1$          |
|----------------|
| -12.1746851607 |
| 4.8201663901   |
| 1.1977798449   |
| -0.9817749997  |
| 4.7371335493   |
| -15.3948373354 |
| 28.0742108778  |
| 10.6550255939  |
| 16.7303724838  |
| -25.1138935098 |
| 28.6533657066  |
| -2.8088692701  |
| 10.5160732618  |
| 14.8996809063  |
| 3.7919674043   |
| 14.5354006947  |
| -2.6754886244  |
| -2.2955673361  |
| -10.2446428201 |
| -3.6835928155  |
| 14.6166829808  |
| 17.3640522957  |
| 16.6183671809  |
| -7.1604125185  |
| 9.6255446517   |
| 19.5357671943  |
| -1.9011071816  |
| 11.7297073013  |
| 8.5857522101   |
| 23.0772318234  |
| -6.6768119338  |
| 5.8027232540   |
| 17.7676038013  |
| 10.5615136514  |
| 18.5474188193  |
| 6.9161875720   |
| -7.1476784966  |
| 3.4520705440   |
| -11.7915801354 |
| 13.5355153249  |
| 5.2884735979   |
| 7.9276666938   |
| -0.6773020829  |
| 7.5671426430   |
| -0.7100800177  |
| 8.3644252798   |
| 2.8966935674   |
| 0.7412264138   |
| 7.7168898003   |
| 20.1980528967  |
| 11.9611459407  |
| -6.4269975035  |
| 7.4592695222   |
| 8.2990138462   |
| -4.1816188396  |
| 5.9829041164   |
| 5.9224147655   |
| -0.2897892447  |
| 14.0932595359  |
| 18.5704870327  |
| 29.7176000344  |
| 6.8381255338   |
| 2.6122963061   |
| 19.2433715204  |
| -3.1706778184  |
| -5.8061471304  |
| -2.2465213845  |
| 0.4387920615   |
| 12.1088297831  |
| 12.4457665423  |

7.7327602731  
 -15.4856586984  
 -7.1674491433  
 3.6323959997  
 -2.6740485435  
 6.6263100369  
 -4.1677917881  
 12.5257735258  
 11.8490185657  
 14.6361533477  
 5.7285335007  
 -5.3373436582  
 5.1453140572  
 8.8360490184  
 -5.1964688720  
 6.2979087756  
 1.9212244071  
 10.6595477684  
 9.4709524638  
 -10.9838044995  
 5.7474667615  
 6.8663615947  
 7.0524814758  
 8.2061372784  
 13.2054768646  
 7.9554384220  
 10.2798819131  
 -7.8195263454  
 4.5645690930  
 4.3720020127

X1 is Logistic(mu=4.000000,sigma=6.000000),



likelihood ratio goodness of fit

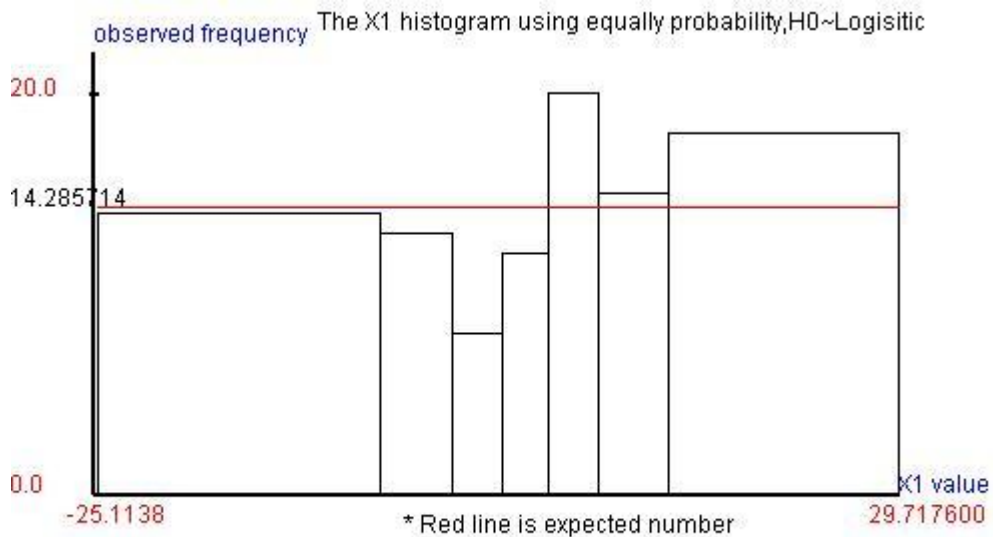
| class       | [ 1 ]    | [ 2 ]    | [ 3 ]    | [ 4 ]    |
|-------------|----------|----------|----------|----------|
| lower limit |          | -5.65468 | -0.83960 | 2.61775  |
| 5.78225     | 9.23960  | 14.05468 |          |          |
| upper limit | -5.65468 | -0.83960 | 2.61775  | 5.78225  |
| 9.23960     | 14.05468 |          |          |          |
| observed no | 14.00000 | 13.00000 | 8.00000  | 12.00000 |
| 20.00000    | 15.00000 | 18.00000 |          |          |
| probability | 0.14286  | 0.14286  | 0.14286  | 0.14286  |
| 0.14286     | 0.14286  | 0.14286  |          |          |
| expected no | 14.28571 | 14.28571 | 14.28571 | 14.28571 |
| 14.28571    | 14.28571 | 14.28571 |          |          |
| chi square  | 0.00583  | 0.12716  | 4.93878  | 0.43537  |
| 1.63265     | 0.03401  | 0.76644  |          |          |

degree of freedom=6

H0:  $X_1 \sim \text{Logistic}(\mu=4.200000, \sigma=5.500000)$ ,

Likelihood ratio chi-square test statistic =7.940246

p-value=0.242500





8.4.19)The population distribution is weibull distribution.

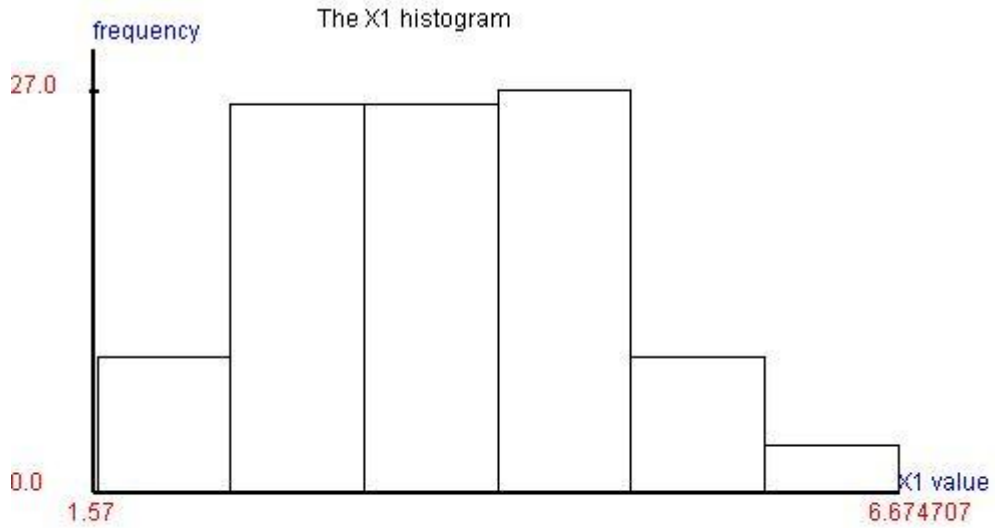
X1 is Weibull(alpha=1.000000,beta=3.000000,gamma=3.000000),

The parameter of alpha value is supposed to 2 and beta is 3 and gamma is 2.

| X1           |
|--------------|
| 1.5743704616 |
| 3.7213122026 |
| 2.5693180627 |
| 3.2044367692 |
| 2.9934836738 |
| 5.2775857349 |
| 4.8508547962 |
| 5.1478577575 |
| 4.0204314423 |
| 1.9534430359 |
| 5.4900670906 |
| 2.1564886051 |
| 3.5811788638 |
| 2.3274472160 |
| 4.4221615178 |
| 3.7935076288 |
| 5.3052521432 |
| 2.6942682257 |
| 3.2865762296 |
| 2.1308838259 |
| 2.2039137621 |
| 2.9276916860 |
| 3.7361516041 |
| 4.4109383681 |
| 4.8454131509 |
| 3.9928532820 |
| 3.2212665599 |
| 4.1973355561 |
| 2.7536889526 |
| 5.3579687998 |
| 4.7899523266 |
| 2.7348294980 |
| 3.6746736631 |
| 3.1475091912 |
| 3.6912629150 |
| 4.4673758003 |
| 3.7762522476 |
| 5.0687725596 |
| 3.7270539621 |
| 4.2041142161 |
| 5.0804404051 |
| 2.5630403502 |
| 6.2684107042 |
| 3.2891226128 |
| 3.1981531699 |
| 4.4770837341 |
| 2.7402832047 |
| 3.6474936359 |
| 2.2914489934 |
| 5.5030188122 |
| 2.8972774949 |
| 3.9619680693 |
| 6.4167715789 |
| 2.4048653671 |
| 3.0198079945 |
| 4.4453479817 |
| 4.1909090555 |
| 3.5889874091 |
| 4.7977470466 |
| 4.4637683959 |
| 2.9467667669 |
| 2.4739420726 |
| 4.9390884666 |
| 4.3074146412 |
| 2.5929743243 |
| 6.6747074402 |
| 4.1925091264 |
| 5.6928902428 |
| 4.4974669419 |
| 4.2407458581 |

4.2644851107  
 3.3718603069  
 2.9832852095  
 3.4894984891  
 3.5626151936  
 2.3408048597  
 3.2718381537  
 3.0880278907  
 4.5445622947  
 3.5612683787  
 3.0485297351  
 2.4488791407  
 4.3549314945  
 4.0277592764  
 3.4418116050  
 3.8117795924  
 3.6391011963  
 3.0401683989  
 4.1839581325  
 4.0165799360  
 4.8894595759  
 4.2083186830  
 2.7776495644  
 3.7089457494  
 3.1030309499  
 3.8622530788  
 2.6338663986  
 4.6452819721  
 4.4577350852  
 4.2789504946

X1 is Weibull(alpha=1.000000,beta=3.000000,gamma=3.000000),



likelihood ratio goodness of fit

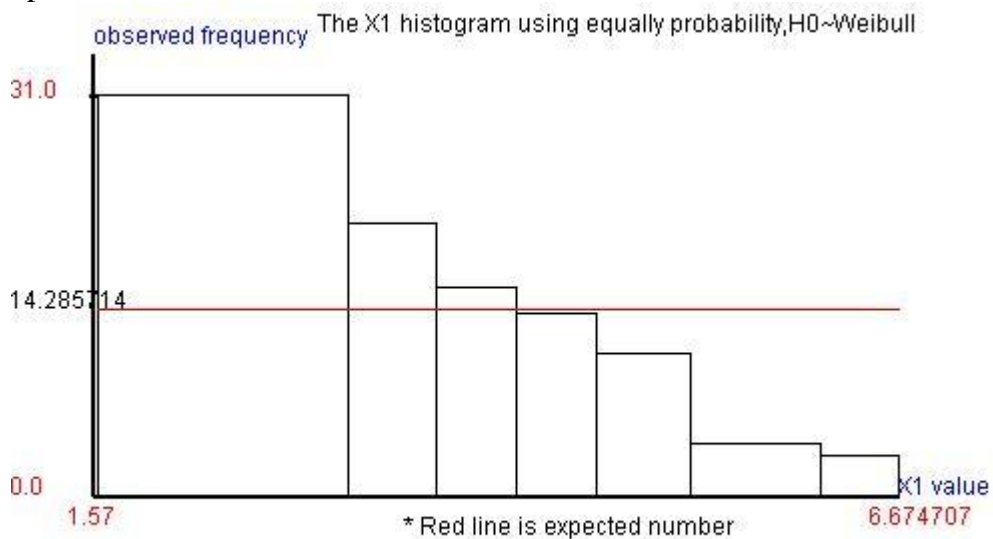
| class       | [ 1 ]    | [ 2 ]    | [ 3 ]    | [ 4 ]    |
|-------------|----------|----------|----------|----------|
| lower limit | 1.57437  | 3.17786  | 3.74019  | 4.24422  |
| 4.76146     | 5.35781  | 6.18488  |          |          |
| upper limit | 3.17786  | 3.74019  | 4.24422  | 4.76146  |
| 5.35781     | 6.18488  |          |          |          |
| observed no | 31.00000 | 21.00000 | 16.00000 | 14.00000 |
| 11.00000    | 4.00000  | 3.00000  |          |          |
| probability | 0.14286  | 0.14286  | 0.14286  | 0.14286  |
| 0.14286     | 0.14286  | 0.14286  |          |          |
| expected no | 14.28571 | 14.28571 | 14.28571 | 14.28571 |
| 14.28571    | 14.28571 | 14.28571 |          |          |
| chi square  | 9.01185  | 2.14674  | 0.18367  | 0.00583  |
| 0.98145     | 26.44898 | 42.45578 |          |          |

degree of freedom=7

H0:  $X_1 \sim \text{Weibull}(\alpha=2.000000, \beta=3.000000, \gamma=2.000000)$ ,

Likelihood ratio chi-square test statistic =81.234308

p-value=0.000000



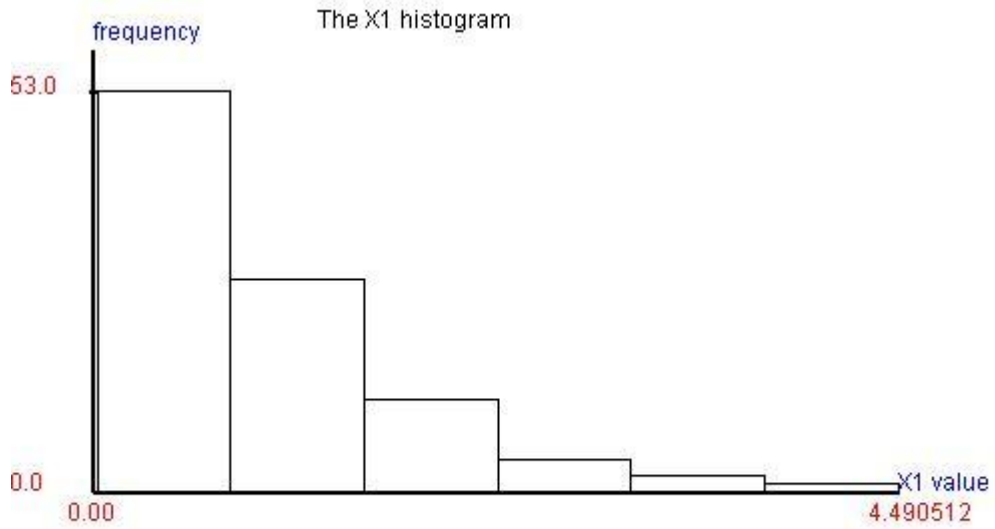
8.4.20)The population distribution is pareto 3 distribution.

X1 is Pareto3(lamda=10.000000,c=10.000000),

| X1           |
|--------------|
| 0.4400612289 |
| 0.6868062546 |
| 1.1708979761 |
| 0.1286921592 |
| 1.3718958292 |
| 1.9778361182 |
| 0.7584322311 |
| 0.3353657004 |
| 0.1481416191 |
| 1.0301599000 |
| 0.3778888979 |
| 0.0792415040 |
| 2.9880554442 |
| 0.1063831144 |
| 1.9603965030 |
| 1.2753457740 |
| 1.9705083228 |
| 0.2736186804 |
| 1.2178959869 |
| 2.4612949652 |
| 1.2738409569 |
| 3.1111230571 |
| 1.3071340510 |
| 3.1575840179 |
| 0.1277257183 |
| 0.6917295616 |
| 0.6680756385 |
| 0.6396896255 |
| 0.2238690157 |
| 0.2423152516 |
| 1.0588834257 |
| 0.5507032279 |
| 0.0212676487 |
| 1.6477362577 |
| 0.1014159388 |
| 0.7537866756 |
| 0.0780331066 |
| 0.0701183956 |
| 0.0092593085 |
| 0.0612214001 |
| 0.3910106995 |
| 0.7672000662 |
| 1.3051788916 |
| 2.0625744113 |
| 0.8323735975 |
| 0.1977022484 |
| 1.6360319298 |
| 0.2082651260 |
| 0.2339964469 |
| 1.7004851388 |
| 0.4167639099 |
| 0.1676196862 |
| 0.3534907544 |
| 0.6651021739 |
| 1.4807623931 |
| 0.5140268387 |
| 0.1454716887 |
| 0.9525548048 |
| 0.3210406820 |
| 0.8772763694 |
| 0.0660055413 |
| 0.1276350536 |
| 0.0223373651 |
| 0.5600213963 |
| 1.3033333584 |
| 0.5380446807 |
| 0.4147386911 |
| 0.1020789290 |
| 1.5097564856 |
| 0.2087240504 |
| 2.6121321160 |

4.4905120062  
 1.3278009812  
 0.2311989239  
 0.9127393439  
 2.5984601742  
 0.5896628179  
 1.3250184663  
 0.2761017101  
 1.1126606920  
 1.4231662647  
 0.3786934045  
 0.3579730314  
 1.4171716043  
 1.5259025439  
 1.0363774350  
 0.9752121916  
 1.9375315022  
 0.2945931949  
 0.3795870965  
 0.9974354488  
 0.5158484057  
 0.4995369385  
 0.5159886494  
 1.6962117005  
 0.7936589593  
 0.3355457824  
 1.8516915781  
 1.4191410980  
 1.1944642738

X1 is Pareto3(lamda=10.000000,c=10.000000),



likelihood ratio goodness of fit

| class       | [ 1 ]    | [ 2 ]    | [ 3 ]    | [ 4 ]    |
|-------------|----------|----------|----------|----------|
| lower limit | 0.00000  | 0.16551  | 0.35342  | 0.57232  |
| 0.83753     | 1.18112  | 1.69534  |          |          |
| upper limit | 0.16551  | 0.35342  | 0.57232  | 0.83753  |
| 1.18112     | 1.69534  | 4.49051  |          |          |
| observed no | 16.00000 | 14.00000 | 16.00000 | 11.00000 |
| 10.00000    | 18.00000 | 15.00000 |          |          |
| probability | 0.14286  | 0.14286  | 0.14286  | 0.14286  |
| 0.14286     | 0.14286  | 0.14286  |          |          |
| expected no | 14.28571 | 14.28571 | 14.28571 | 14.28571 |
| 14.28571    | 14.28571 | 14.28571 |          |          |
| chi square  | 0.18367  | 0.00583  | 0.18367  | 0.98145  |
| 1.83673     | 0.76644  | 0.03401  |          |          |

degree of freedom=4

H0:  $X_1 \sim \text{Pareto } 3(\lambda, c)$ ,  $\lambda, c$  are unknown

$\lambda$  point estimated value=4.104678 (MLE)

$c$  point estimated value=4.490512 (MLE)

Likelihood ratio chi-square test statistic =3.991813

p-value=0.407100

