

## Chapter six The goodness of fit test

7).Q Q plot:

7.1)The process.

$H_0$  : Population distribution is a continuous probability distribution.

There are sample data and the sample size is n.

The  $x$  is sample value after ascending sorting.

$P(X \leq x^*) = \text{number of samples} / n \sim H_0$  : Population distribution

$(x, x^*)$

which scatter diagram is Q Q plot.

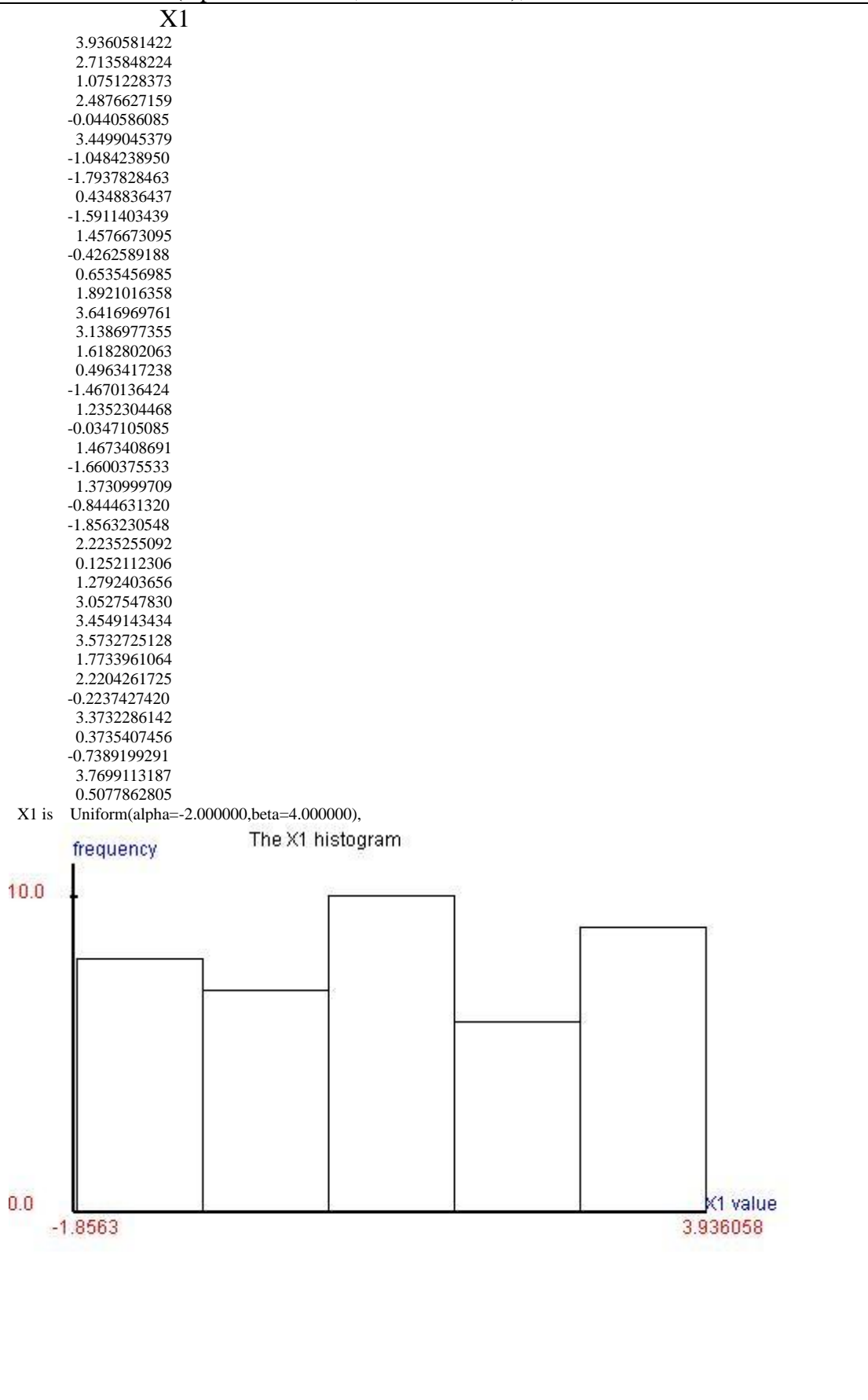
7.2) Example (The simulated sample data and computing the result by the P\_S\_CCC)

The Q-Q plot, Q Q is Quantity value from samples, Q is Quantity value from under  $H_0$

1.H0:Uniform distribution	13.H0:Gumbel distribution
2.H0:Normal distribution	14.H0:Triangular 1 distribution
3.H0:Shifted exponential distribution	15.H0:Trapezoid distribution
4.H0:Pareto 1 distribution	16.H0:U-quadratic distribution
5.H0:Pareto 2 distribution	17.H0:Semi-circle distribution
6.H0:Rayleigh distribution	18.H0:Logistic distribution
7.H0:Double exponential distribution	19.H0:Weibull distribution
8.H0:Log normal distribution	20.H0:Pareto 3 distribution
9.H0:Gamma distribution	** Above $H_0$ population all do once
10.H0:Beta distribution	
11.H0:Cauchy distribution	
12.H0:Arcsin distribution	

7.2.1)The population distribution is uniform distribution.

X1 is Uniform(alpha=-2.000000,beta=4.000000),



H0:  $X_1 \sim \text{Uniform}(\alpha, \beta)$ ,  $\alpha, \beta$  are unknown

$\alpha$  point estimated value = -1.856323 (MLE)

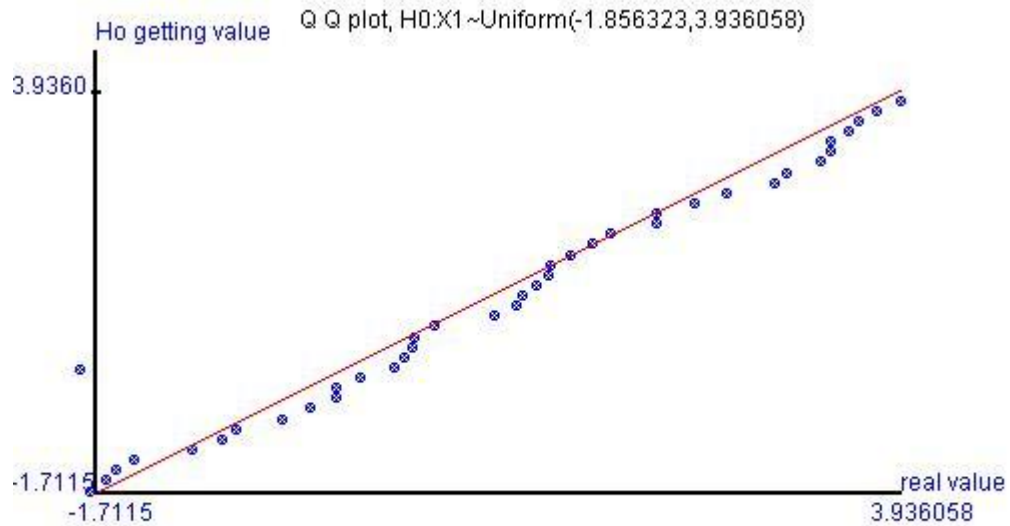
$\beta$  point estimated value = 3.936058 (MLE)

Q Q plot

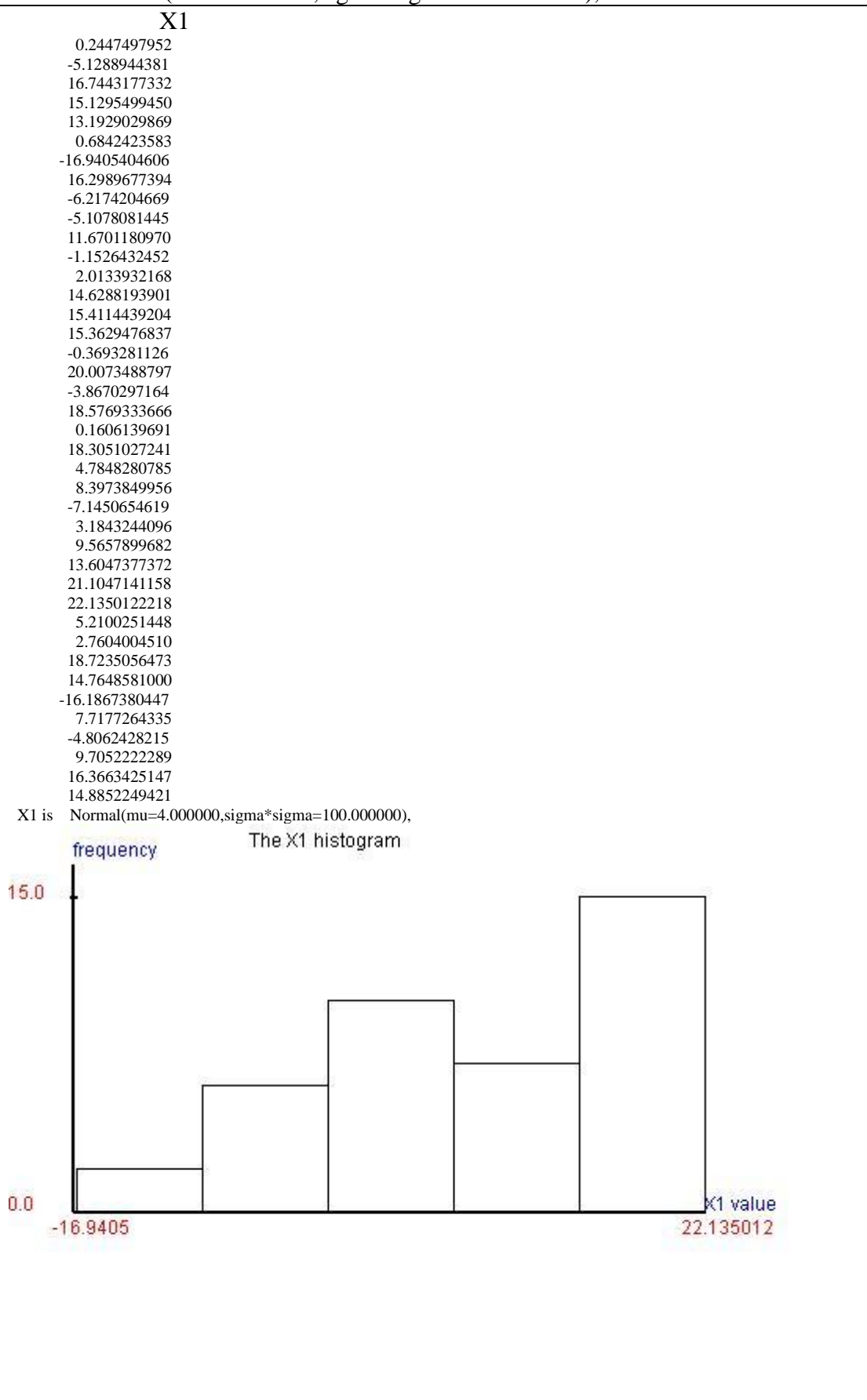
horizontal axis is samples value,

vertical axis is the value in according H0 population distribution.

The Q Q plot images is stored in c:\book\_01\QQ\_plot01\_image.jpg



7.2.2)The population distribution is normal distribution.  
 X1 is Normal( $\mu=4.000000$ , $\sigma^2=100.000000$ ),



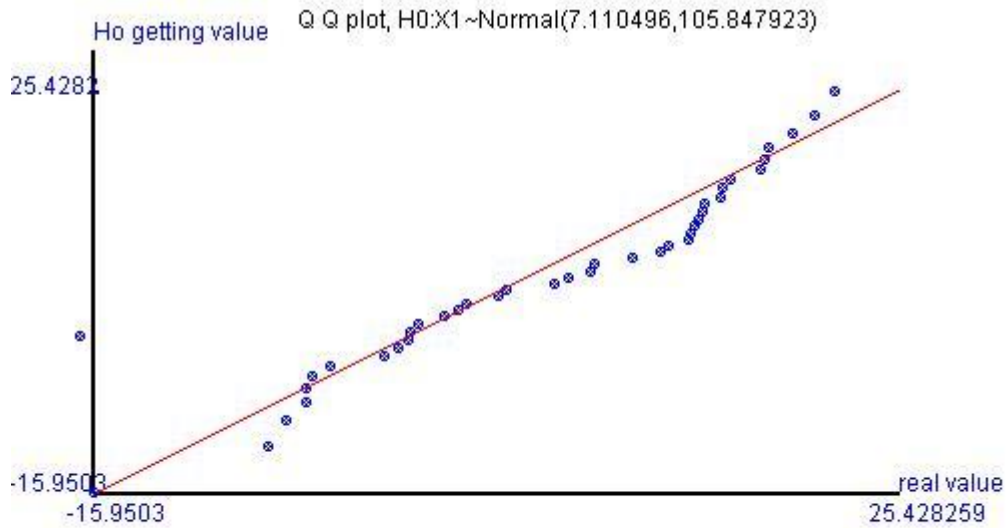
H0:  $X_1 \sim \text{Normal}(\mu, \sigma^2)$ ,  $\mu, \sigma$  are unknown  
population mean( $\mu$ ) point estimated value=7.110496 (MLE,UMVUE)  
population variance( $\sigma^2$ ) which point estimated value=105.847923 (UMVUE)

Q Q plot

horizontal axis is samples value,

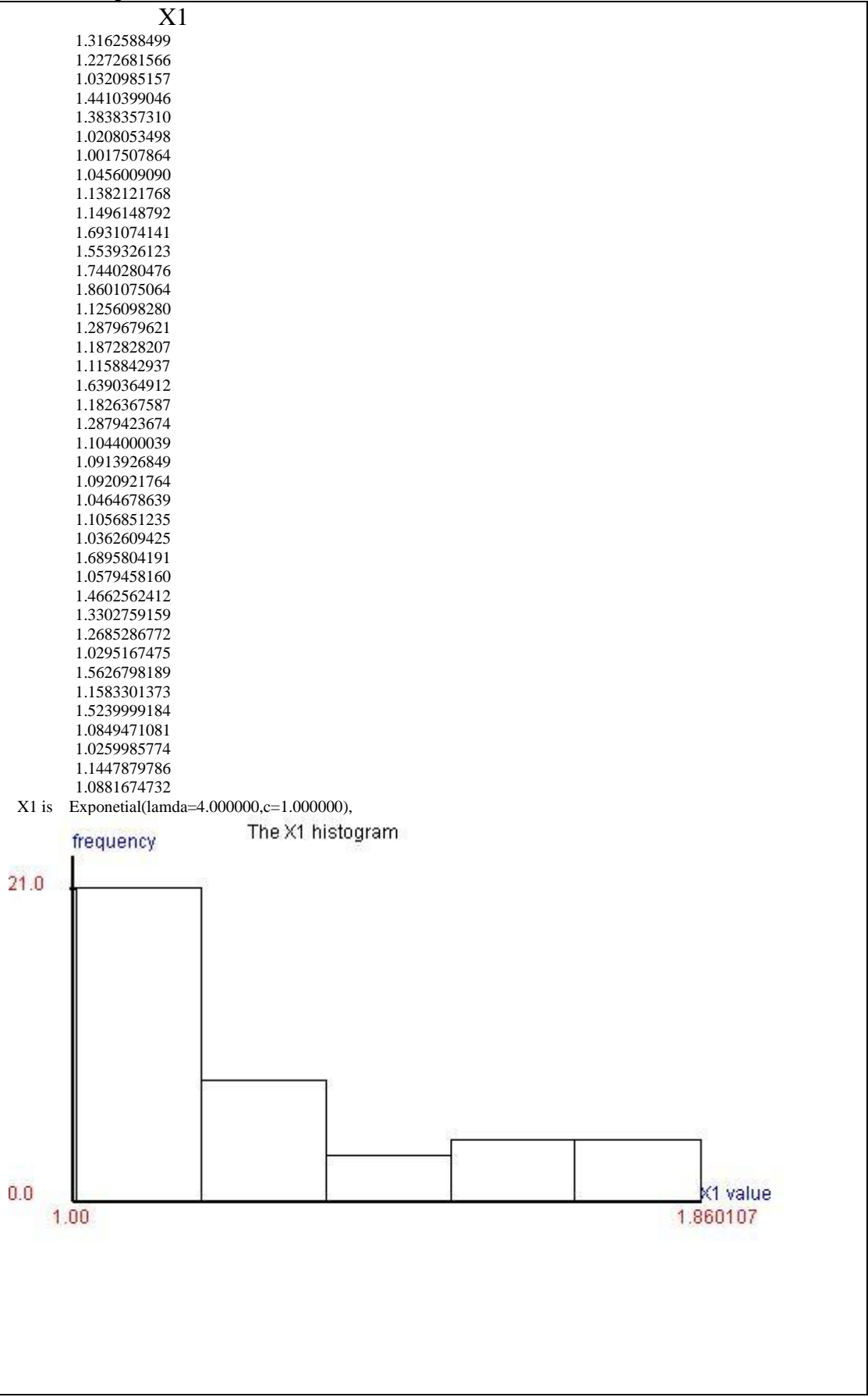
vertical axis is the value in according Ho population distribution.

The Q Q plot images is stored in c:\book\_01\QQ\_plot02\_image.jpg



7.2.3)The population distribution is shifted exponential distribution.

X1 is Exponential( $\lambda=4.000000, c=1.000000$ ),



H0:  $X_1 \sim \text{Shifted exponential}(\lambda, c)$ ,  $\lambda, c$  are unknown

$\lambda$  point estimated value = 3.894345 (MLE)

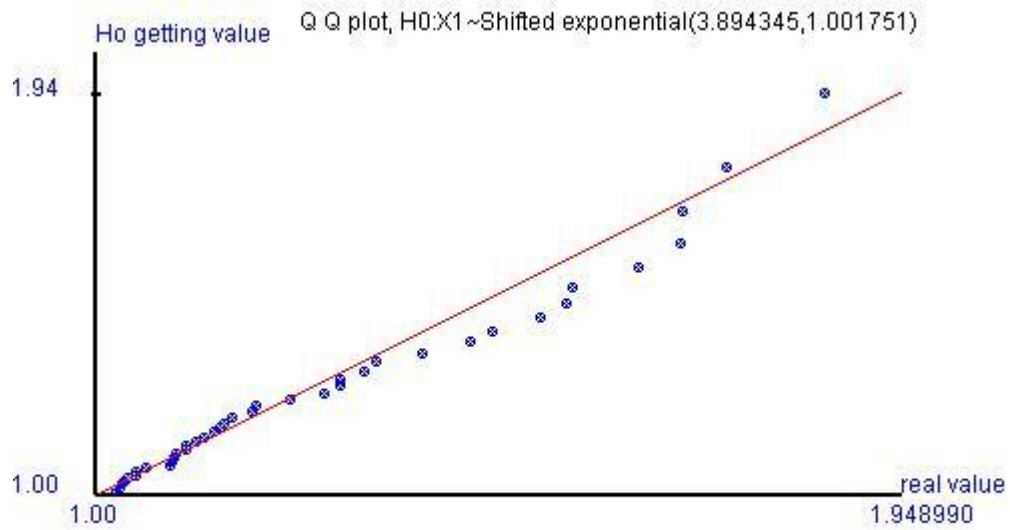
$c$  point estimated value = 1.001751 (MLE)

Q Q plot

horizontal axis is samples value,

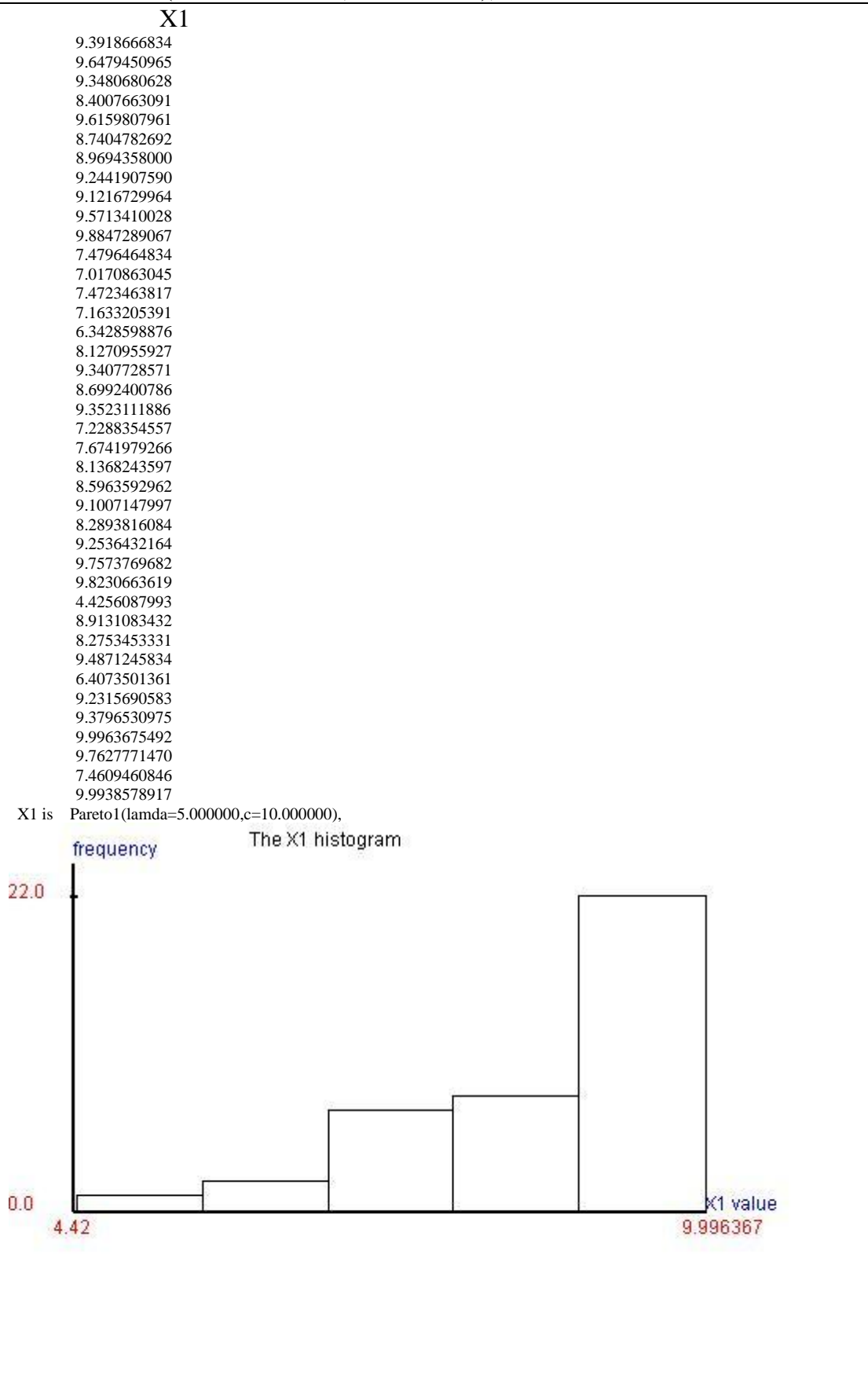
vertical axis is the value in according H0 population distribution.

The Q Q plot images is stored in c:\book\_01\QQ\_plot03\_image.jpg



7.2.4)The population distribution is pareto1 distribution.

X1 is Pareto1(lamda=5.000000,c=10.000000),





H0:  $X_1 \sim \text{Pareto 1}(\lambda, c)$ ,  $\lambda, c$  are unknown

$\lambda$  point estimated value = 6.183656 (MLE)

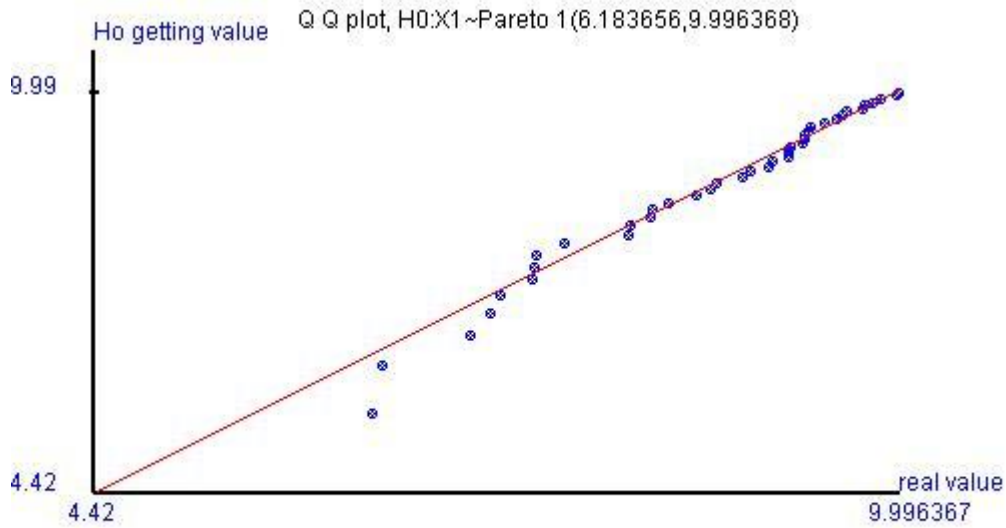
$c$  point estimated value = 9.996368 (MLE)

Q Q plot

horizontal axis is samples value,

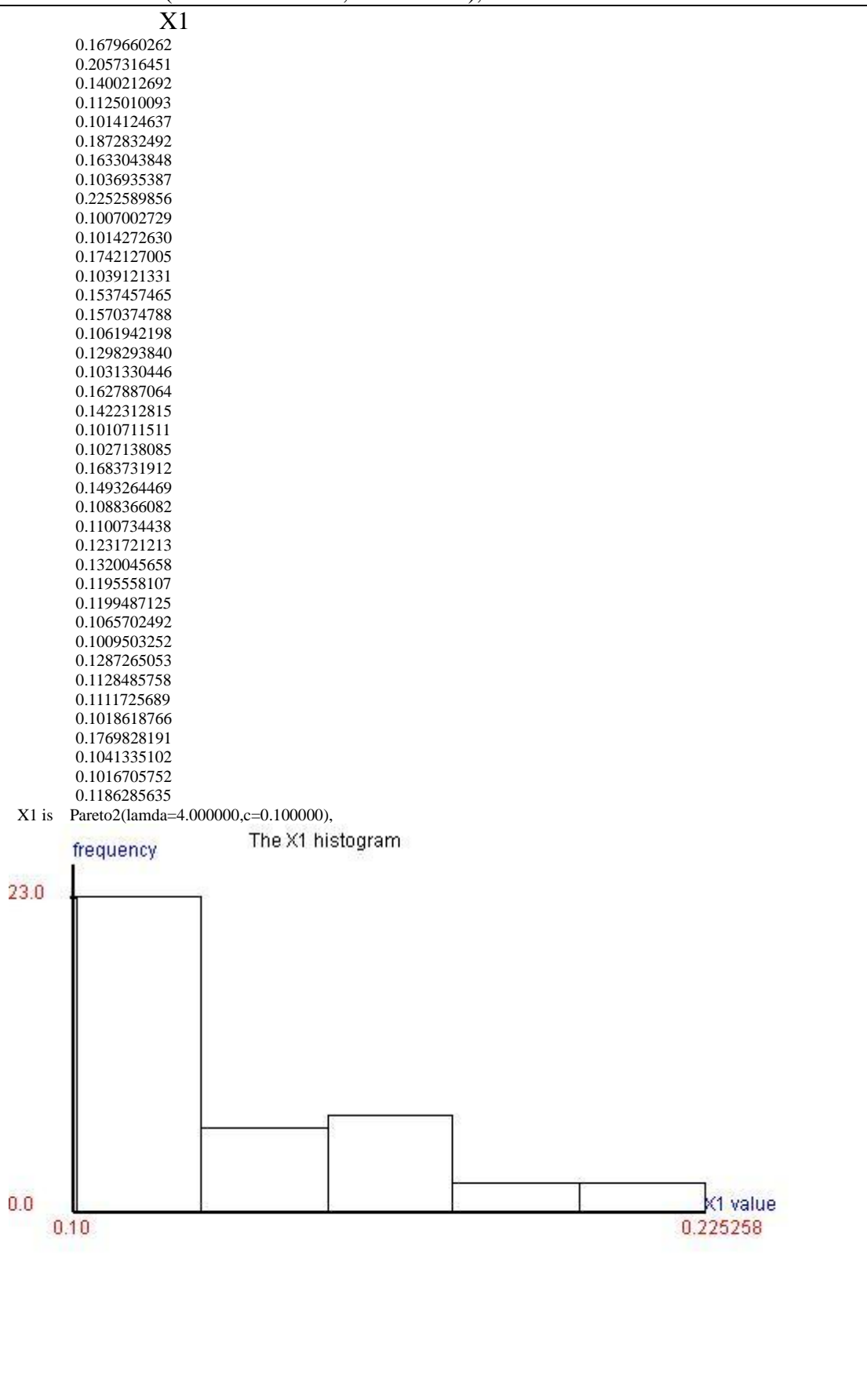
vertical axis is the value in according H0 population distribution.

The Q Q plot images is stored in c:\book\_01\QQ\_plot04\_image.jpg



7.2.5)The population distribution is pareto2 distribution.

X1 is Pareto2(lamda=4.000000,c=0.100000),



H0:  $X_1 \sim \text{Pareto 2}(\lambda, c)$ ,  $\lambda, c$  are unknown

$\lambda$  point estimated value = 4.237524 (MLE)

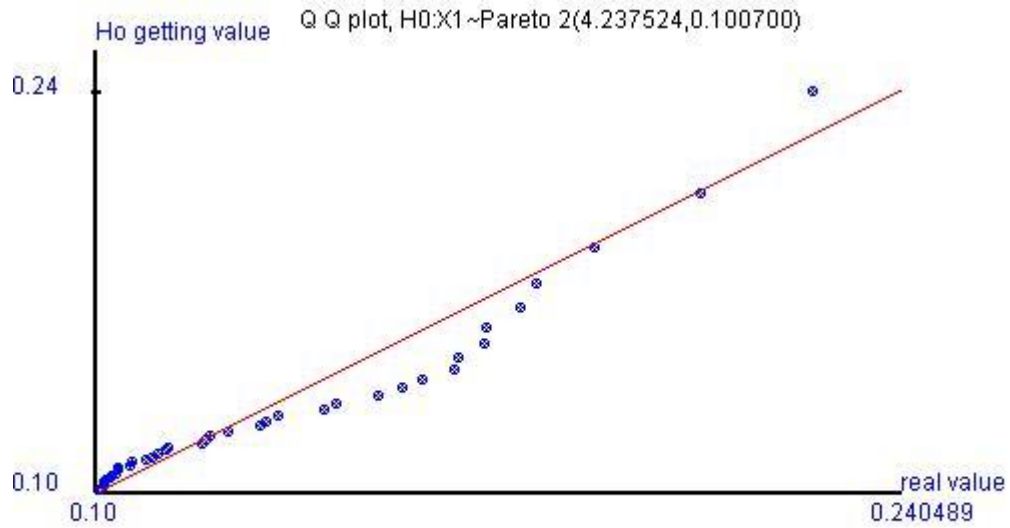
$c$  point estimated value = 0.100700 (MLE)

Q Q plot

horizontal axis is samples value,

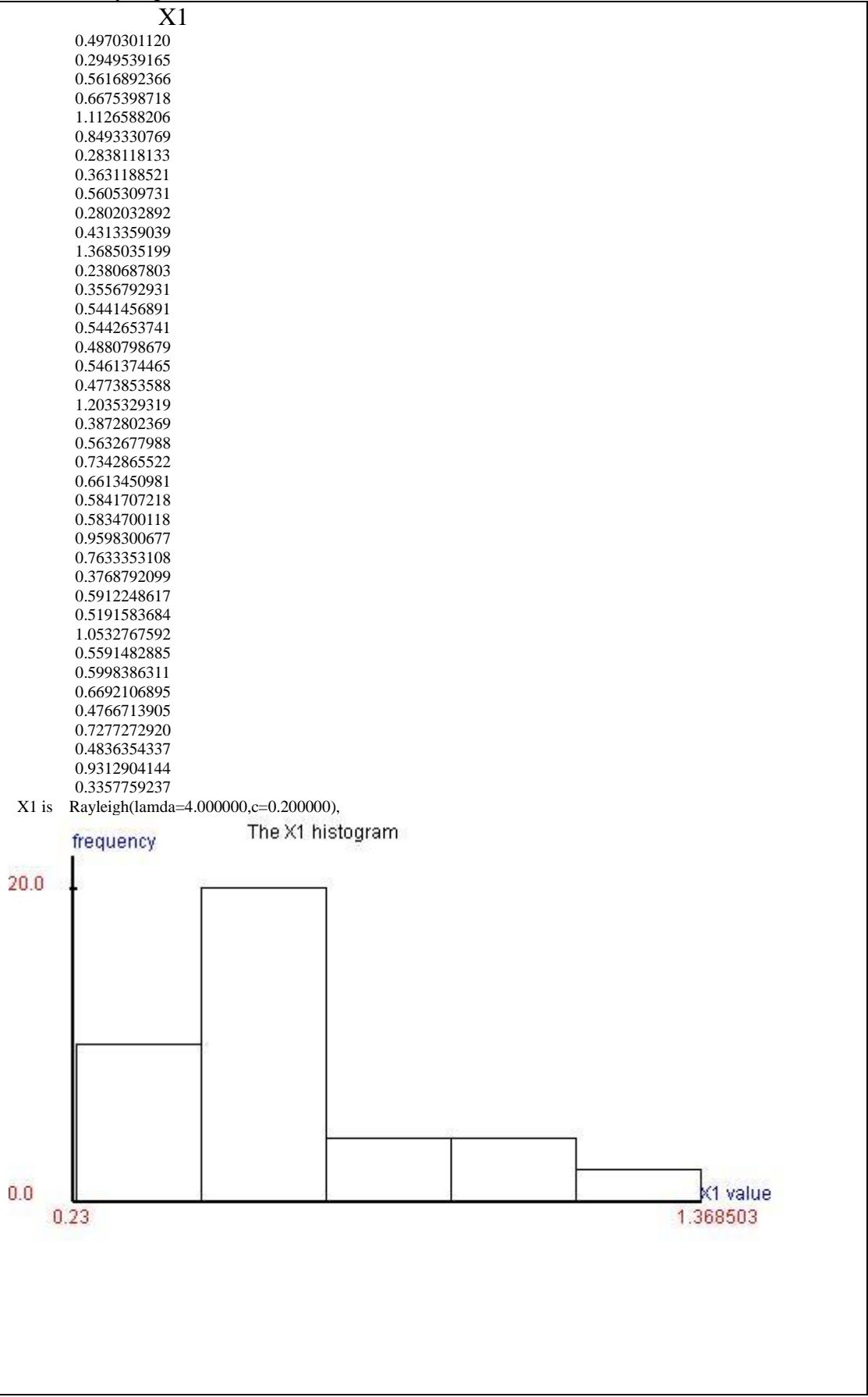
vertical axis is the value in according H0 population distribution.

The Q Q plot images is stored in c:\book\_01\QQ\_plot05\_image.jpg



7.2.6)The population distribution is rayleigh distribution.

X1 is Rayleigh(lamda=4.000000,c=0.200000),



H0:  $X_1 \sim \text{Rayleigh}(\lambda, c)$ ,  $\lambda, c$  are unknown

$\lambda$  point estimated value = 4.982134 (MLE)

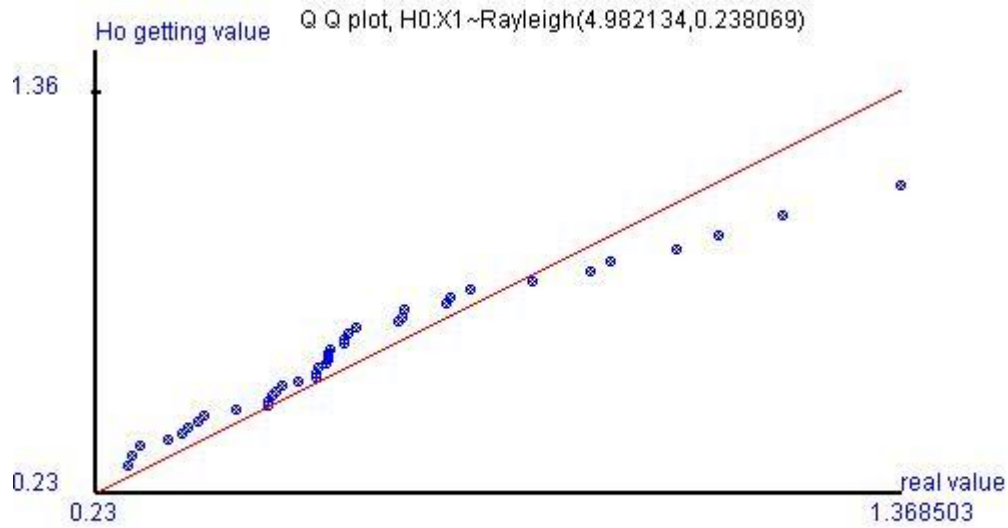
$c$  point estimated value = 0.238069 (MLE)

Q Q plot

horizontal axis is samples value,

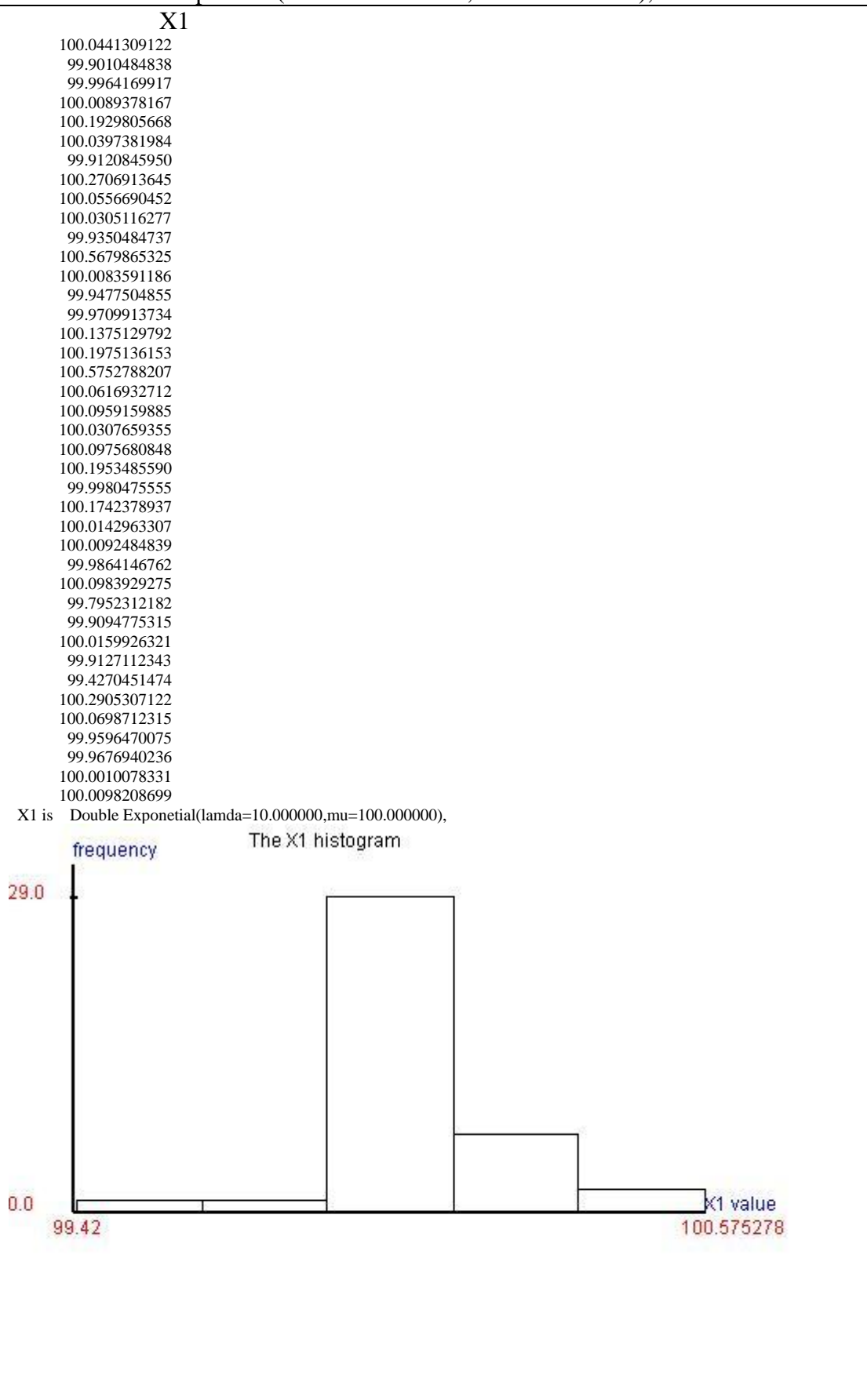
vertical axis is the value in according H0 population distribution.

The Q Q plot images is stored in c:\book\_01\QQ\_plot06\_image.jpg



7.2.7) The population distribution is double exponential distribution.

X1 is Double Exponential( $\lambda=10.000000, \mu=100.000000$ ),



H0:  $X_1 \sim \text{Double exponential}(\lambda, \mu)$ ,  $\lambda, \mu$  are unknown

$\lambda$  point estimated value = 8.750722 (MLE)

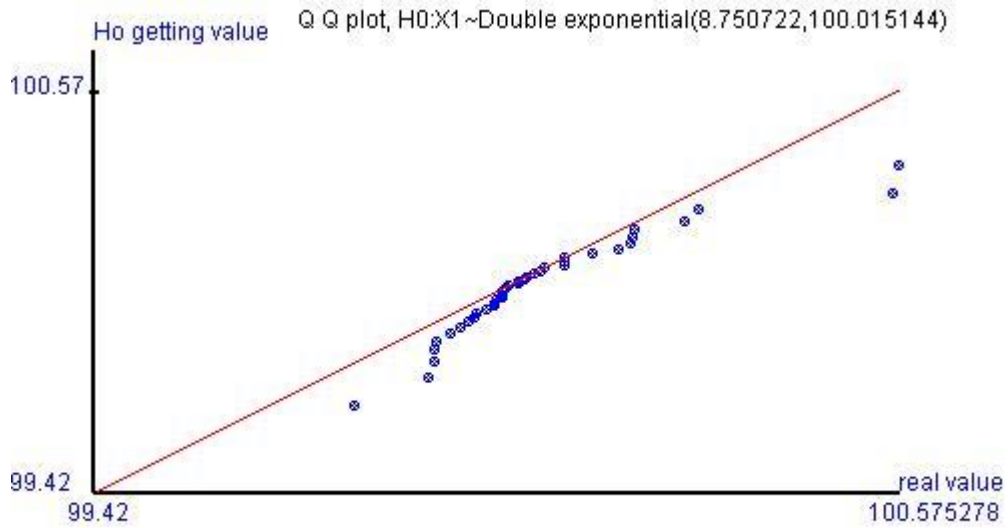
$\mu$  point estimated value = 100.015144 (MLE)

Q Q plot

horizontal axis is samples value,

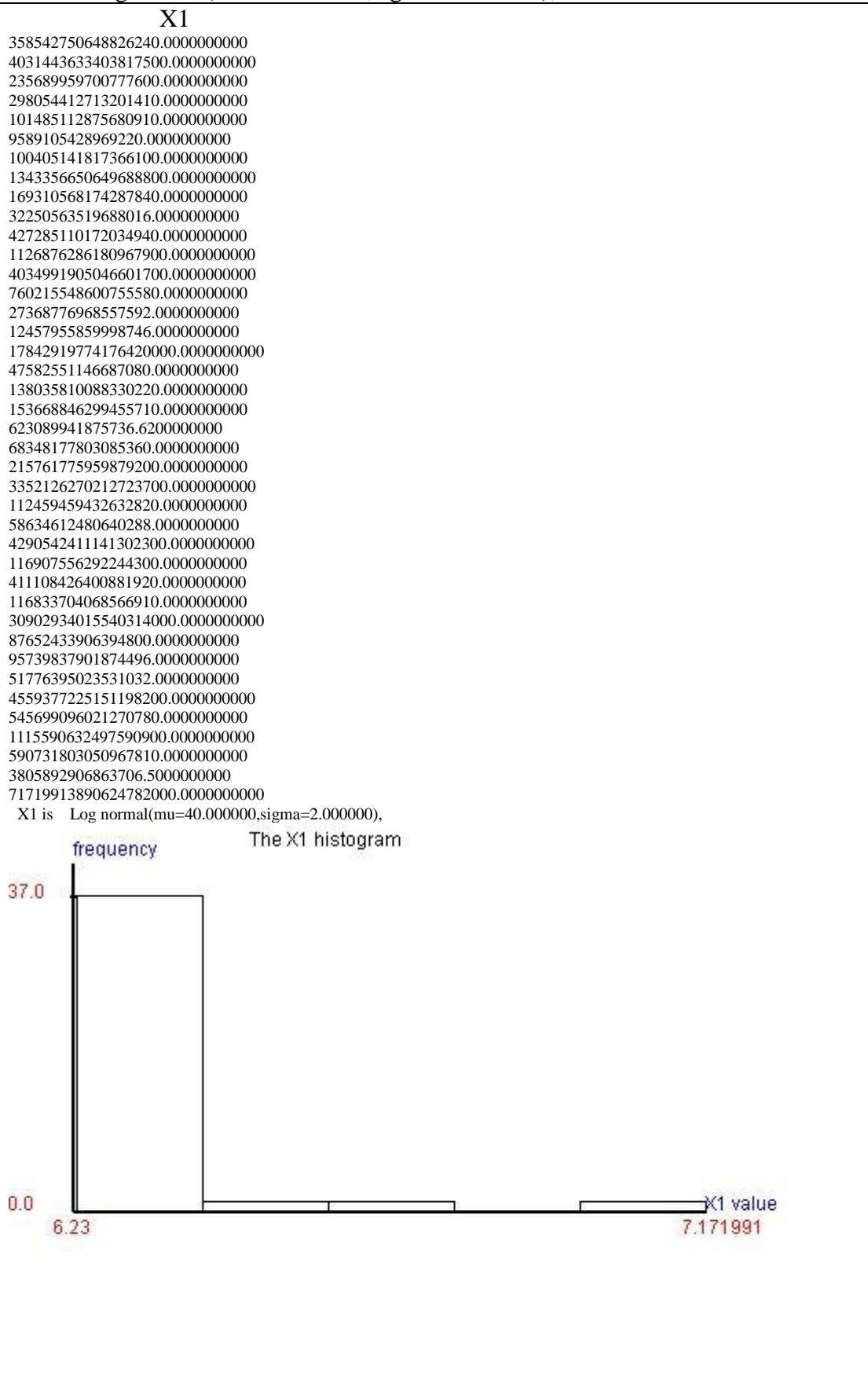
vertical axis is the value in according H0 population distribution.

The Q Q plot images is stored in c:\book\_01\QQ\_plot07\_image.jpg



7.2.8)The population distribution is lognormal distribution.

X1 is Log normal( $\mu=40.000000$ , $\sigma=2.000000$ ),





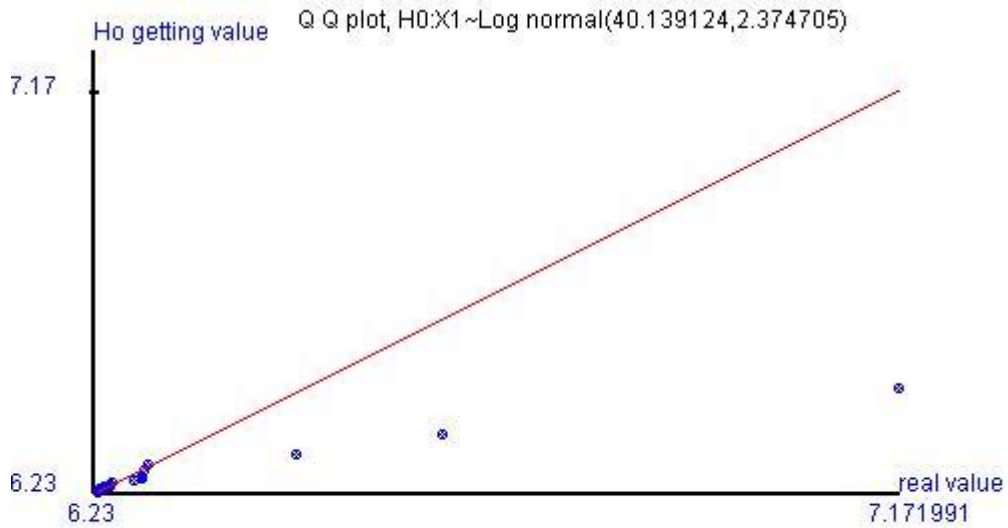
H0:  $X_1 \sim \text{Log\_Normal}(\mu, \sigma^2)$ ,  $\mu, \sigma$  are unknown  
population mean( $\mu$ ) point estimated value=40.139124 (MLE,UMVUE)  
population variance( $\sigma^2$ ) which point estimated value=5.639224 (UMVUE)

Q Q plot

horizon axis is samples value,

vertical axis is the value in according Ho population distribution.

The Q Q plot images is stored in c:\book\_01\QQ\_plot08\_image.jpg

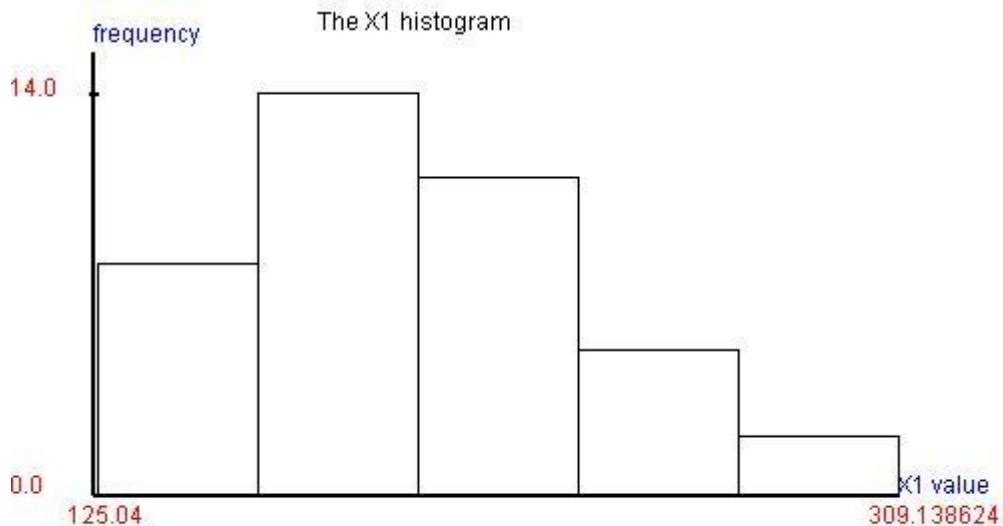


7.2.9)The population distribution is gamma distribution.

X1 is Gamma(alpha=20.500000,beta=10.000000),

X1  
194.1851972510  
254.0390186317  
161.1566815441  
196.3251958032  
152.1107067111  
198.7642083002  
234.2597694799  
156.5760457549  
216.4045690653  
259.1831046908  
196.3593753509  
209.4496260850  
213.5860230137  
258.1216978321  
195.2568797913  
159.4383240672  
161.4210775866  
212.5135223020  
216.0249516547  
218.3270223556  
201.4763617179  
248.9674258817  
185.8928801739  
161.9431460162  
171.2910147035  
232.2448724254  
155.2413268017  
309.1386248331  
188.4551054475  
125.0432100331  
184.4469704810  
220.3100210619  
297.9414161914  
181.1532056071  
160.8491930243  
195.1257097984  
251.6738843612  
170.2080368913  
192.9014029124  
189.8807168171

X1 is Gamma(alpha=20.500000,beta=10.000000),



H0:  $X_1 \sim \text{Gamma}(\alpha, \beta)$ ,  $\alpha, \beta$  are unknown

alpha point estimated value=26.000000 (MME)

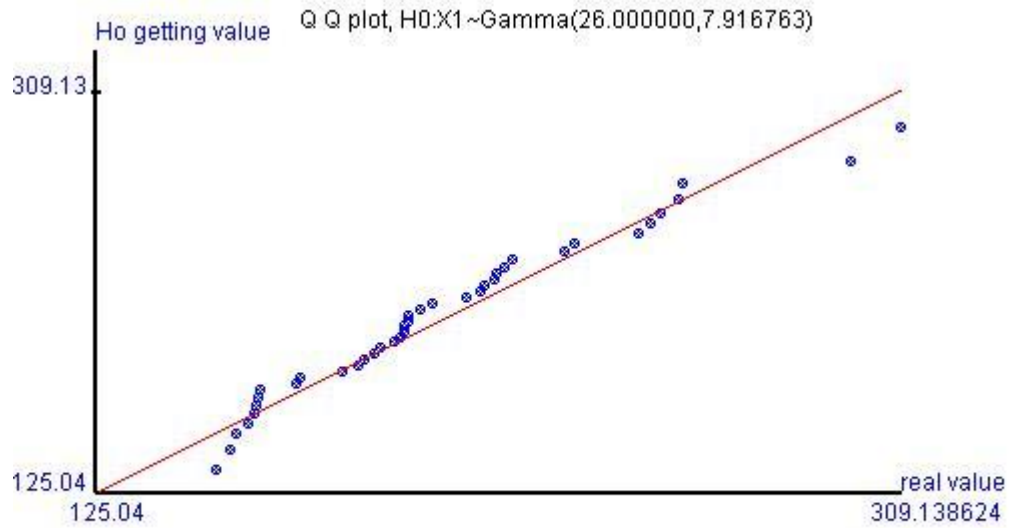
beta point estimated value=7.916763 (MME)

Q Q plot

horizontal axis is samples value,

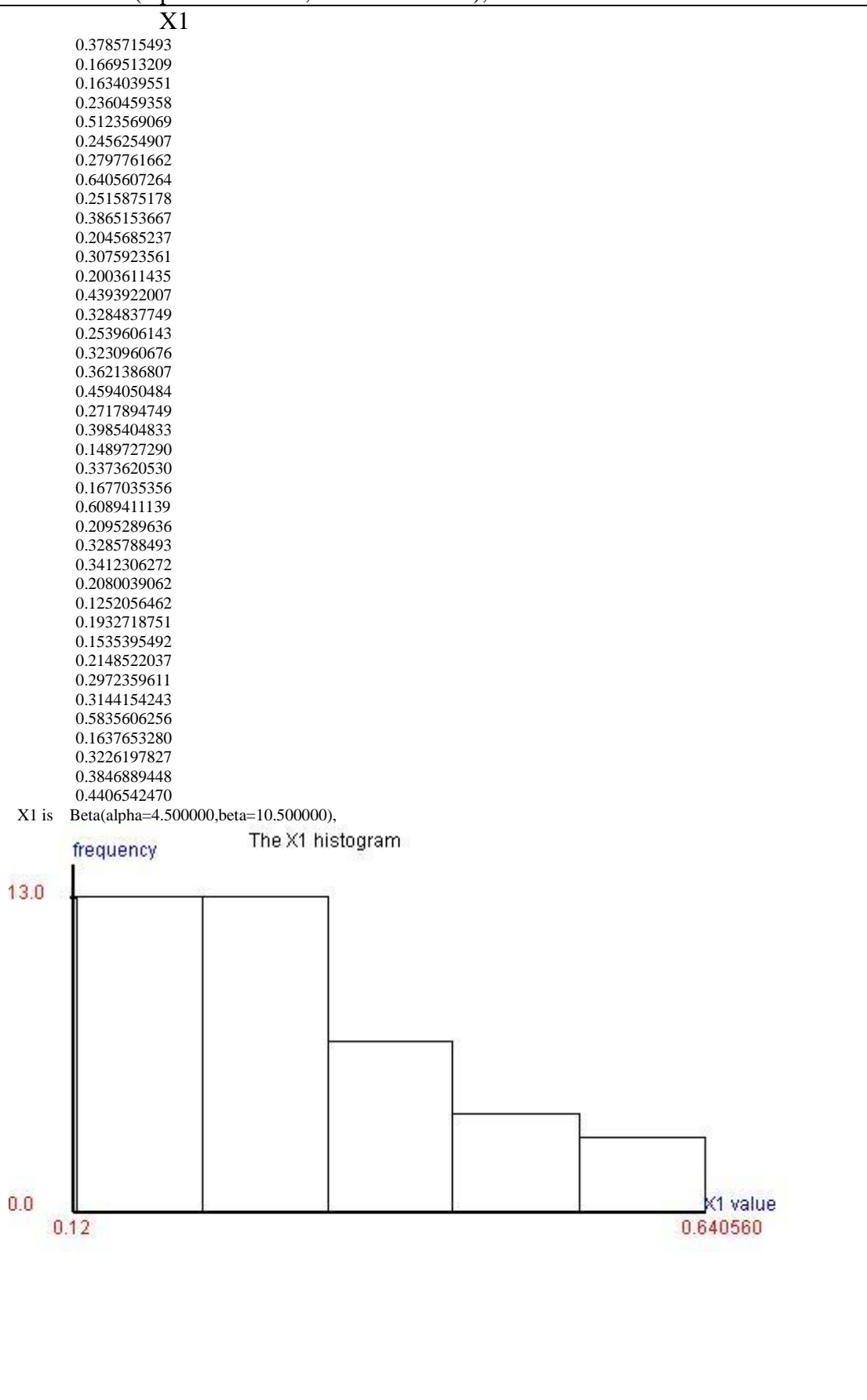
vertical axis is the value in according H0 population distribution.

The Q Q plot images is stored in c:\book\_01\QQ\_plot09\_image.jpg



7.2.10)The population distribution is beta distribution.

X1 is  $\text{Beta}(\alpha=4.500000,\beta=10.500000)$ ,



H0:  $X_1 \sim \text{Beta}(\alpha, \beta)$ ,  $\alpha, \beta$  are unknown

$\alpha$  point estimated value = 3.500000 (MME)

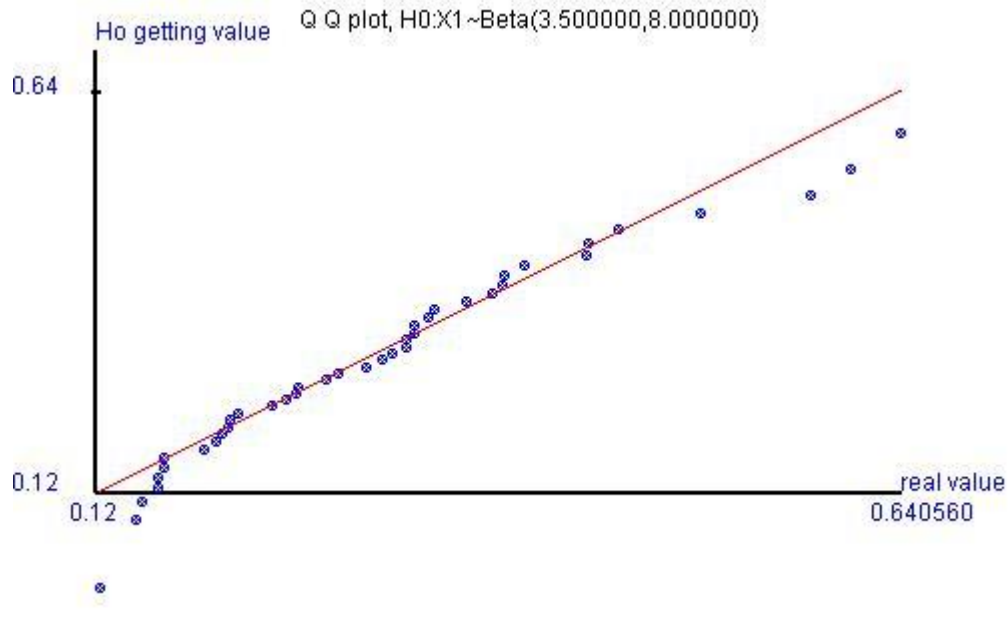
$\beta$  point estimated value = 8.000000 (MME)

Q Q plot

horizontal axis is samples value,

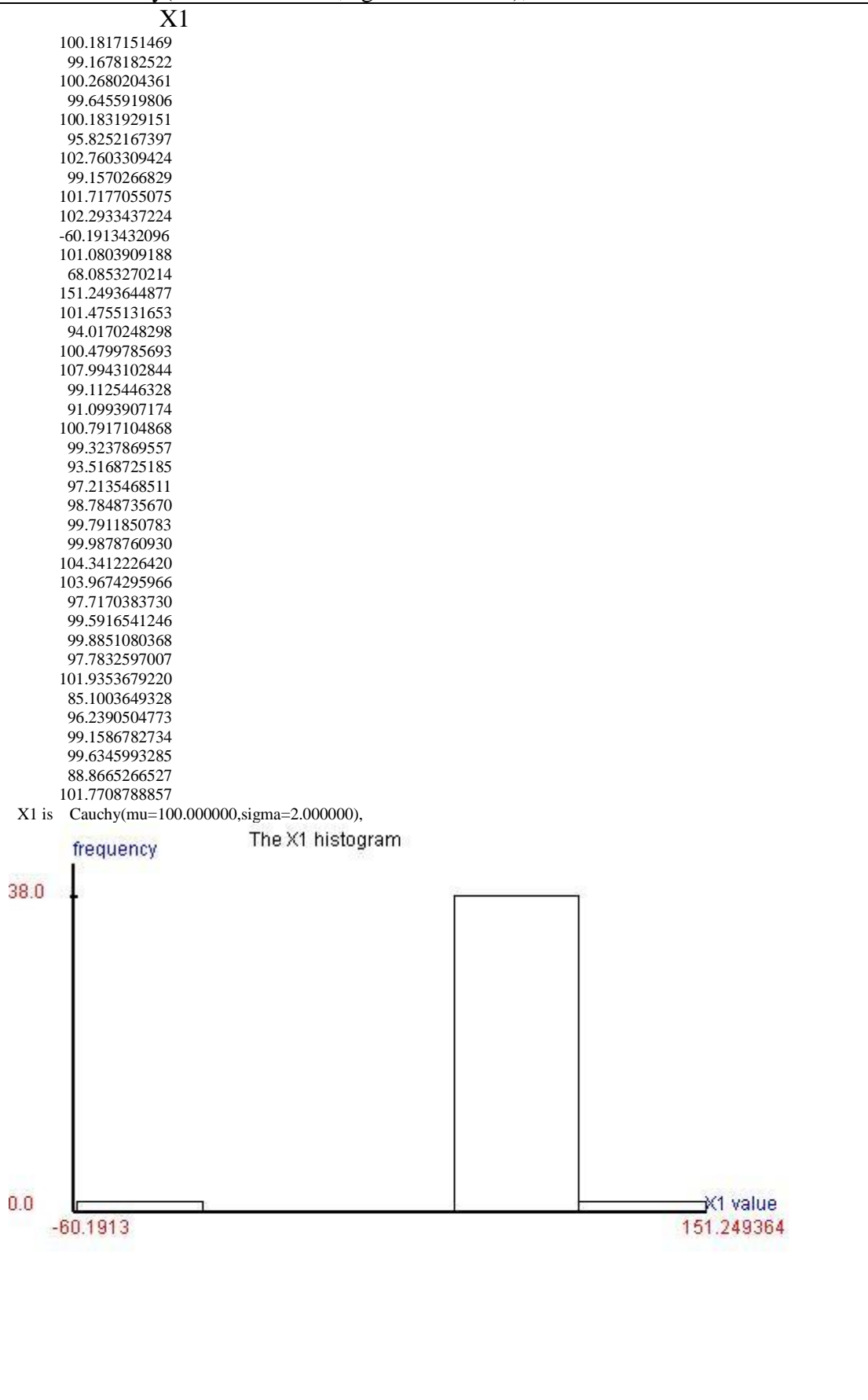
vertical axis is the value in according H0 population distribution.

The Q Q plot images is stored in c:\book\_01\QQ\_plot10\_image.jpg



7.2.11)The population distribution is cauchy distribution.

X1 is Cauchy( $\mu=100.000000,\sigma=2.000000$ ),



H0:  $X_1 \sim \text{Cauchy}(\mu, \sigma)$ ,  $\mu, \sigma$  are unknown

$\mu$  point estimated value=99.640096

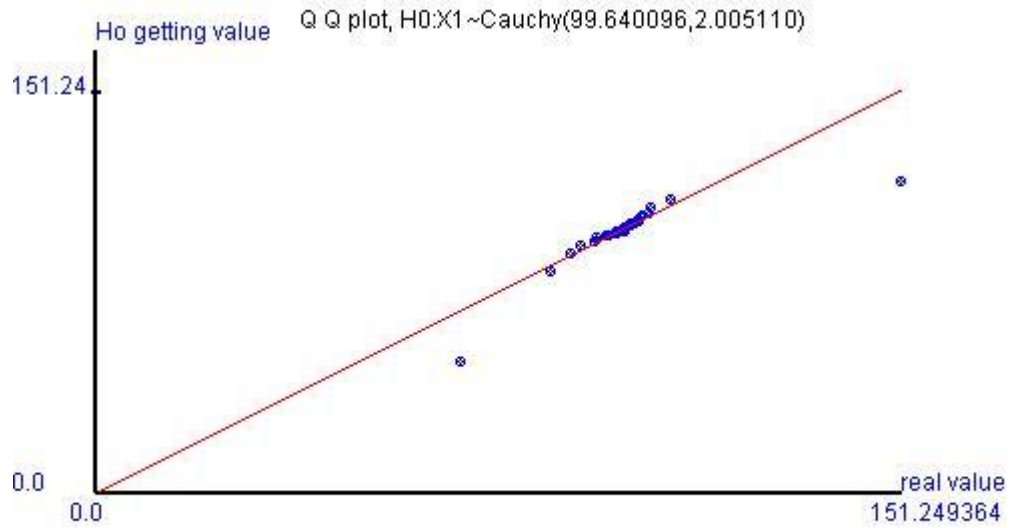
$\sigma$  point estimated value=2.005110

Q Q plot

horizontal axis is samples value,

vertical axis is the value in according H0 population distribution.

The Q Q plot images is stored in c:\book\_01\QQ\_plot11\_image.jpg

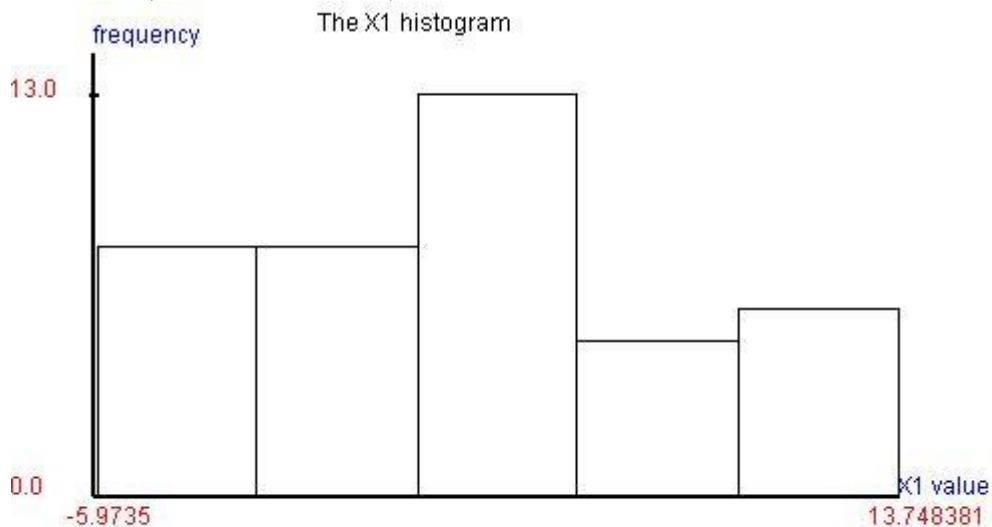


7.2.12) The population distribution is arcsin distribution.

X1 is  $\text{Arcsin}(\mu=4.000000, c=10.000000)$ ,

X1  
 3.5156556172  
 1.8733948480  
 12.9375555439  
 13.7483814631  
 1.5194291474  
 0.1658224019  
 1.8578701035  
 3.7255916619  
 -2.7322345732  
 11.2000530252  
 11.2560746730  
 4.9080034646  
 6.5650235851  
 9.5800058655  
 -3.9843121335  
 5.7364186928  
 3.2670243965  
 0.7009355146  
 -0.2955536993  
 -2.9209000576  
 12.8260112153  
 3.4406154957  
 6.5808699366  
 6.7036202687  
 -2.9639275037  
 13.1909238023  
 -3.7873106643  
 3.7962750006  
 2.5705180864  
 -1.5638442648  
 0.3685175640  
 -2.8022605405  
 3.8157018338  
 5.7444233083  
 1.9525372031  
 3.7011385684  
 3.5167160491  
 -5.1137974831  
 -5.9735945936  
 9.5494508631

X1 is  $\text{Arcsin}(\mu=4.000000, c=10.000000)$ ,





H0:  $X_1 \sim \text{Arcsin}(\mu, c)$ ,  $\mu, c$  are unknown

$\mu$  point estimated value=3.454421

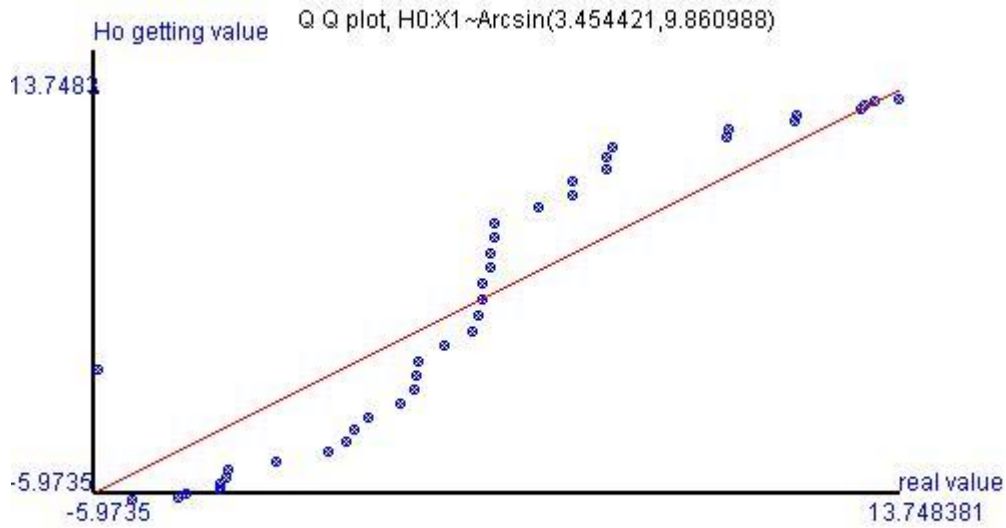
$c$  point estimated value=9.860988 (MLE)

Q Q plot

horizontal axis is samples value,

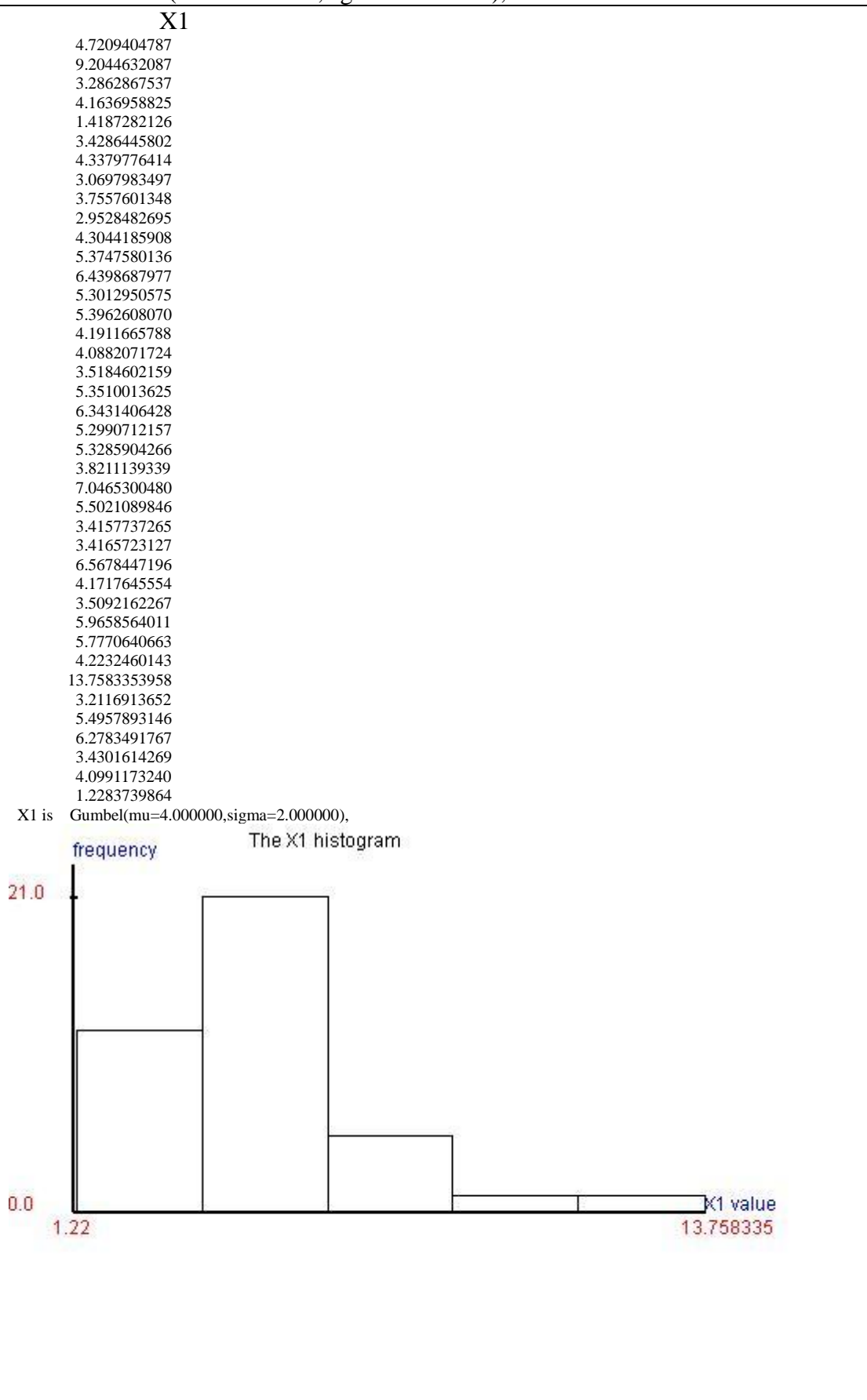
vertical axis is the value in according  $H_0$  population distribution.

The Q Q plot images is stored in c:\book\_01\QQ\_plot12\_image.jpg



7.2.13)The population distribution is gumbel distribution.

X1 is Gumbel( $\mu=4.000000,\sigma=2.000000$ ),



H0:  $X_1 \sim \text{Gumbel}(\mu, \sigma)$ ,  $\mu, \sigma$  are unknown

$\mu$  point estimated value=3.861053 (MME)

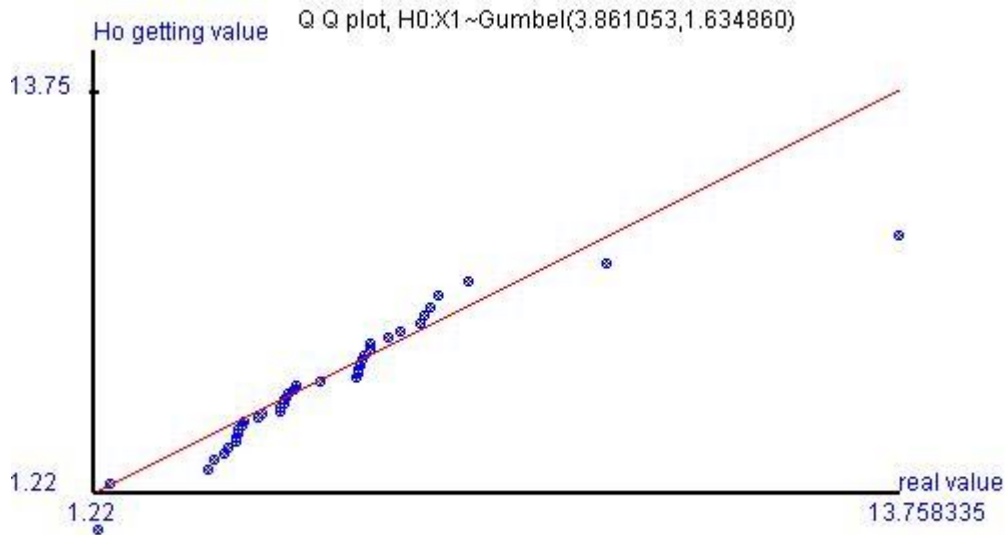
$\sigma$  point estimated value=1.634860 (MME)

Q Q plot

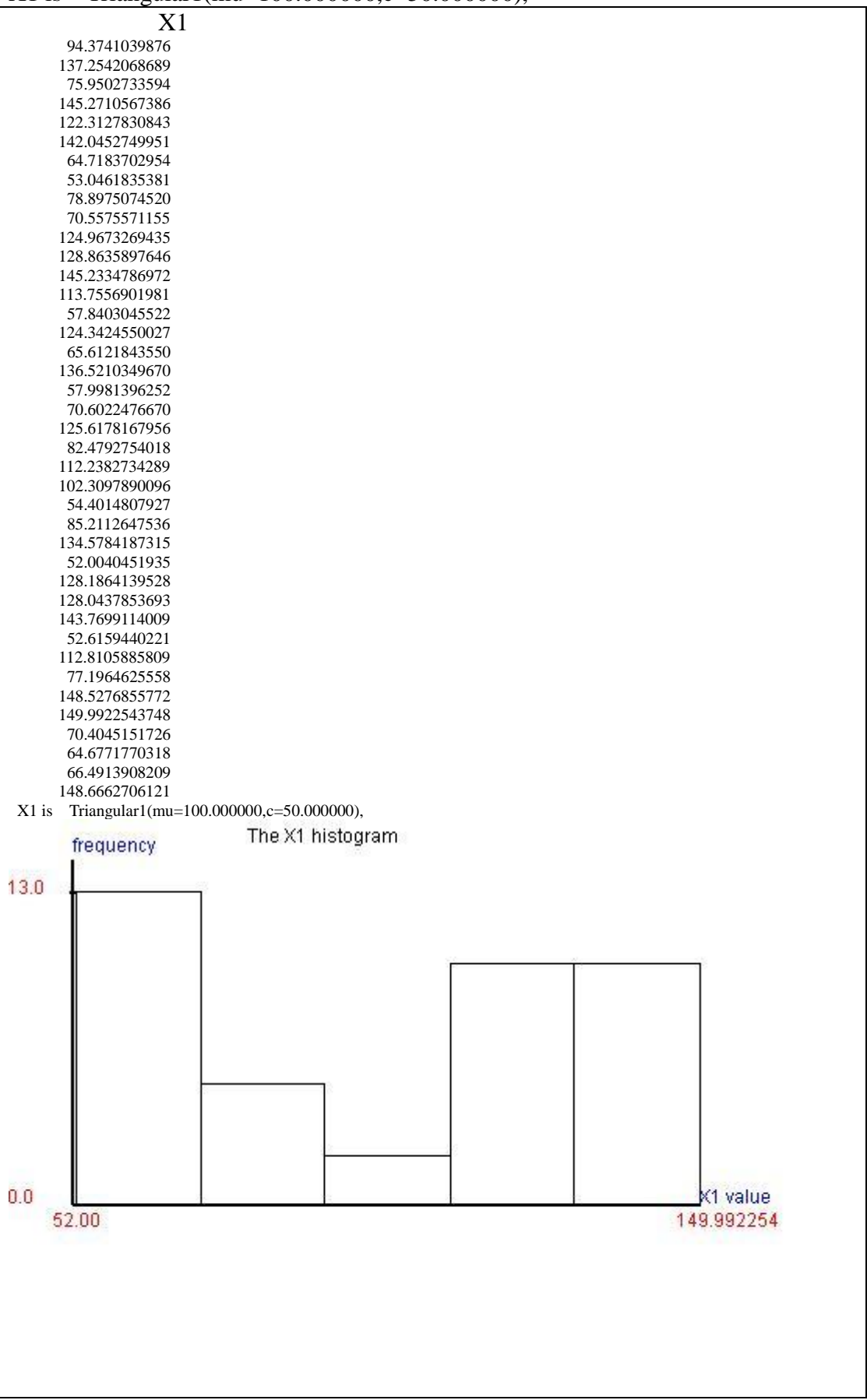
horizontal axis is samples value,

vertical axis is the value in according H0 population distribution.

The Q Q plot images is stored in c:\book\_01\QQ\_plot13\_image.jpg



7.2.14) The population distribution is triangular 1 distribution.  
 $X_1$  is  $\text{Triangular1}(\mu=100.000000, c=50.000000)$ ,



H0:  $X_1 \sim \text{Triangular } 1(\mu, c)$ ,  $\mu, c$  are unknown

$\mu$  point estimated value=101.259663

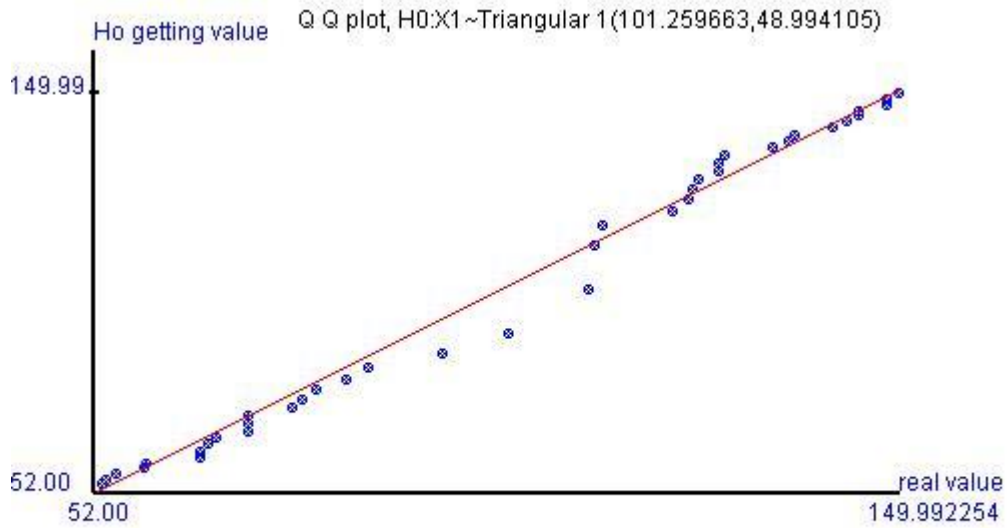
$c$  point estimated value=48.994105 (MLE)

Q Q plot

horizontal axis is samples value,

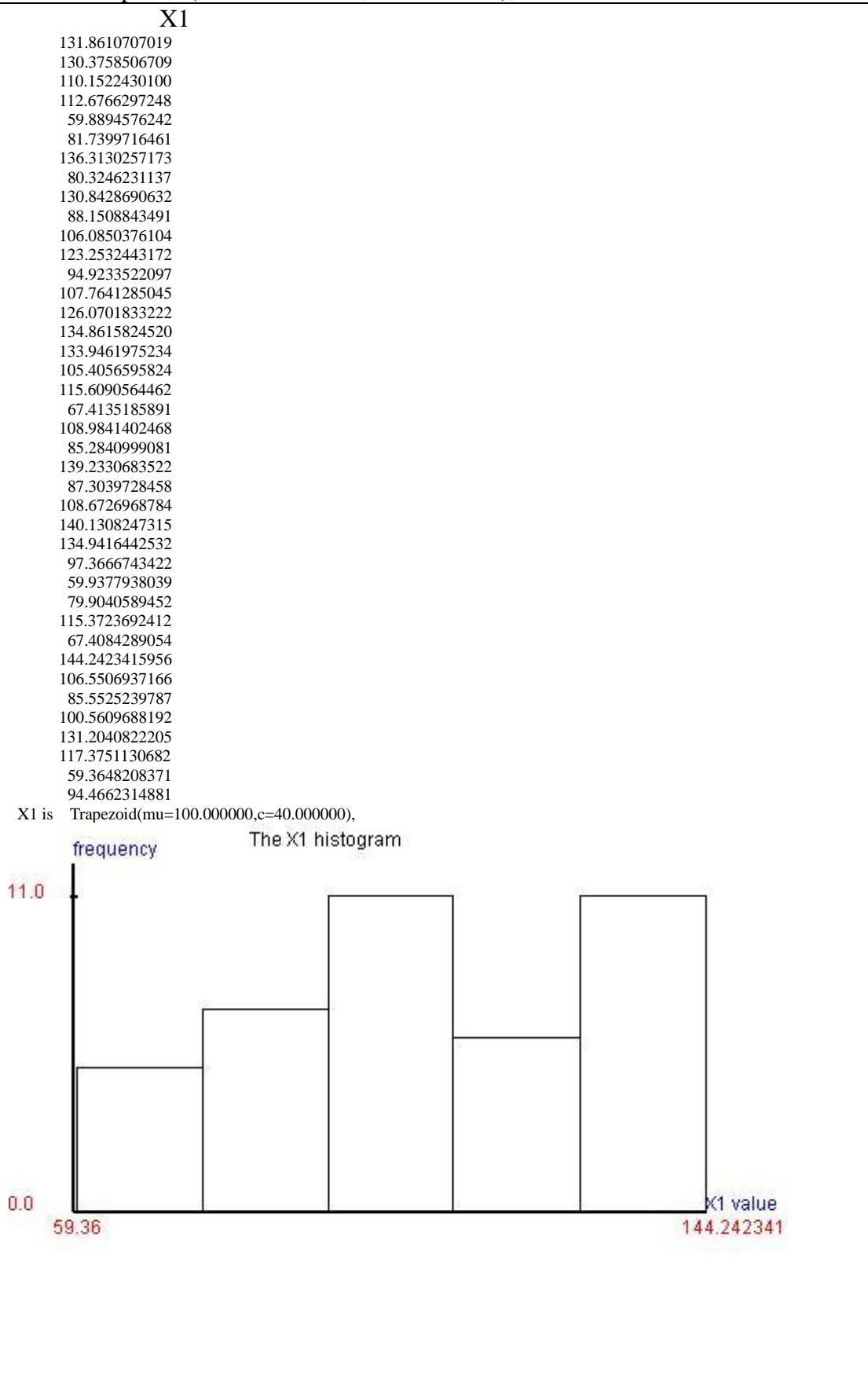
vertical axis is the value in according Ho population distribution.

The Q Q plot images is stored in c:\book\_01\QQ\_plot14\_image.jpg



7.2.15) The population distribution is trapezoid distribution.

X1 is Trapezoid( $\mu=100.000000, c=40.000000$ ),



H0:  $X_1 \sim \text{Trapezoid}(\mu, c)$ ,  $\mu, c$  are unknown

$\mu$  point estimated value=106.037878

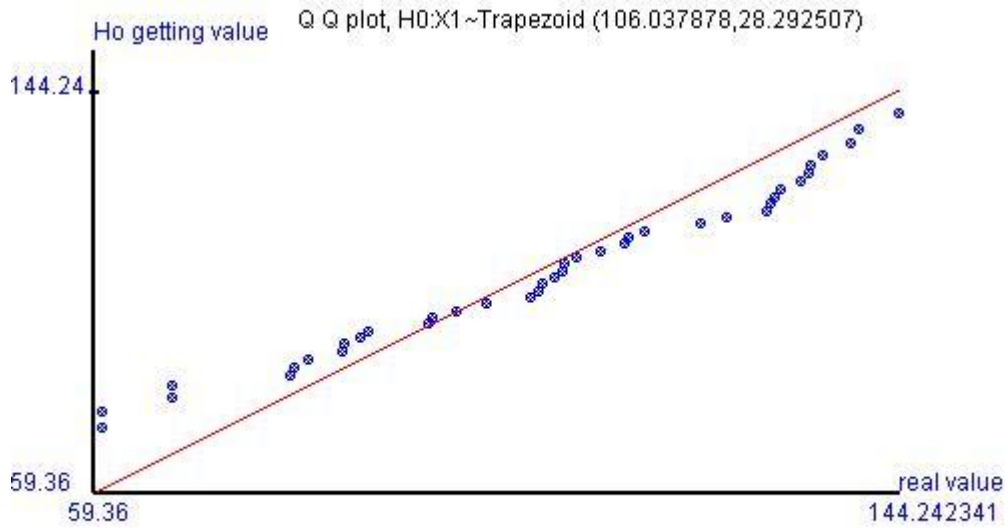
$c$  point estimated value=28.292507 (MLE)

Q Q plot

horizontal axis is samples value,

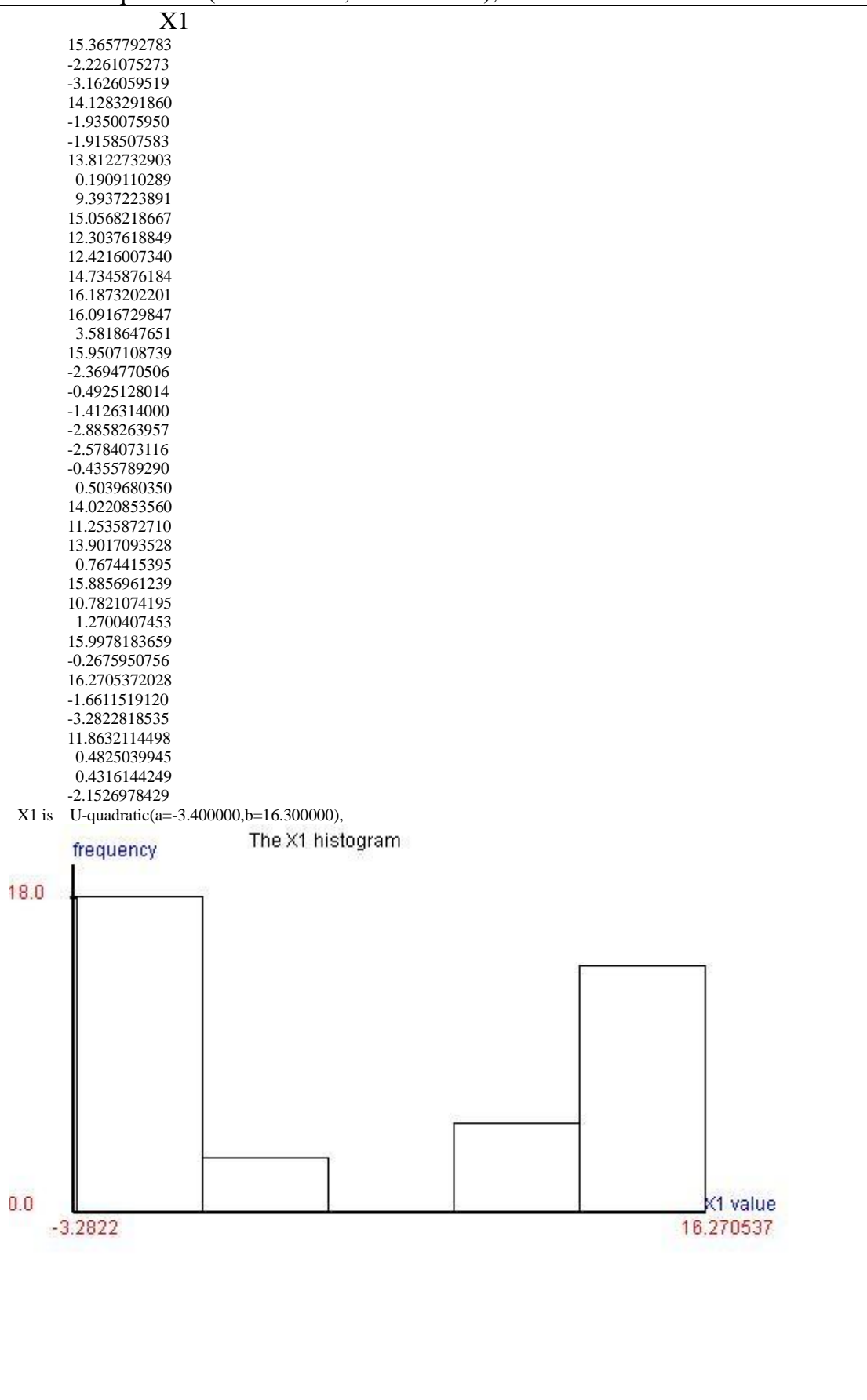
vertical axis is the value in according H0 population distribution.

The Q Q plot images is stored in c:\book\_01\QQ\_plot15\_image.jpg



7.2.16)The population distribution is U quadratic distribution.

X1 is U-quadratic(a=-3.400000,b=16.300000),





H0:  $X_1 \sim U_{\text{quadratic}}(a,b)$ ,  $a,b$  are unknown

$a$  point estimated value = -3.282282 (MLE)

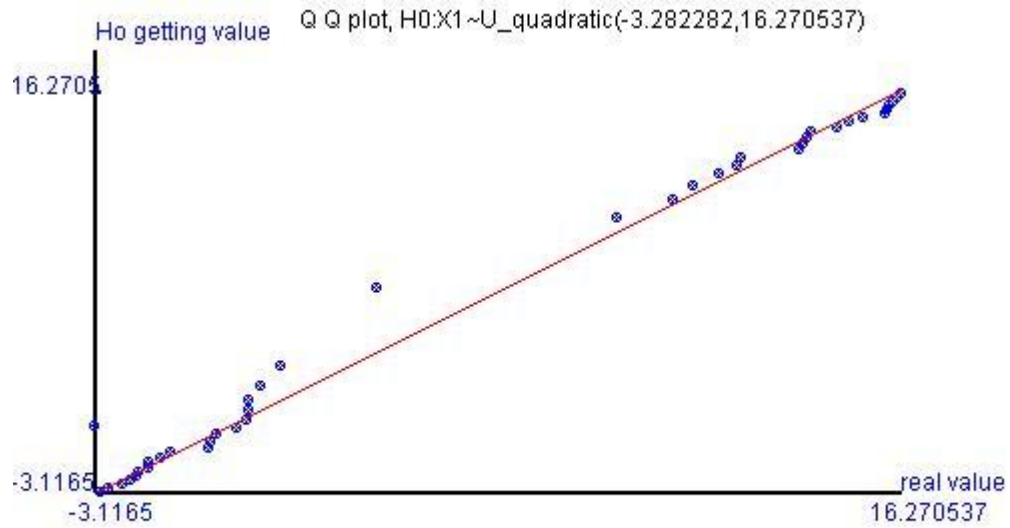
$b$  point estimated value = 16.270537 (MLE)

Q Q plot

horizontal axis is samples value,

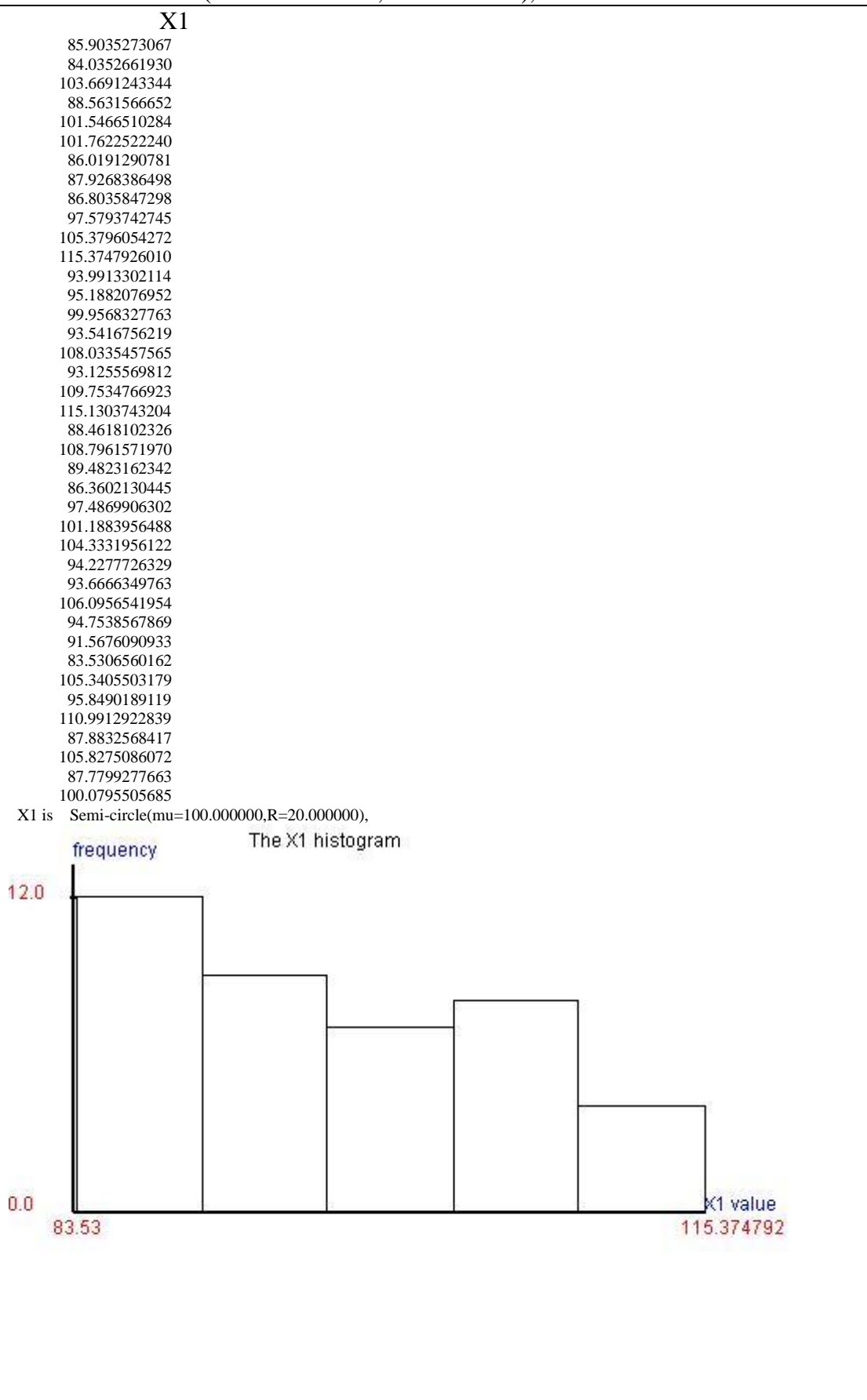
vertical axis is the value in according  $H_0$  population distribution.

The Q Q plot images is stored in `c:\book_01\QQ_plot16_image.jpg`



7.2.17)The population distribution is semi circle distribution.

X1 is Semi-circle( $\mu=100.000000,R=20.000000$ ),



H0:  $X_1 \sim \text{Semi-circle}(\mu, R)$ ,  $\mu, R$  are unknown

$\mu$  point estimated value = 97.174667

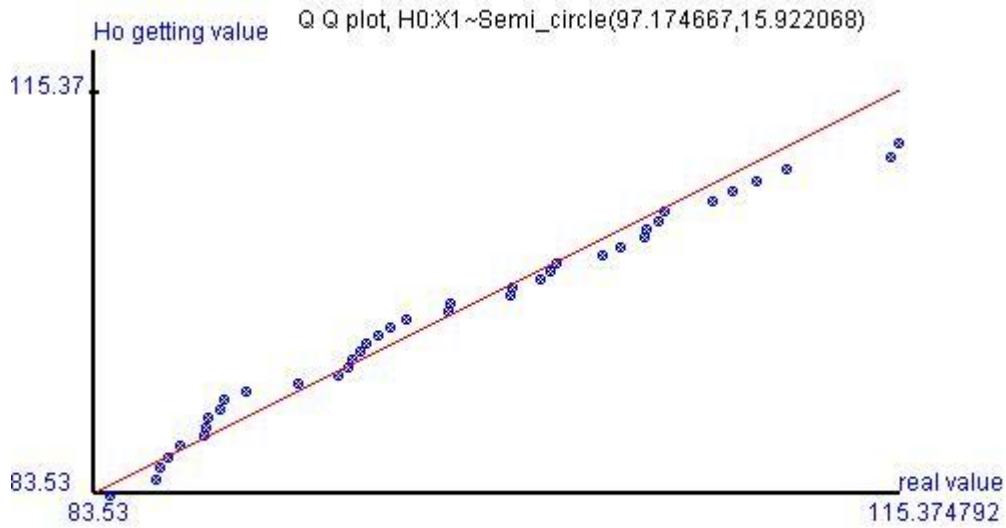
$R$  point estimated value = 15.922068 (MLE)

Q Q plot

horizontal axis is samples value,

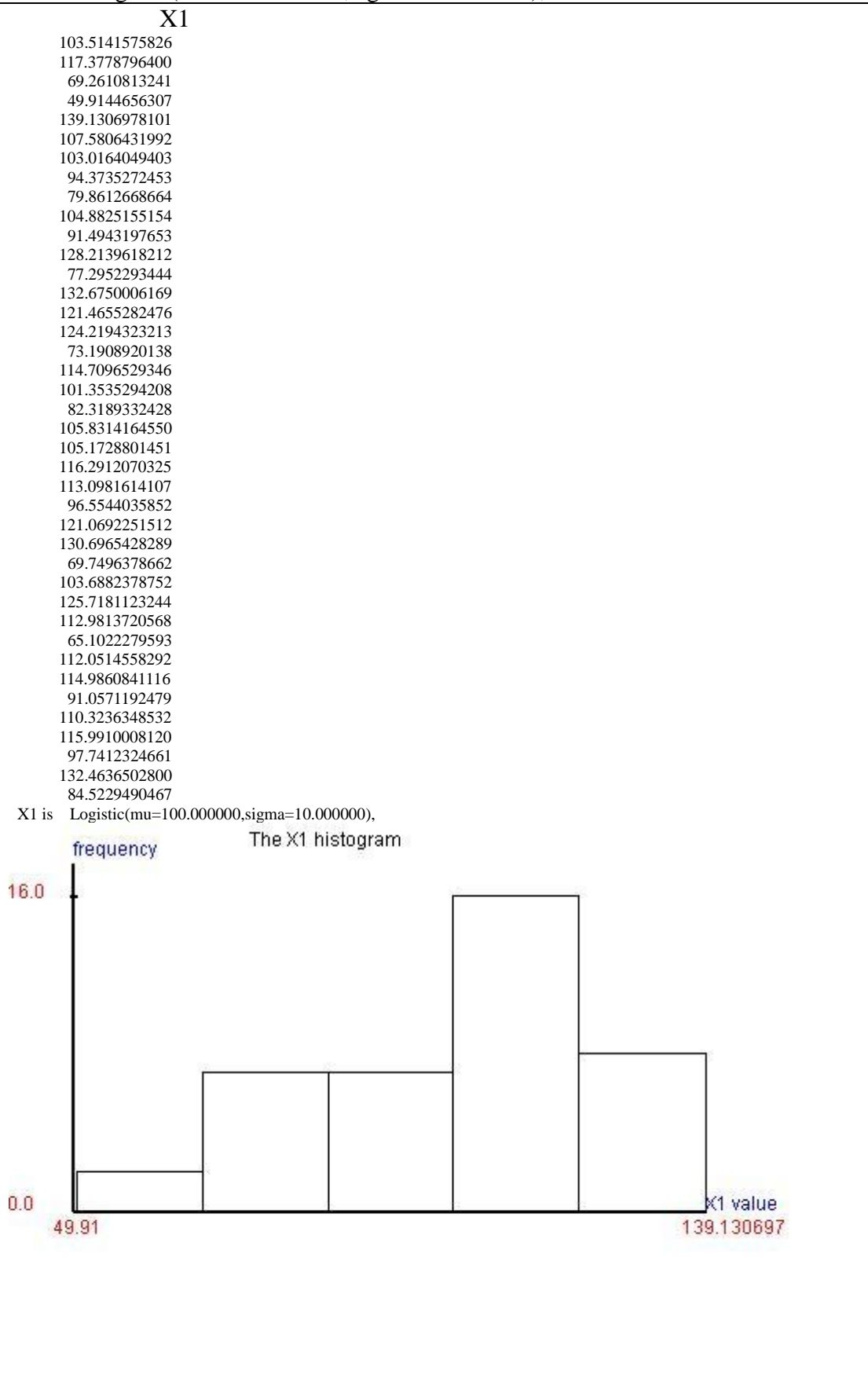
vertical axis is the value in according H0 population distribution.

The Q Q plot images is stored in c:\book\_01\QQ\_plot17\_image.jpg



7.2.18)The population distribution is logistic distribution.

X1 is Logistic( $\mu=100.000000,\sigma=10.000000$ ),



H0:  $X_1 \sim \text{Logistic}(\mu, \sigma)$ ,  $\mu, \sigma$  are unknown

$\mu$  point estimated value=103.523492 (MME)

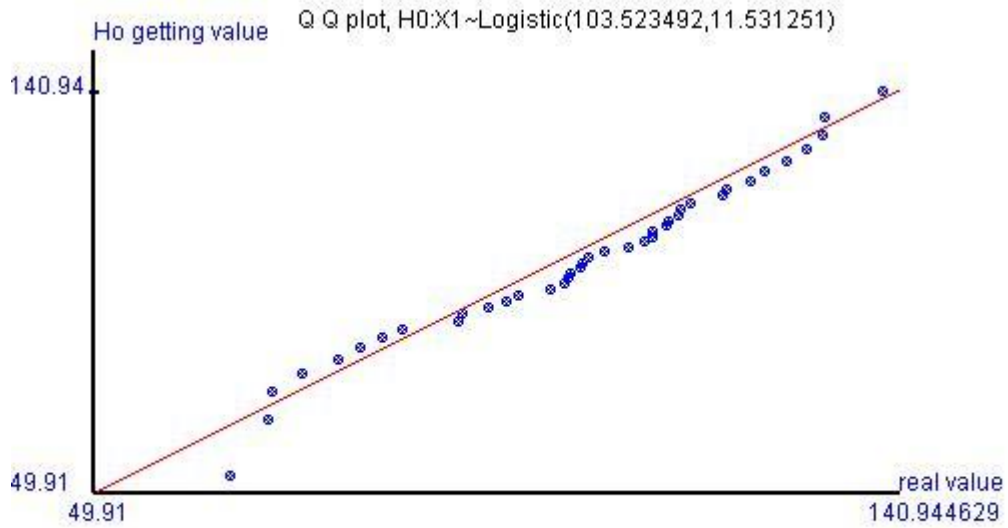
$\sigma$  point estimated value=11.531251 (MME)

Q Q plot

horizontal axis is samples value,

vertical axis is the value in according H0 population distribution.

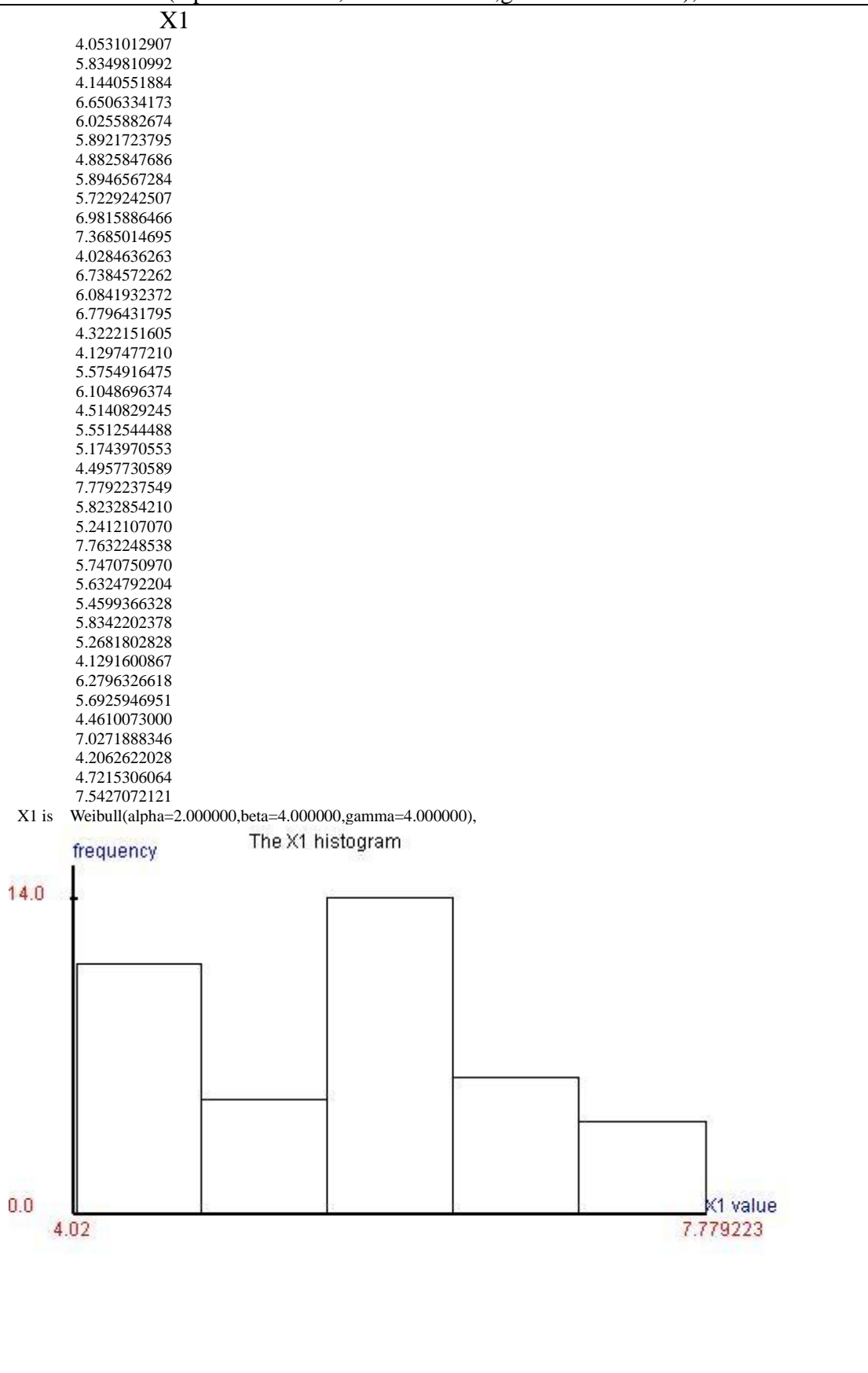
The Q Q plot images is stored in c:\book\_01\QQ\_plot18\_image.jpg



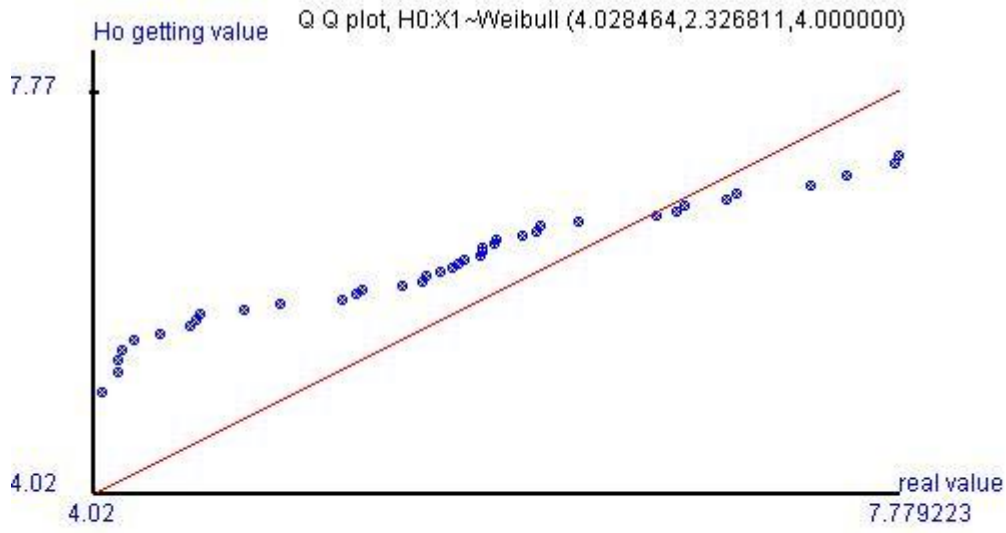
7.2.19)The population distribution is weibull distribution.

The gamma values is supposed to 4.

X1 is Weibull(alpha=2.000000,beta=4.000000,gamma=4.000000),



H0:  $X_1 \sim \text{Weibull}(\alpha, \beta, \gamma = 4.000000)$ ,  $\alpha, \beta$  are unknown  
alpha point estimated value = 4.028464 (MLE)  
beta point estimated value = 2.326811 (MLE)  
gamma value = 4.000000 (hypothesis value)  
Q Q plot  
horizon axis is samples value,  
vertical axis is the value in according Ho population distribution.  
The Q Q plot images is stored in c:\book\_01\QQ\_plot19\_image.jpg







H0:  $X_1 \sim \text{Pareto } 3(\lambda, c)$ ,  $\lambda, c$  are unknown

$\lambda$  point estimated value = 1.859225 (MLE)

$c$  point estimated value = 2.242741 (MLE)

Q Q plot

horizontal axis is samples value,

vertical axis is the value in according  $H_0$  population distribution.

The Q Q plot images is stored in c:\book\_01\QQ\_plot20\_image.jpg

