

## Chapter six The goodness of fit test

6).P P plot:

6.1)The process.

$H_0$  : Population distribution is a continuous probability distribution.

There are sample data and the sample size is n.

The  $x$  is sample value after ascending sorting.

(number(less than and equal  $x$ )/n,  $P(X \leq x)$ ,  $X \sim H_0$  : Population distribution)

which scatter diagram is P P plot.

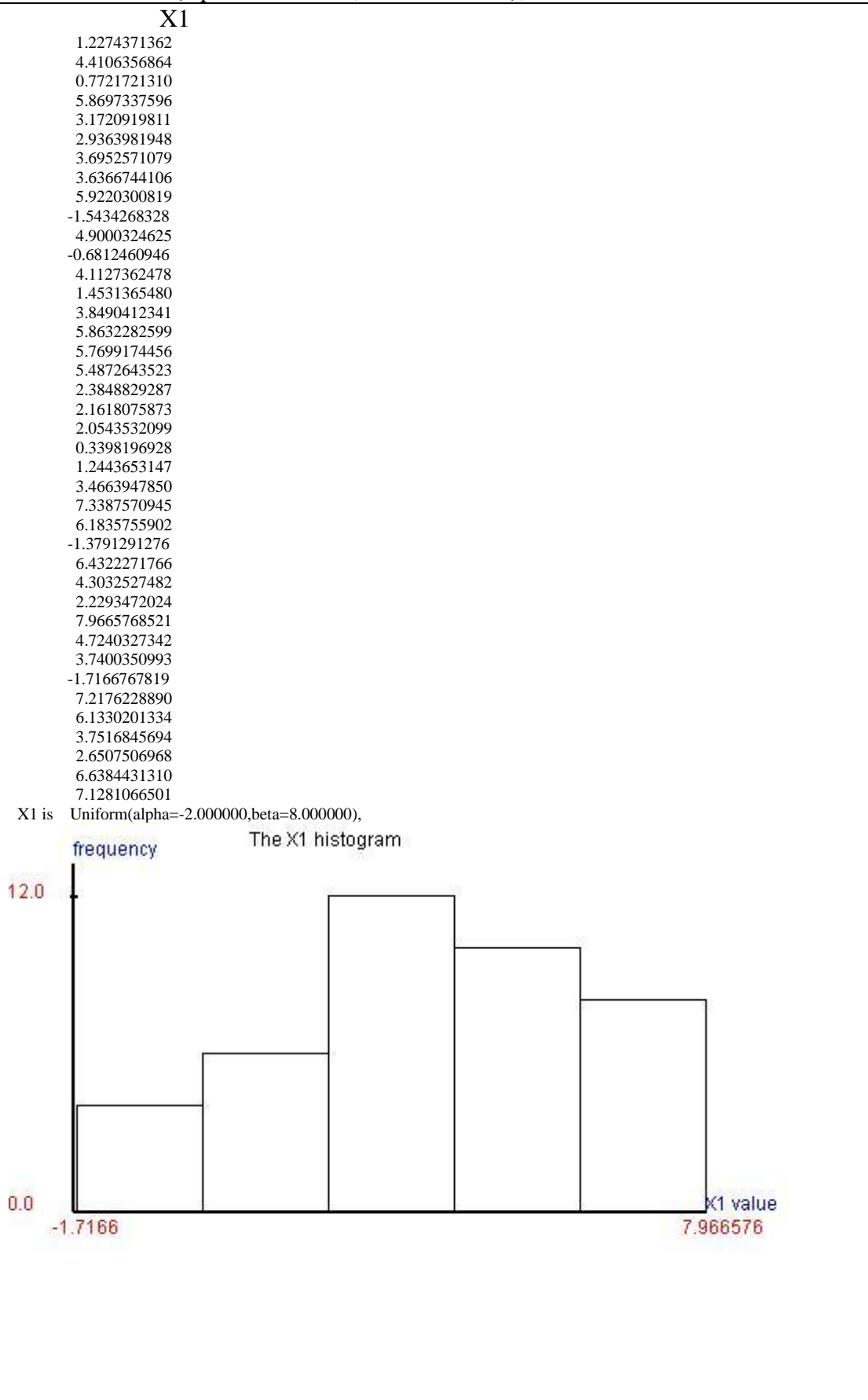
6.2) Example (The simulated sample data and computing the result by the P\_S\_CCC)

The P-P plot, P P is probability(cumulaitve relative frequency), probability(under H0 df)

1.H0:Uniform distribution	13.H0:Gumbel distribution
2.H0:Normal distribution	14.H0:Triangular 1 distribution
3.H0:Shifted exponential distribution	15.H0:Trapezoid distribution
4.H0:Pareto 1 distribution	16.H0:U-quadractic distribution
5.H0:Pareto 2 distribution	17.H0:Semi-circle distribution
6.H0:Rayleigh distribution	18.H0:Logistic distribution
7.H0:Double expoenoential distribution	19.H0:Weibull distribution
8.H0:Log normal distribution	20.H0:Pareto 3 distribution
9.H0:Gamma distribution	** Above Ho population all do once
10.H0:Beta distribution	
11.H0:Cauchy distribution	
12.H0:Arcsin distribution	

6.2.1)The population distribution is uniform distribution.

X1 is Uniform(alpha=-2.000000,beta=8.000000),



H0:  $X_1 \sim \text{Uniform}(\alpha, \beta)$ ,  $\alpha, \beta$  are unknown

$\alpha$  point estimated value = -1.716677 (MLE)

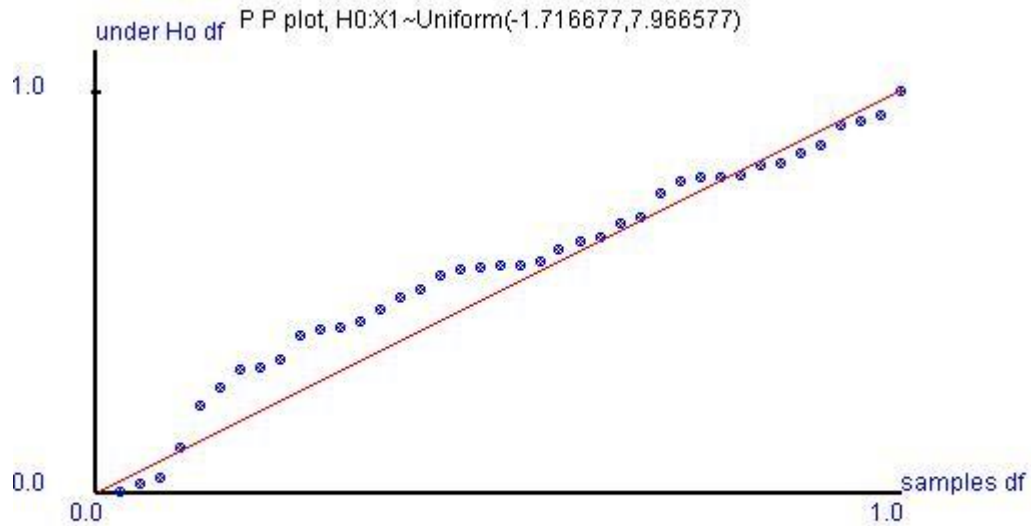
$\beta$  point estimated value = 7.966577 (MLE)

p p plot

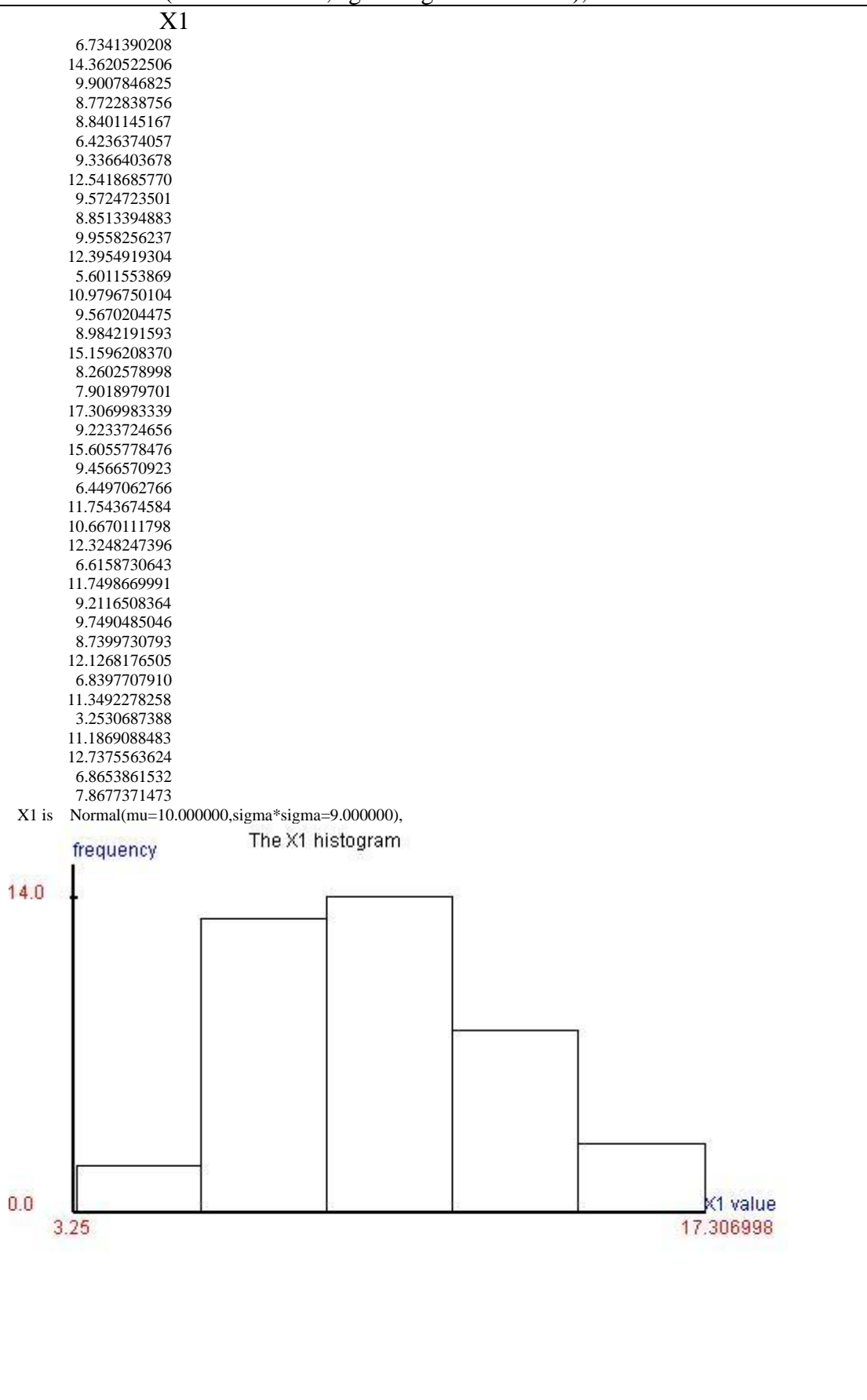
horizontal axis is samples cumulative relative frequency,

vertical axis is cumulative probability under H0 from sample values.

The p p plot image is stored in c:\book\_01\pp\_plot01\_image.jpg



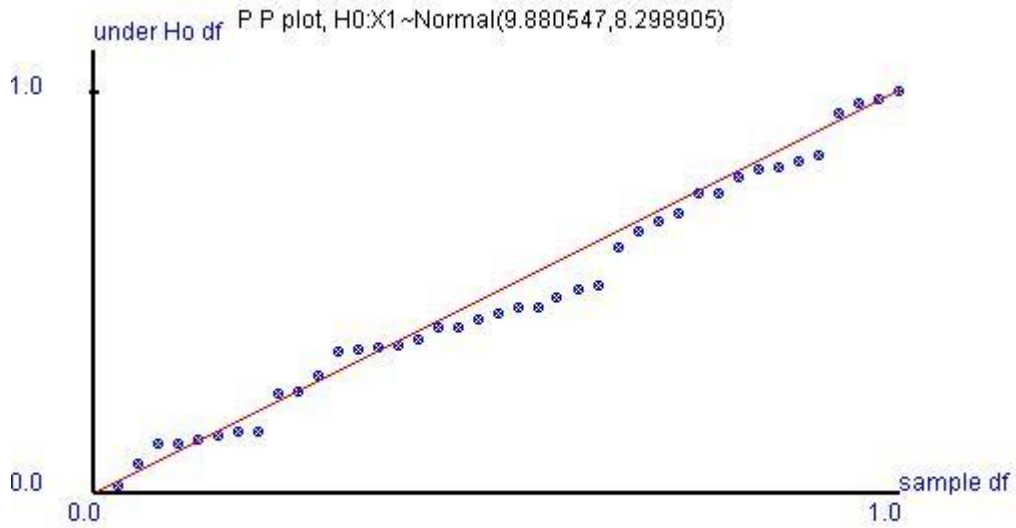
6.2.2)The population distribution is normal distribution.  
 X1 is Normal( $\mu=10.000000$ , $\sigma^2=9.000000$ ),



H0:  $X_1 \sim \text{Normal}(\mu, \sigma^2)$ ,  $\mu, \sigma$  are unknown  
population mean( $\mu$ ) point estimated value=9.880547 (MLE,UMVUE)  
population variance( $\sigma^2$ ) which point estimated value=8.298905 (UMVUE)

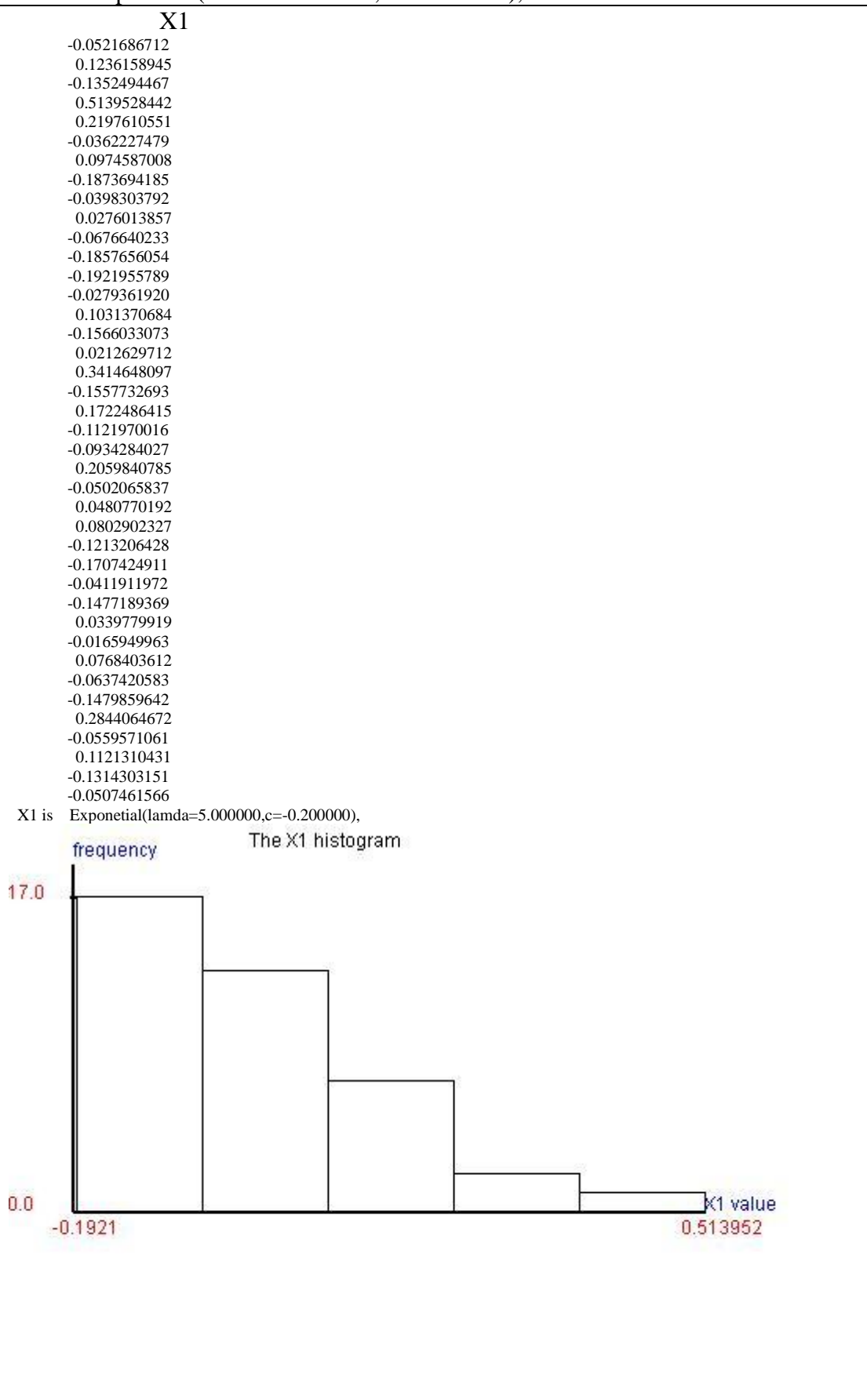
horizontal axis is samples cumulative relative frequency,  
vertical axis is cumulative probability under H0 from sample values.

The p p plot images is stored in c:\book\_01\pp\_plot02\_image.jpg

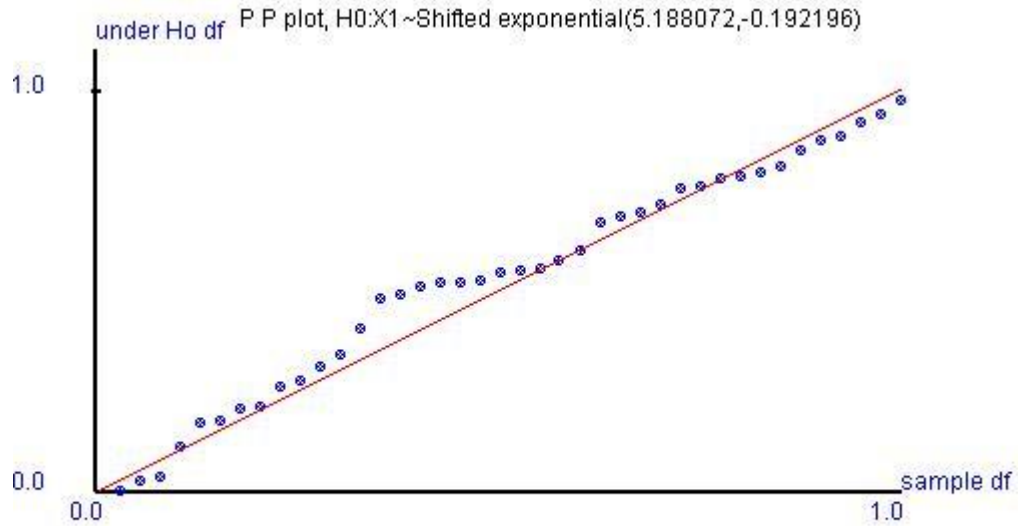


6.2.3) The population distribution is shifted exponential distribution.

X1 is Exponential( $\lambda=5.000000, c=-0.200000$ ),

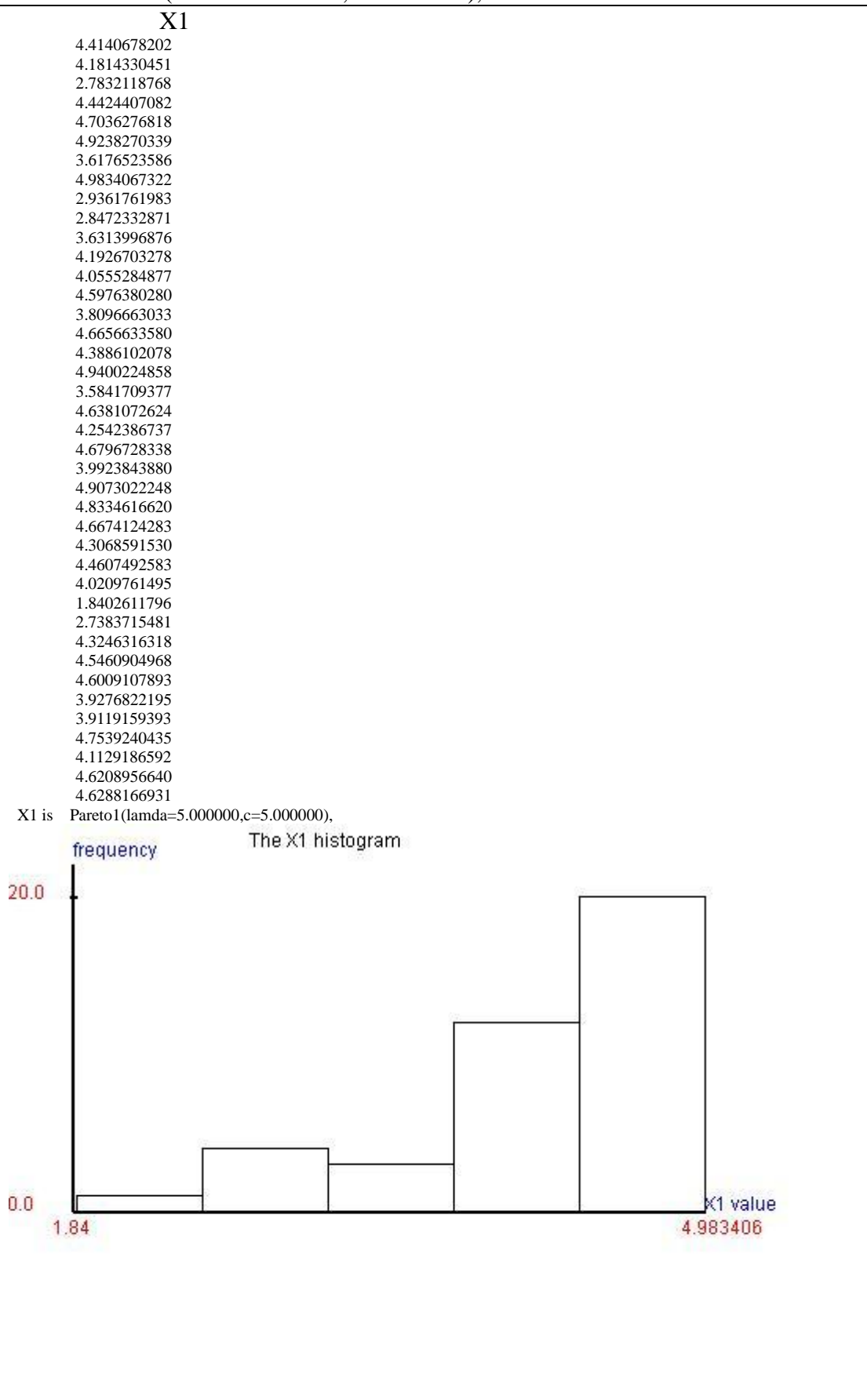


H0:  $X_1 \sim \text{Shifted exponential}(\lambda, c)$ ,  $\lambda, c$  are unknown  
 $\lambda$  point estimated value = 5.188072 (MLE)  
 $c$  point estimated value = -0.192196 (MLE)  
horizon axis is samples cumulative relative frequency,  
vertical axis is cumulative probability under H0 from sample values.  
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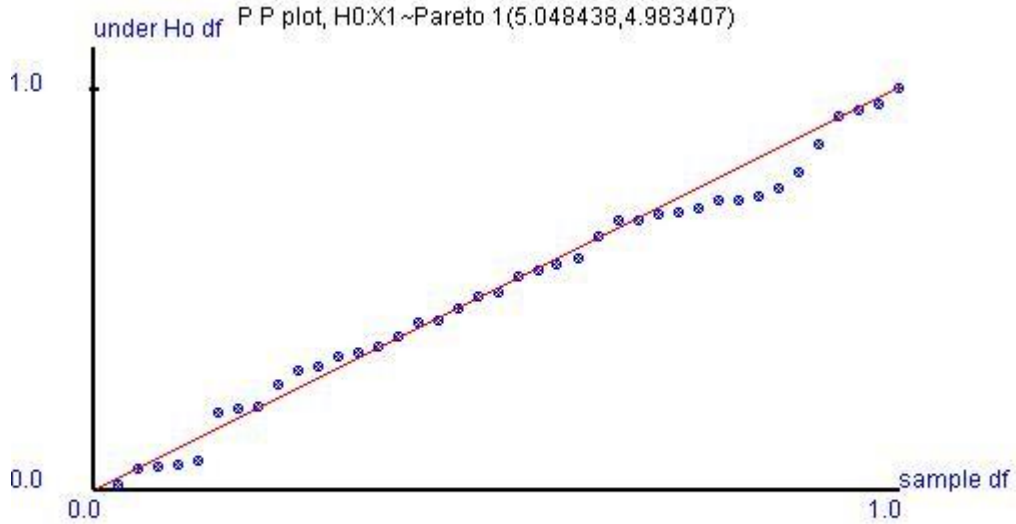
6.2.4)The population distribution is pareto1 distribution.

X1 is Pareto1(lamda=5.000000,c=5.000000),



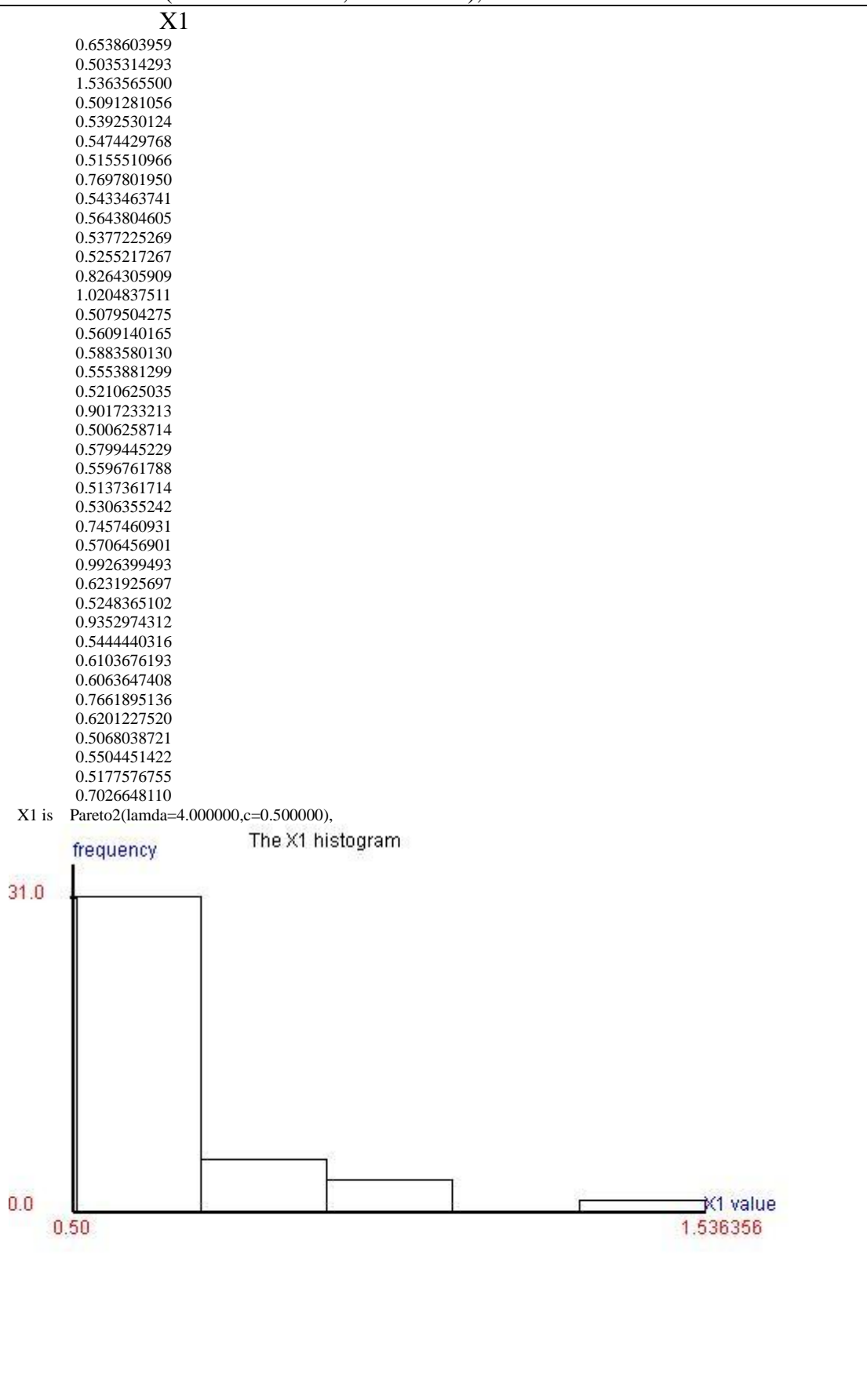


$H_0: X_1 \sim \text{Pareto } 1(\lambda, c)$ ,  $\lambda, c$  are unknown  
 $\lambda$  point estimated value = 5.048438 (MLE)  
 $c$  point estimated value = 4.983407 (MLE)  
horizontal axis is samples cumulative relative frequency,  
vertical axis is cumulative probability under  $H_0$  from sample values.  
The p p plot images is stored in c:\book\_01\pp\_plot04\_image.jpg



6.2.5)The population distribution is pareto2 distribution.

X1 is Pareto2(lamda=4.000000,c=0.500000),



H0:  $X_1 \sim \text{Pareto 2}(\lambda, c)$ ,  $\lambda, c$  are unknown

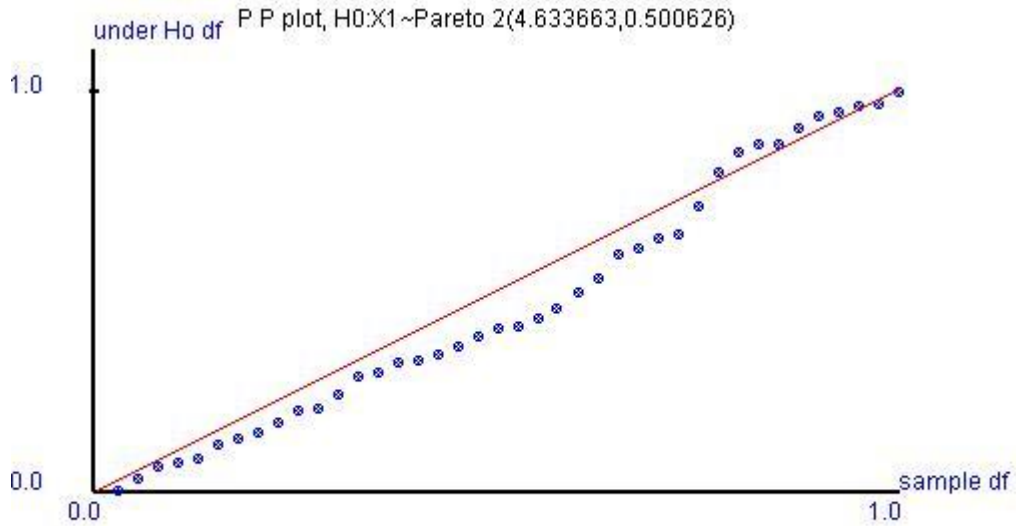
$\lambda$  point estimated value=4.633663 (MLE)

$c$  point estimated value=0.500626 (MLE)

horizon axis is samples cumulative relative frequency,

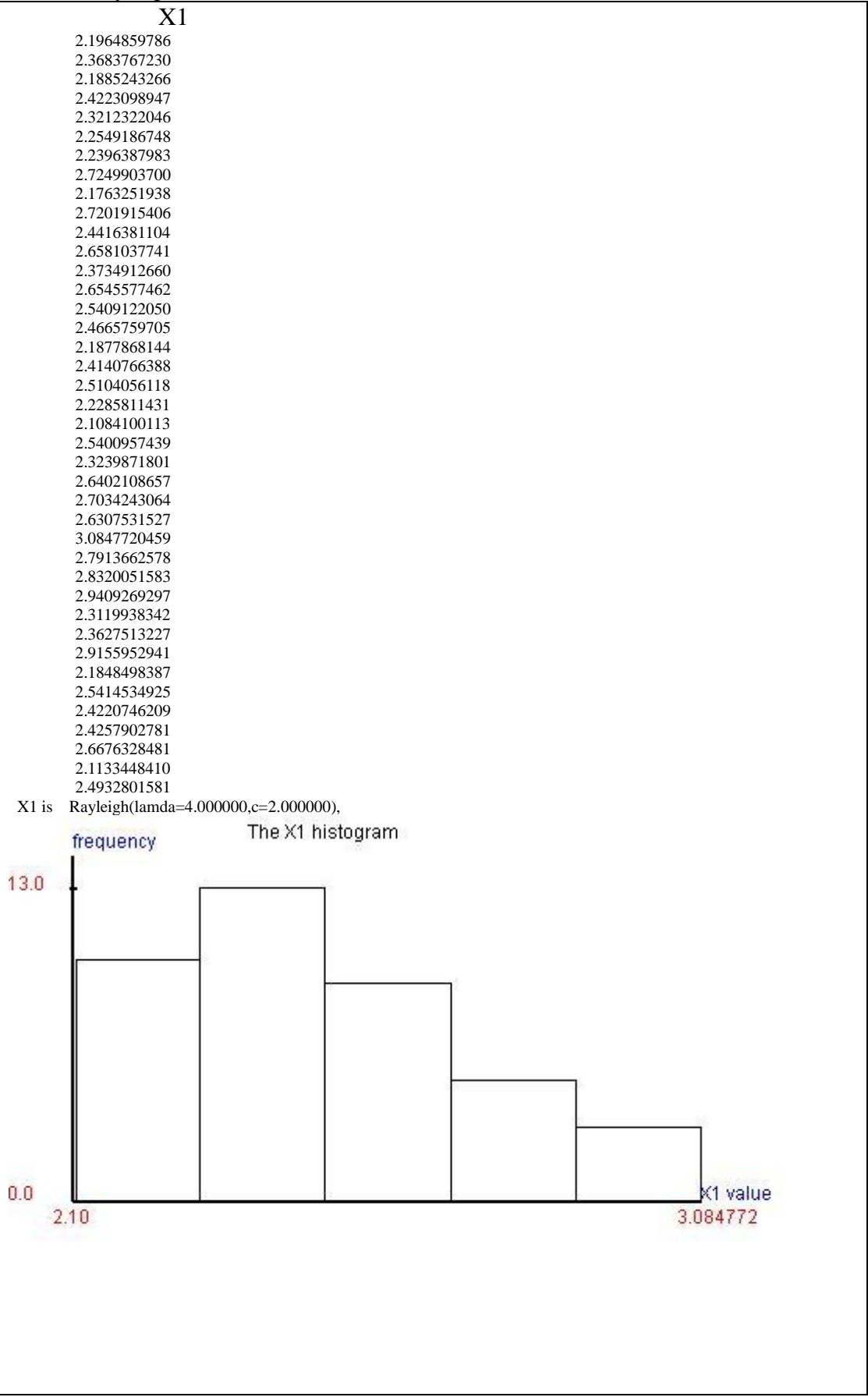
vertical axis is cumulative probability under H0 from sample values.

The p p plot images is stored in c:\book\_01\pp\_plot05\_image.jpg

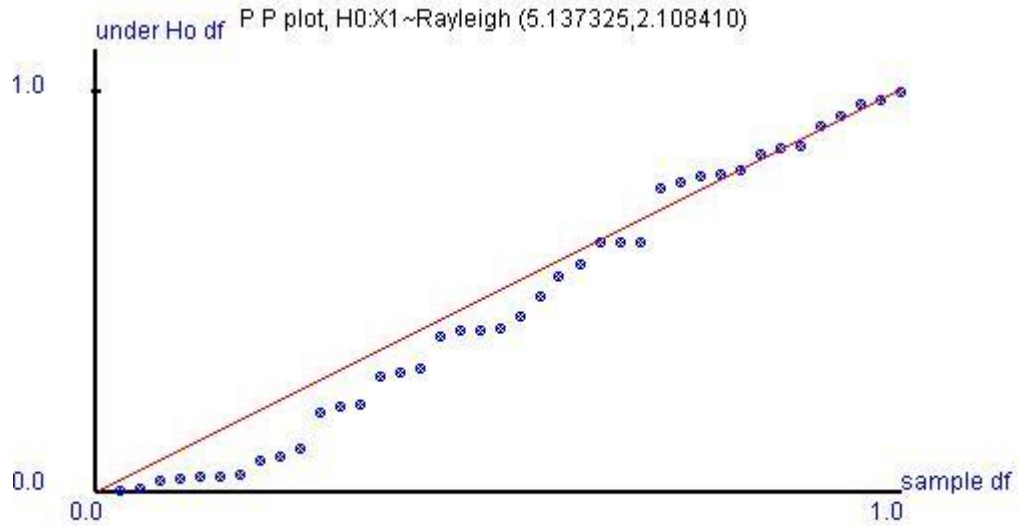


6.2.6)The population distribution is rayleigh distribution.

X1 is Rayleigh(lamda=4.000000,c=2.000000),

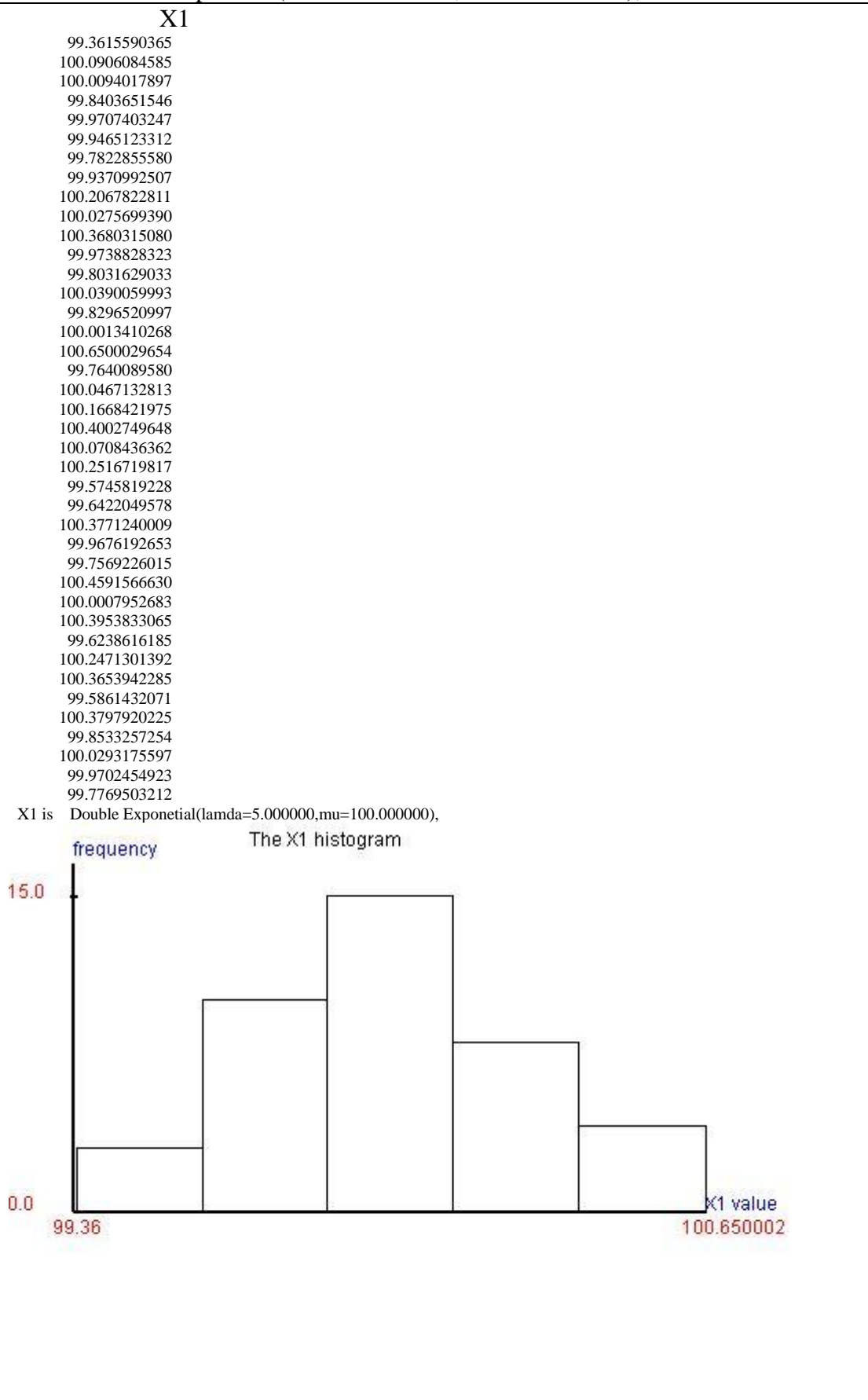


H0:  $X_1 \sim \text{Rayleigh}(\lambda, c)$ ,  $\lambda, c$  are unknown  
 $\lambda$  point estimated value=5.137325 (MLE)  
 $c$  point estimated value=2.108410 (MLE)  
horizon axis is samples cumulative relative frequency,  
vertical axis is cumulative probability under H0 from sample values.  
The p p plot images is stored in c:\book\_01\pp\_plot06\_image.jpg

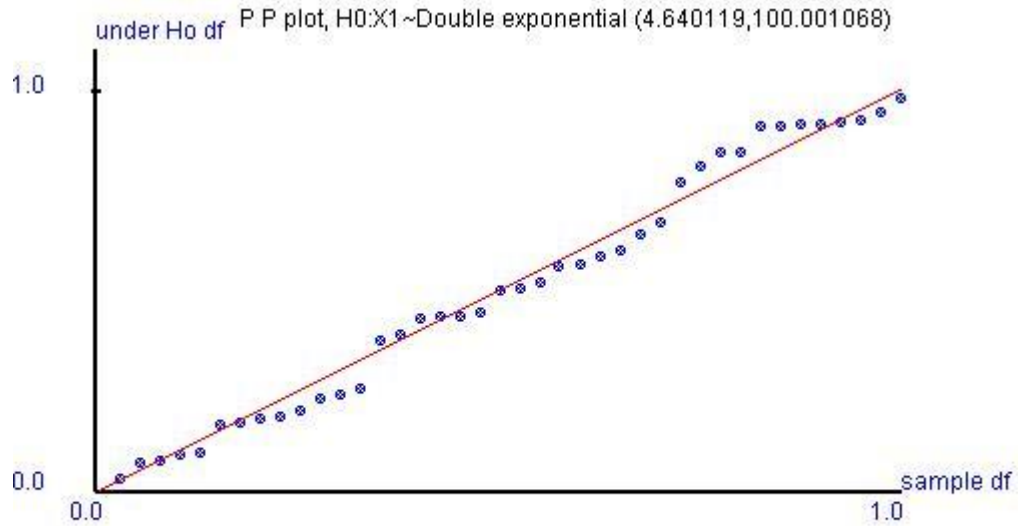


6.2.7) The population distribution is double exponential distribution.

X1 is Double Exponential( $\lambda=5.000000, \mu=100.000000$ ),

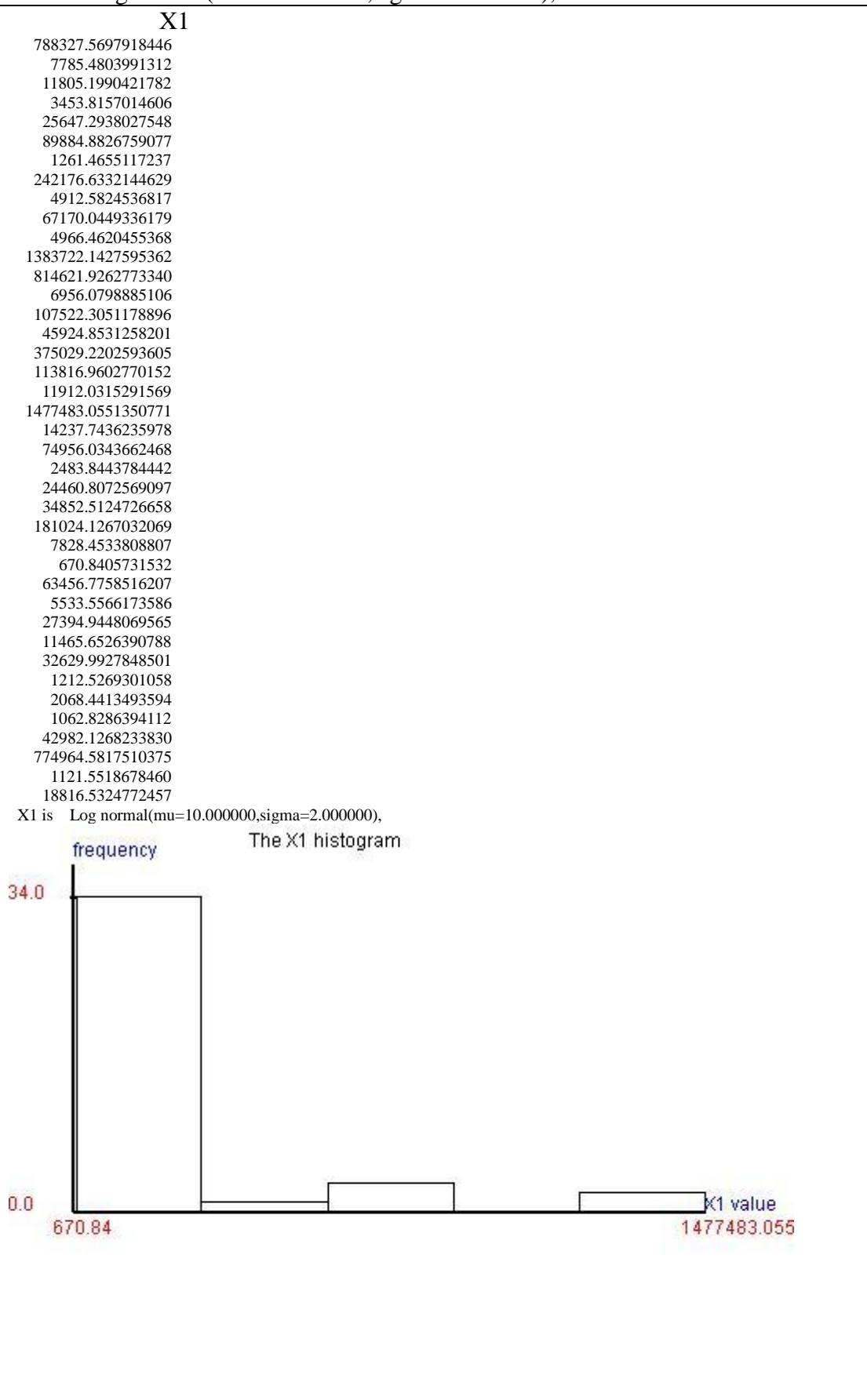


H0:  $X_1 \sim \text{Double exponential}(\lambda, \mu)$ ,  $\lambda, \mu$  are unknown  
 $\lambda$  point estimated value = 4.640119 (MLE)  
 $\mu$  point estimated value = 100.001068 (MLE)  
horizontal axis is samples cumulative relative frequency,  
vertical axis is cumulative probability under H0 from sample values.  
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6.2.8)The population distribution is lognormal distribution.

X1 is Log normal( $\mu=10.000000,\sigma=2.000000$ ),

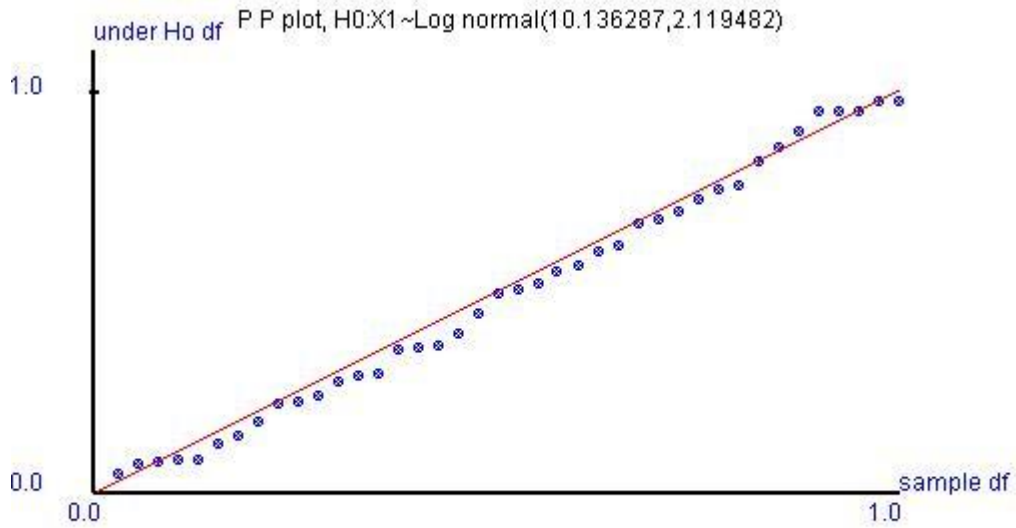




H0:  $X_1 \sim \text{Log\_Normal}(\mu, \sigma^2)$ ,  $\mu, \sigma$  are unknown  
population mean( $\mu$ ) point estimated value=10.136287 (MLE,UMVUE)  
population variance( $\sigma^2$ ) which point estimated value=4.492204 (UMVUE)

horizontal axis is samples cumulative relative frequency,  
vertical axis is cumulative probability under H0 from sample values.

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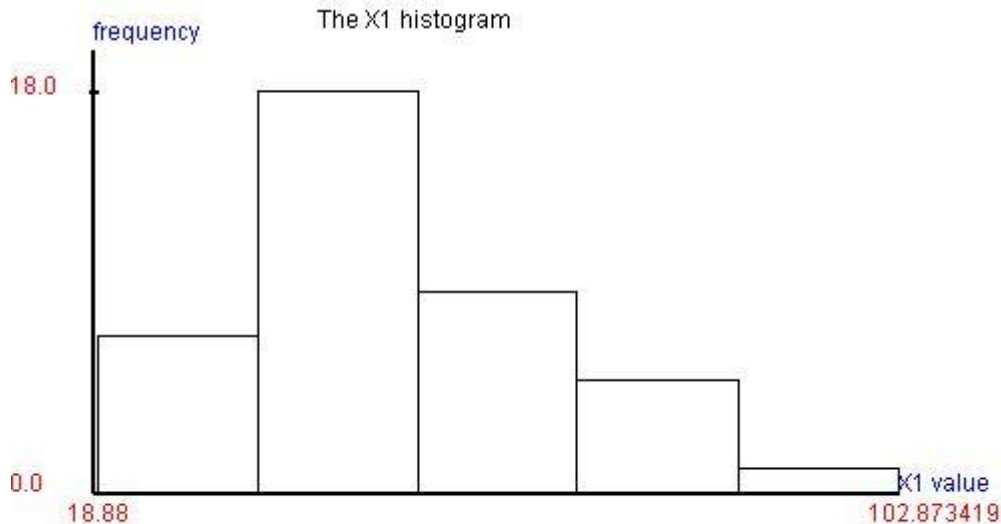


6.2.9)The population distribution is gamma distribution.

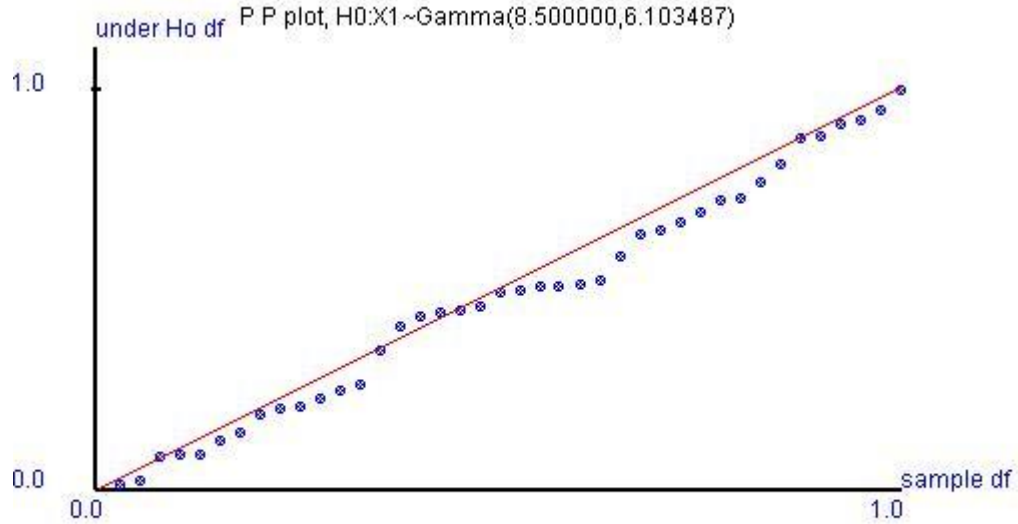
X1 is  $\text{Gamma}(\alpha=10.000000, \beta=5.000000)$ ,

X1  
 78.3296126571  
 72.7720826490  
 76.4577028930  
 18.8874908637  
 60.5132921992  
 21.7028628525  
 38.7119760096  
 53.3304681297  
 35.8067081706  
 29.1342802368  
 45.6598849494  
 43.1121564332  
 72.6233005620  
 33.2679724215  
 63.4292095898  
 66.4148999733  
 49.2226647645  
 36.4860602517  
 29.6365716180  
 50.2428345984  
 47.2699238872  
 37.7540475562  
 50.5885970860  
 57.4865544154  
 32.0859471303  
 56.5168684386  
 58.8736733058  
 60.9174112594  
 47.4338484936  
 102.8734193504  
 83.0405908711  
 29.5301452096  
 50.0321663378  
 55.9229323654  
 36.8466768181  
 49.9717108340  
 39.4468036281  
 47.7765560704  
 46.6432012936  
 49.5441760434

X1 is  $\text{Gamma}(\alpha=10.000000, \beta=5.000000)$ ,

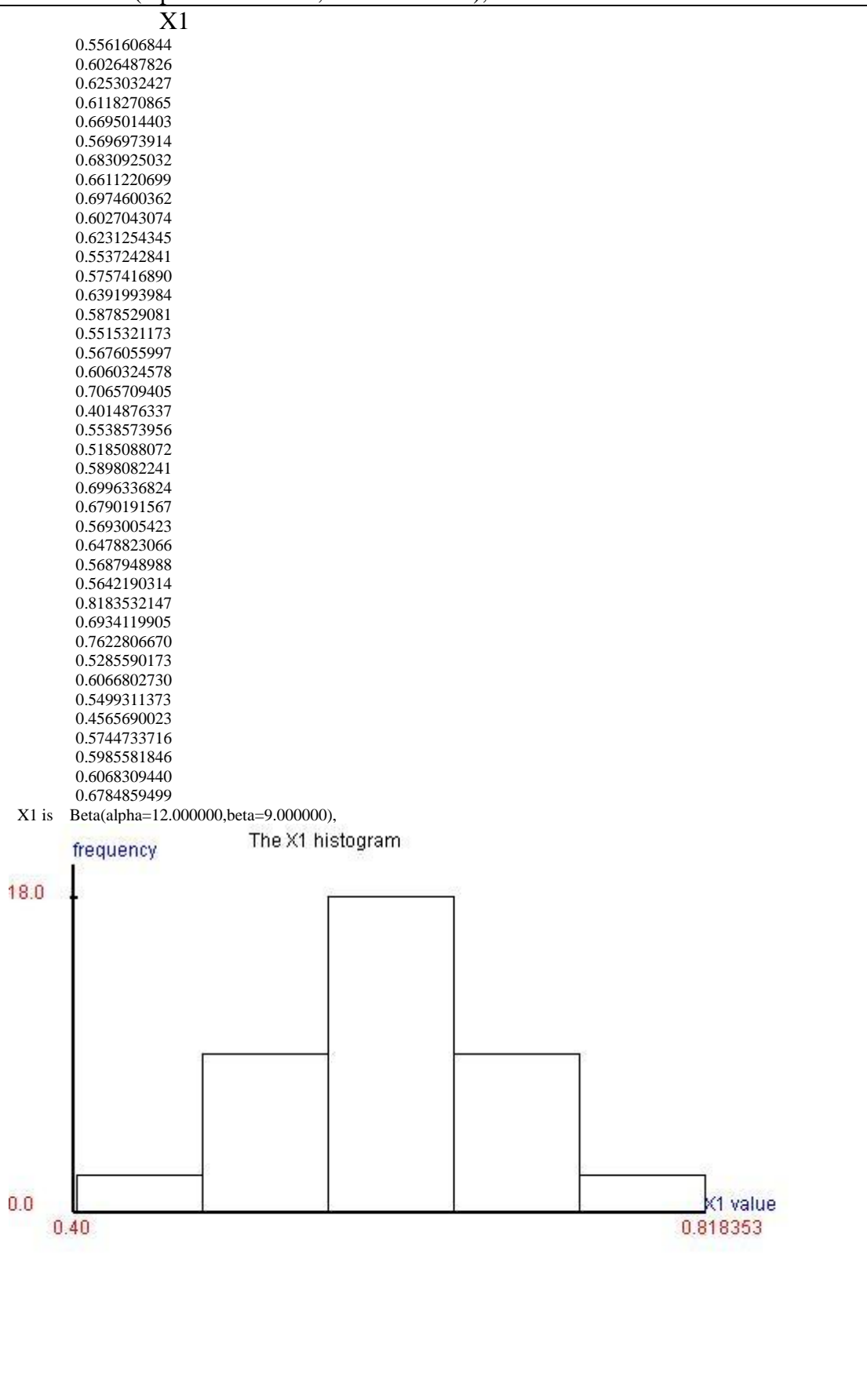


H0:  $X_1 \sim \text{Gamma}(\alpha, \beta)$ ,  $\alpha, \beta$  are unknown  
alpha point estimated value=8.500000 (MME)  
beta point estimated value=6.103487 (MME)  
horizon axis is samples cumulative relative frequency,  
vertical axis is cumulative probability under H0 from sample values.  
The p p plot images is stored in c:\book\_01\pp\_plot09\_image.jpg

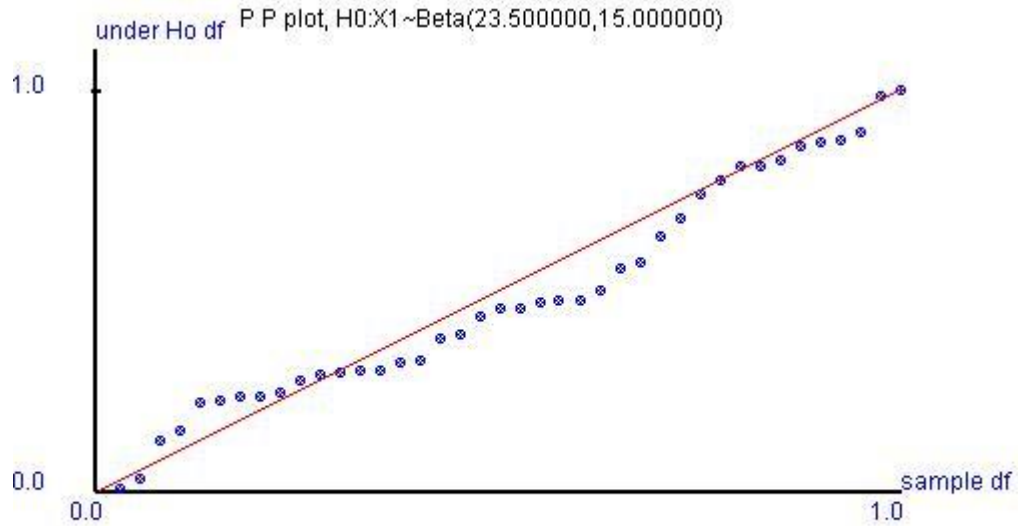


6.2.10)The population distribution is beta distribution.

X1 is Beta(alpha=12.000000,beta=9.000000),

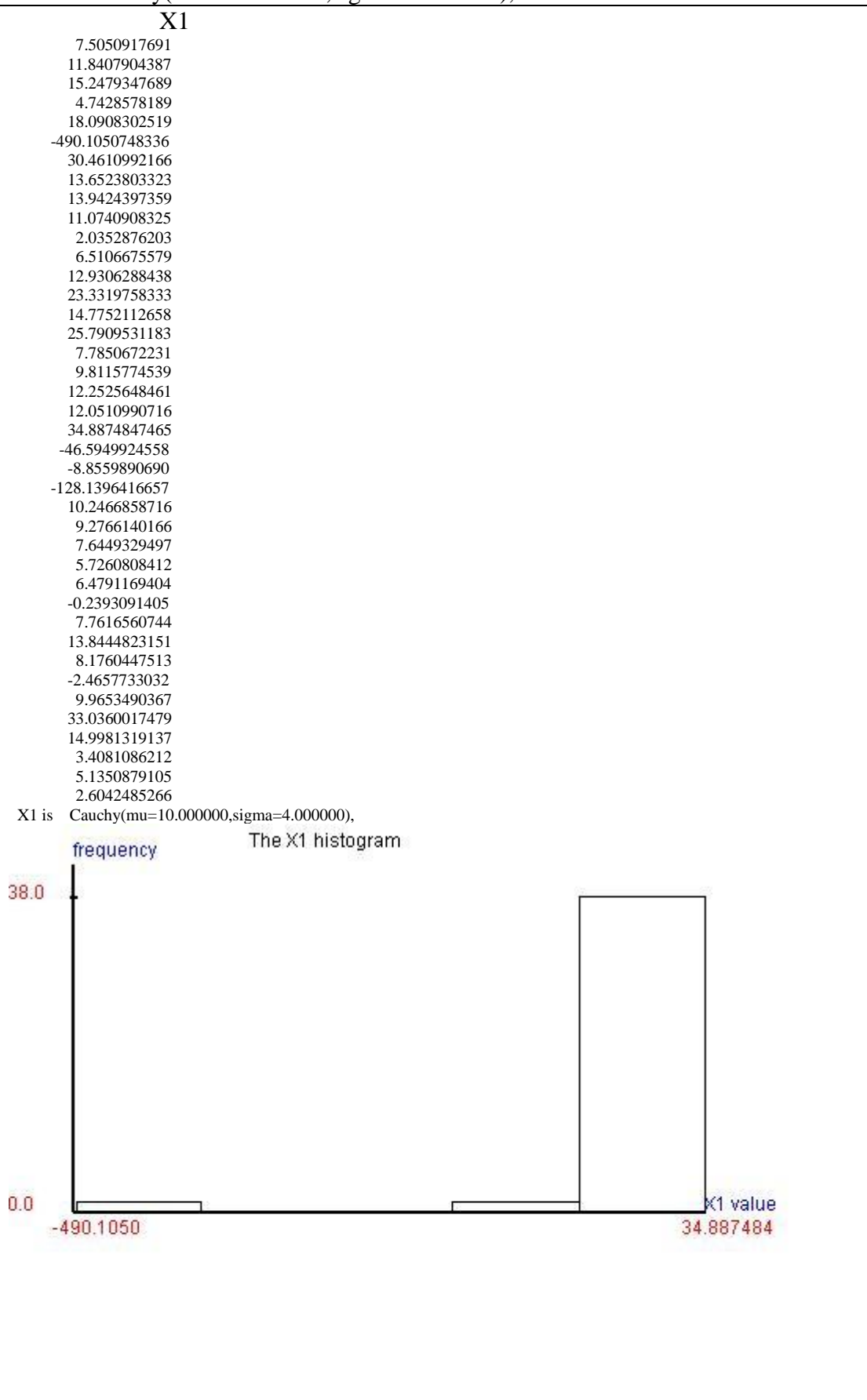


H0:  $X_1 \sim \text{Beta}(\alpha, \beta)$ ,  $\alpha, \beta$  are unknown  
alpha point estimated value=23.500000 (MME)  
beta point estimated value=15.000000 (MME)  
horizon axis is samples cumulative relative frequency,  
vertical axis is cumulative probability under H0 from sample values.  
The p p plot images is stored in c:\book\_01\pp\_plot10\_image.jpg

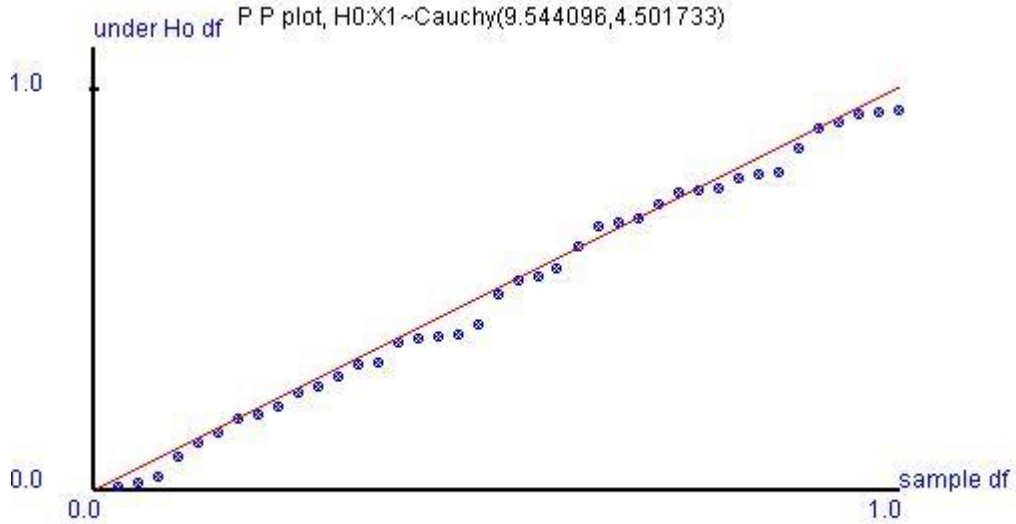


6.2.11)The population distribution is cauchy distribution.

X1 is Cauchy( $\mu=10.000000,\sigma=4.000000$ ),

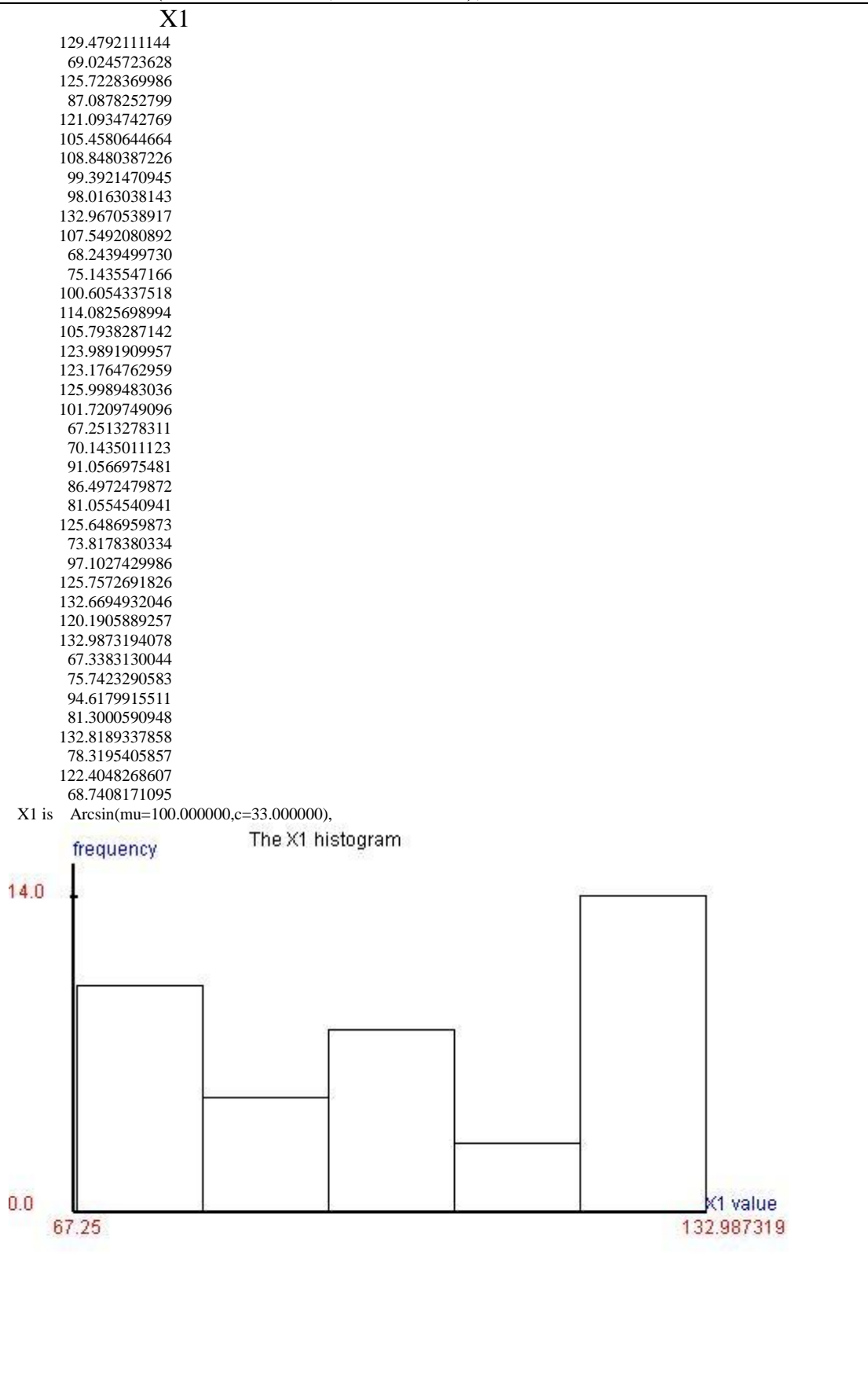


H0:  $X_1 \sim \text{Cauchy}(\mu, \sigma)$ ,  $\mu, \sigma$  are unknown  
mu point estimated value=9.544096  
sigma point estimated value=4.501733  
horizon axis is samples cumulative relative frequency,  
vertical axis is cumulative probability under H0 from sample values.  
The p p plot images is stored in c:\book\_01\pp\_plot11\_image.jpg



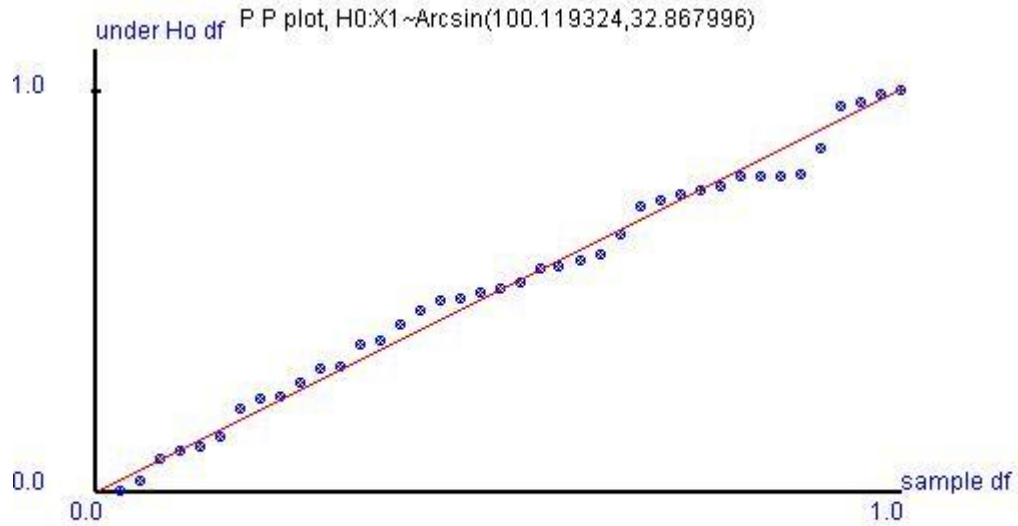
6.2.12)The population distribution is arcsin distribution.

X1 is  $\text{Arcsin}(\mu=100.000000,c=33.000000)$ ,



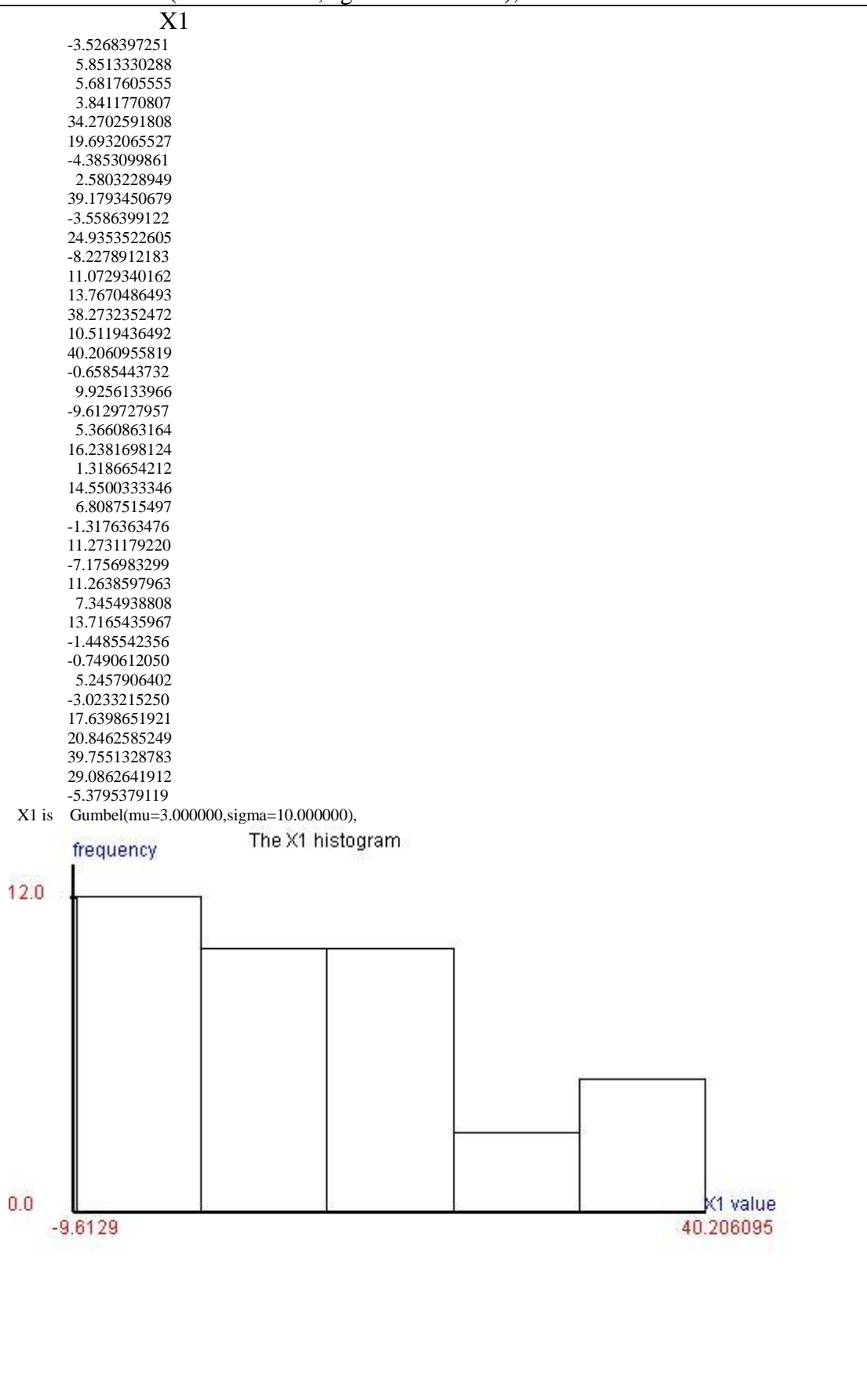


$H_0: X_1 \sim \text{Arcsin}(\mu, c)$ ,  $\mu, c$  are unknown  
 $\mu$  point estimated value=100.119324 (MLE)  
 $c$  point estimated value=32.867996 (MLE)  
horizon axis is samples cumulative relative frequency,  
vertical axis is cumulative probability under  $H_0$  from sample values.  
The p p plot images is stored in c:\book\_01\pp\_plot12\_image.jpg

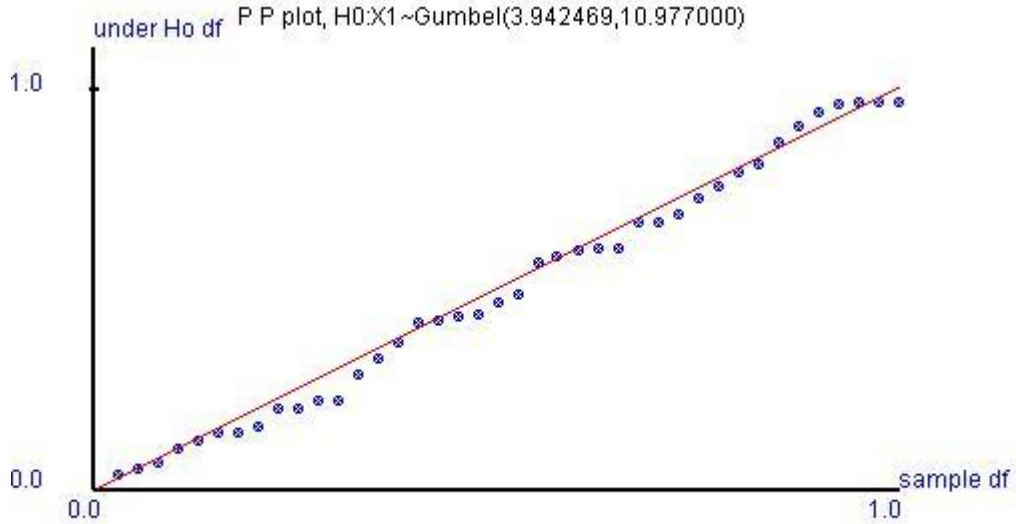


6.2.13)The population distribution is gumbel distribution.

X1 is Gumbel( $\mu=3.000000,\sigma=10.000000$ ),

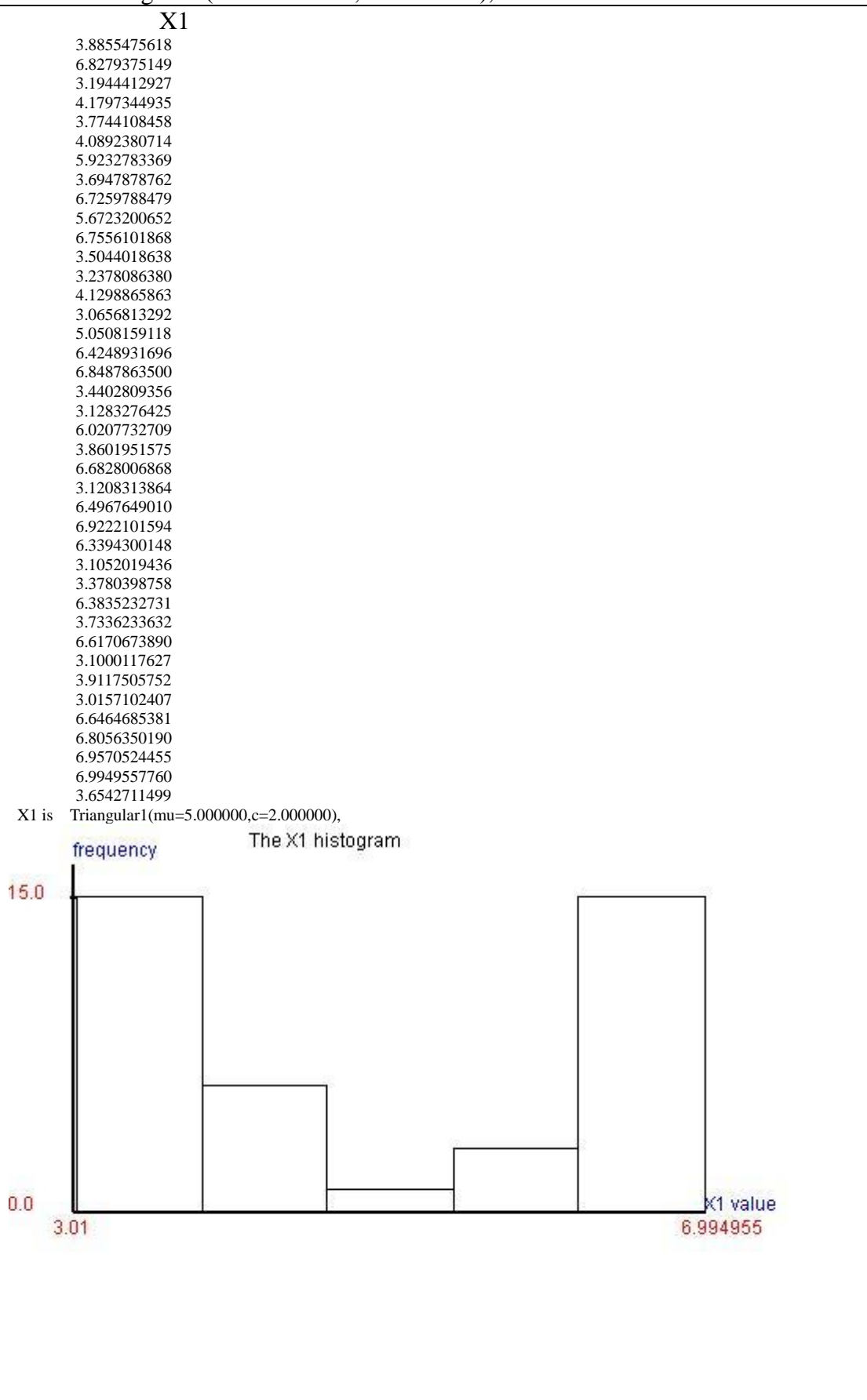


H0:  $X_1 \sim \text{Gumbel}(\mu, \sigma)$ ,  $\mu, \sigma$  are unknown  
mu point estimated value=3.942469 (MME)  
sigma point estimated value=10.977000 (MME)  
horizon axis is samples cumulative relative frequency,  
vertical axis is cumulative probability under H0 from sample values.  
The p p plot images is stored in c:\book\_01\pp\_plot13\_image.jpg

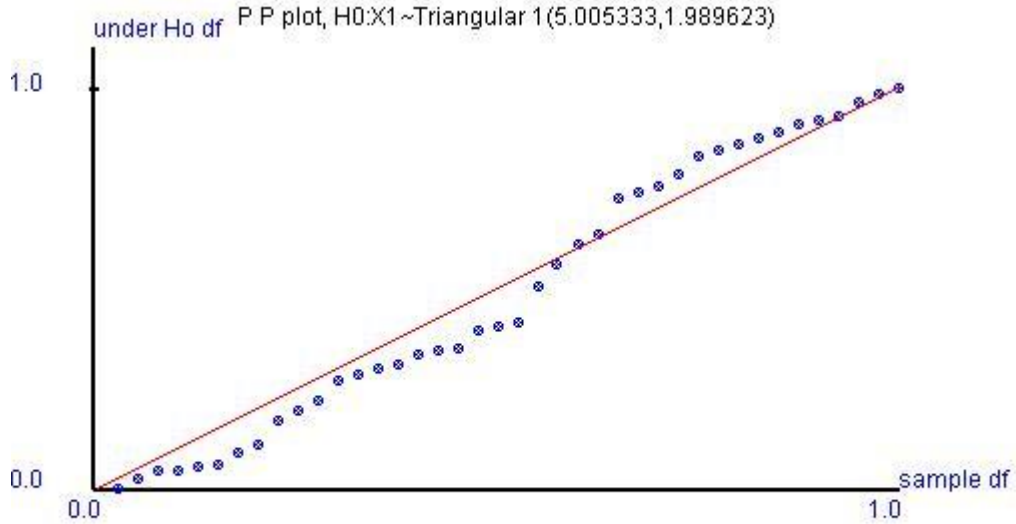


6.2.14) The population distribution is triangular 1 distribution.

X1 is  $\text{Triangular1}(\mu=5.000000, c=2.000000)$ ,

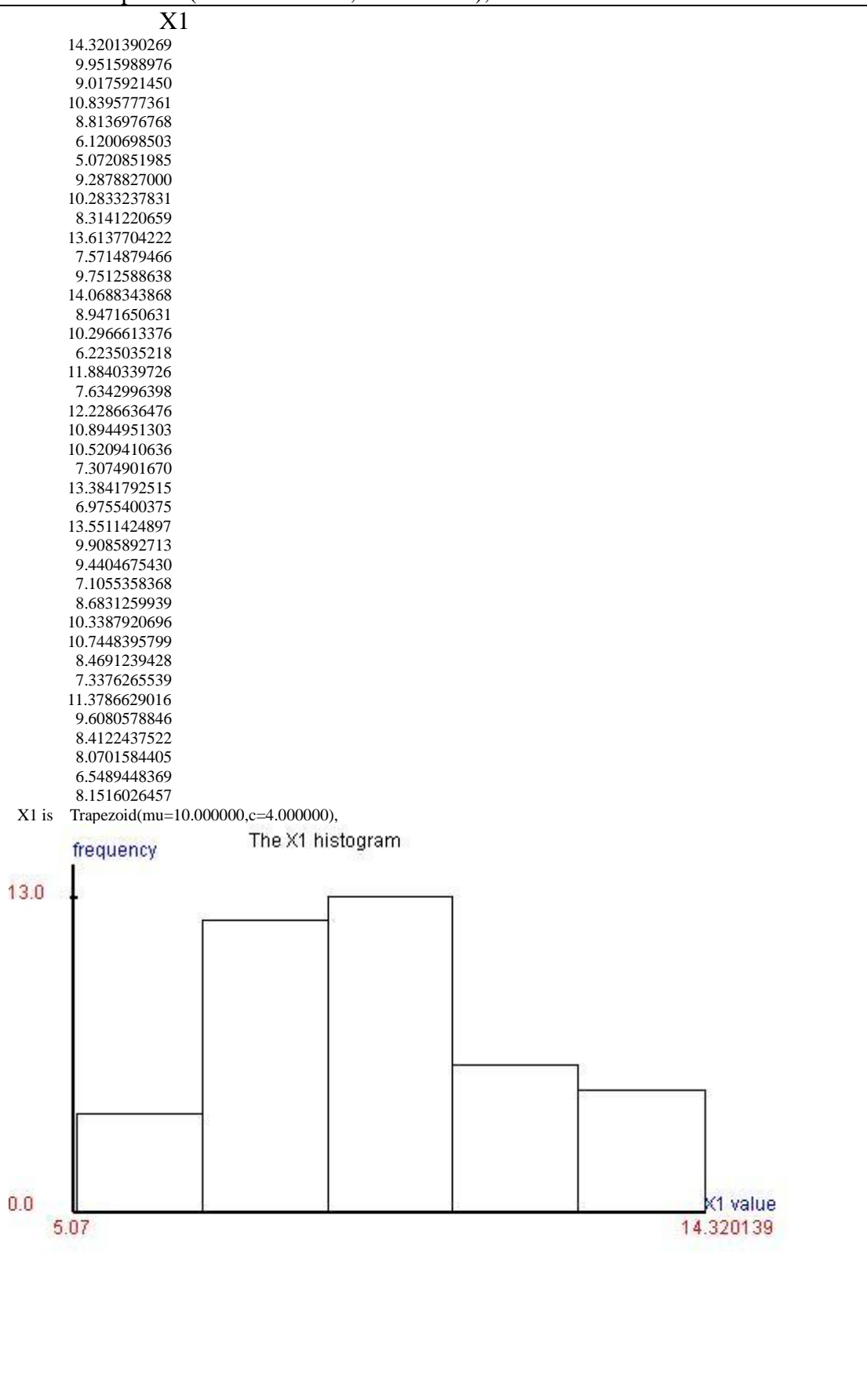


H0:  $X_1 \sim \text{Triangular } 1(\mu, c)$ ,  $\mu, c$  are unknown  
 $\mu$  point estimated value=5.005333 (MLE)  
 $c$  point estimated value=1.989623 (MLE)  
horizon axis is samples cumulative relative frequency,  
vertical axis is cumulative probability under H0 from sample values.  
The p p plot images is stored in c:\book\_01\pp\_plot14\_image.jpg

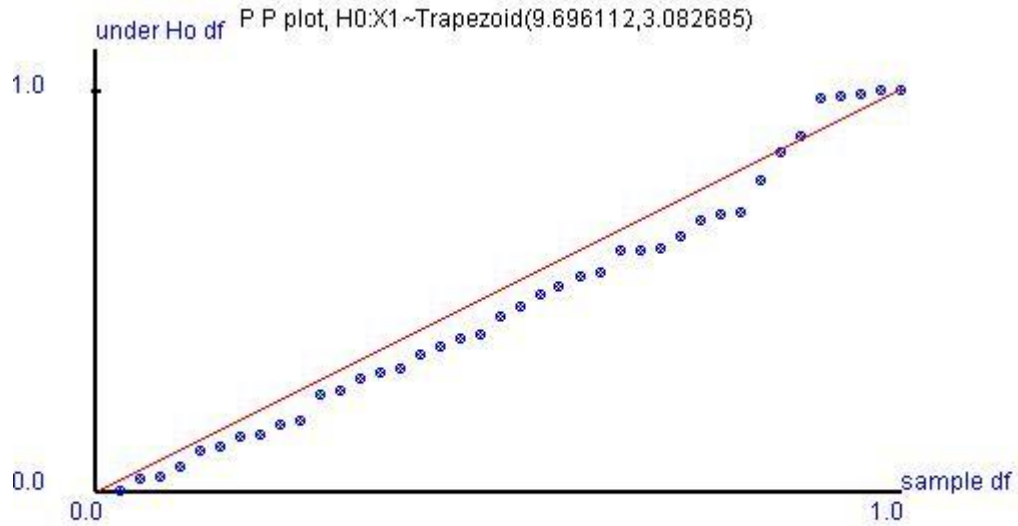


6.2.15) The population distribution is trapezoid distribution.

X1 is Trapezoid( $\mu=10.000000, c=4.000000$ ),

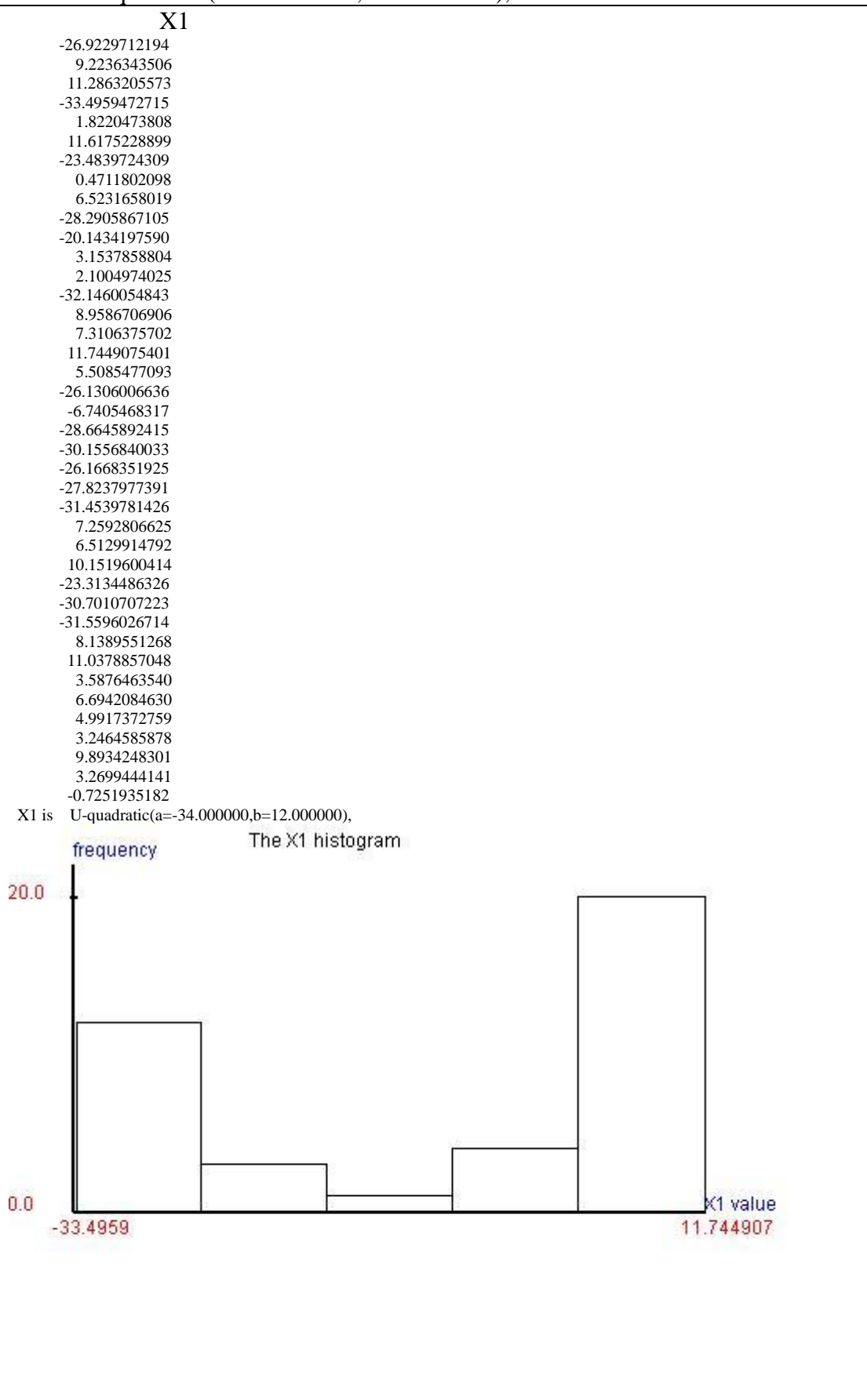


H0:  $X_1 \sim \text{Trapezoid}(\mu, c)$ ,  $\mu, c$  are unknown  
mu point estimated value=9.696112 (MLE)  
c point estimated value=3.082685 (MLE)  
horizon axis is samples cumulative relative frequency,  
vertical axis is cumulative probability under H0 from sample values.  
The p p plot images is stored in c:\book\_01\pp\_plot15\_image.jpg



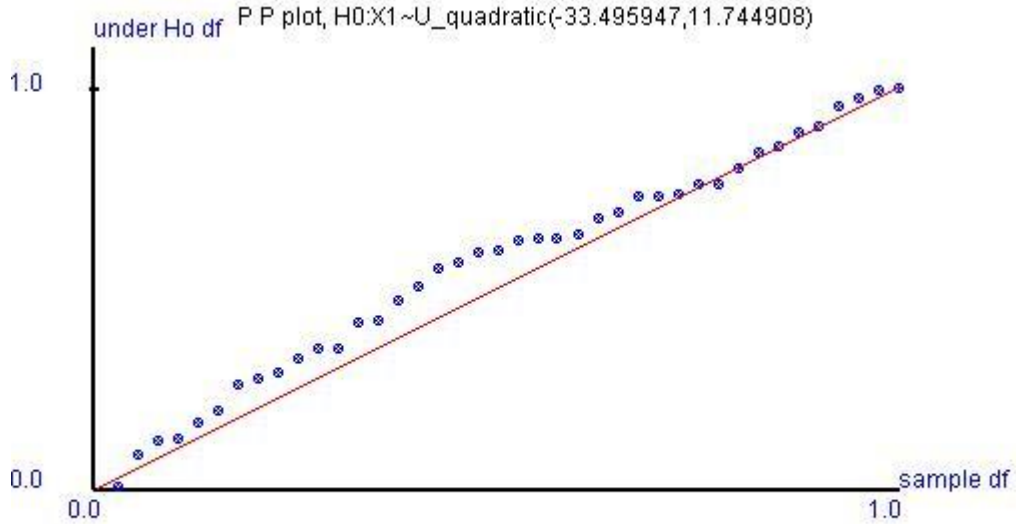
6.2.16) The population distribution is U quadratic distribution.

X1 is U-quadratic(a=-34.000000,b=12.000000),



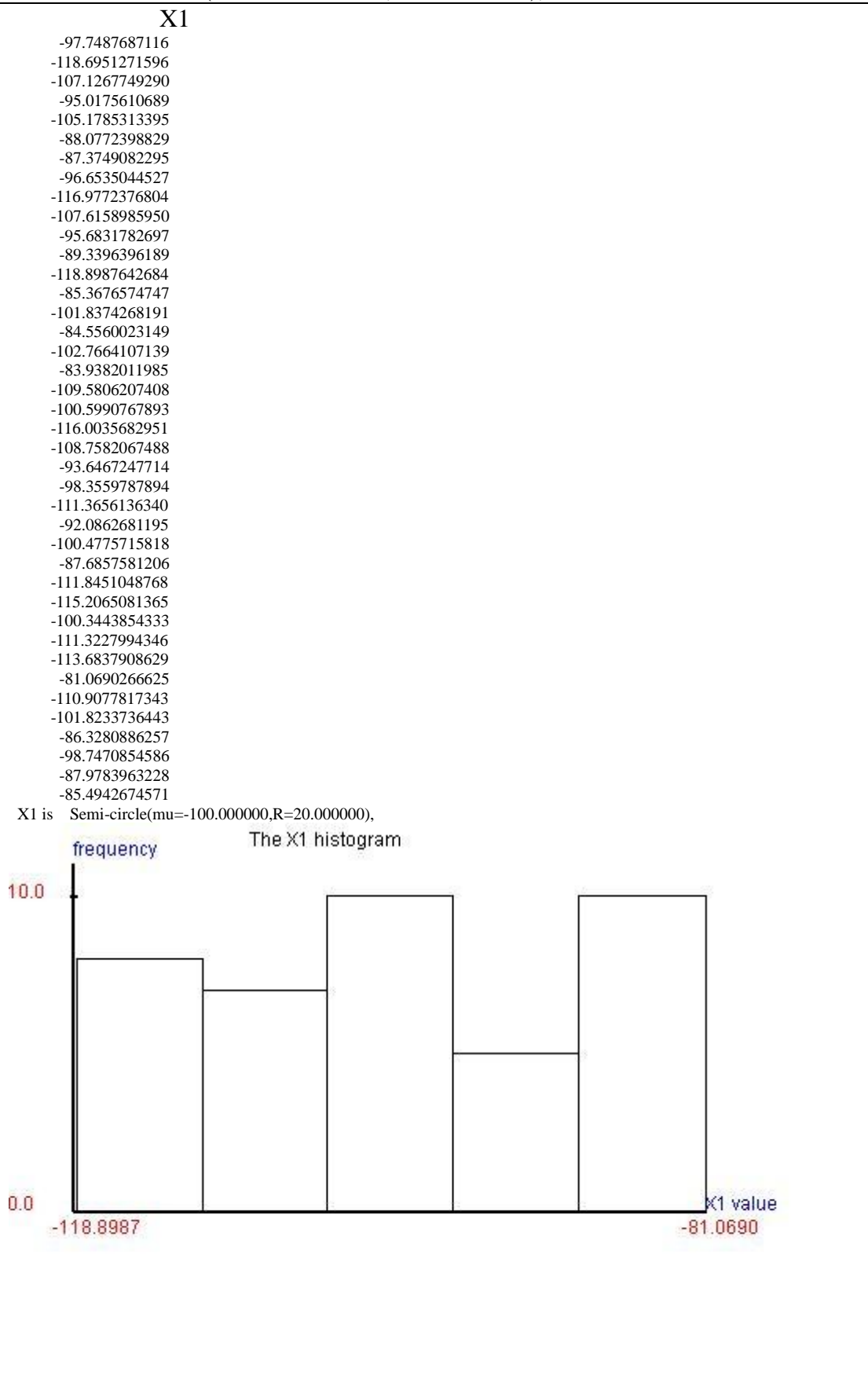


H0:  $X_1 \sim U_{\text{quadratic}}(a,b)$ ,  $a,b$  are unknown  
a point estimated value=-33.495947 (MLE)  
b point estimated value=11.744908 (MLE)  
horizon axis is samples cumulative relative frequency,  
vertical axis is cumulative probability under H0 from sample values.  
The p p plot images is stored in c:\book\_01\pp\_plot16\_image.jpg

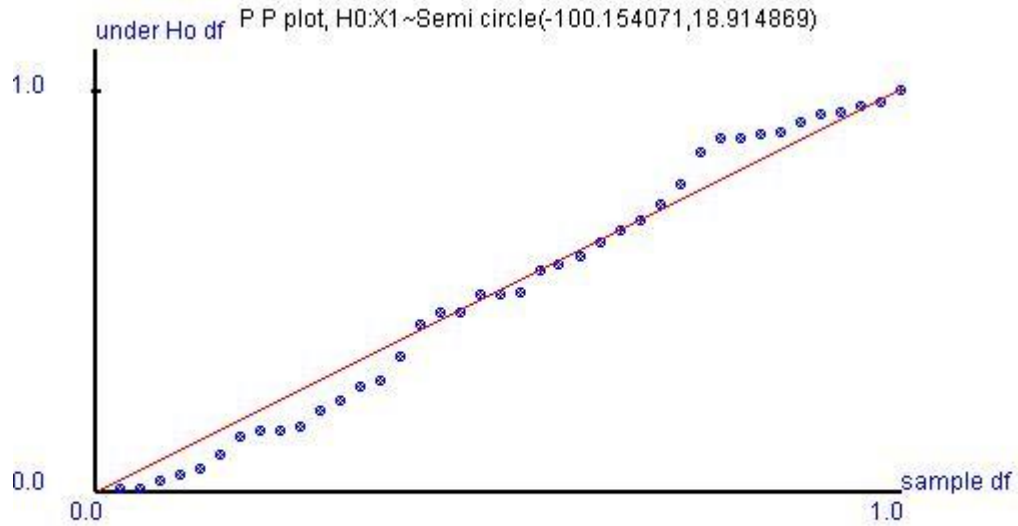


6.2.17) The population distribution is semi circle distribution.

X1 is Semi-circle( $\mu=-100.000000, R=20.000000$ ),

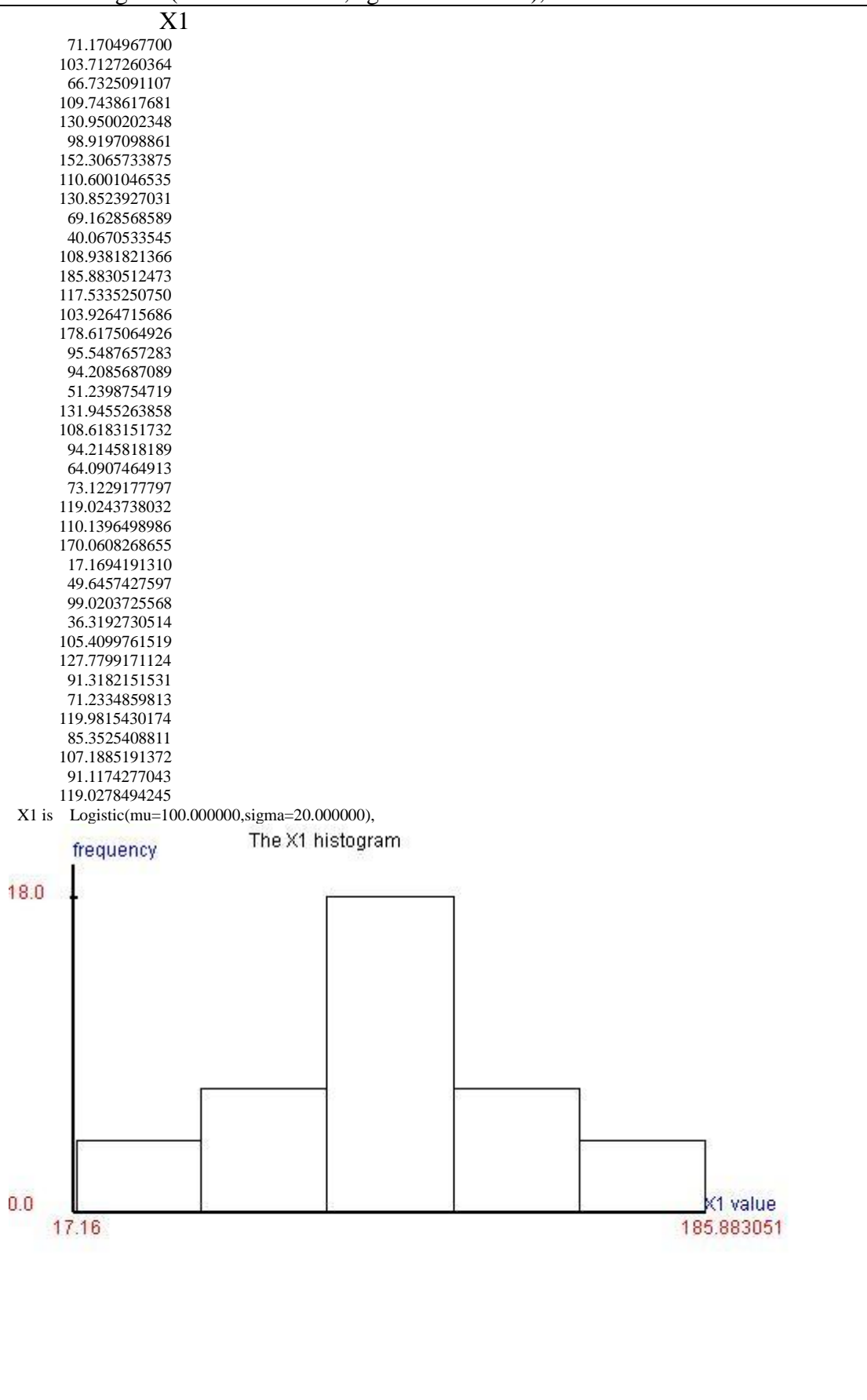


H0:  $X_1 \sim \text{Semi-circle}(\mu, R)$ ,  $\mu, R$  are unknown  
 $\mu$  point estimated value = -100.154071 (MLE)  
 $R$  point estimated value = 18.914869 (MLE)  
horizontal axis is samples cumulative relative frequency,  
vertical axis is cumulative probability under H0 from sample values.  
The p p plot images is stored in c:\book\_01\pp\_plot17\_image.jpg

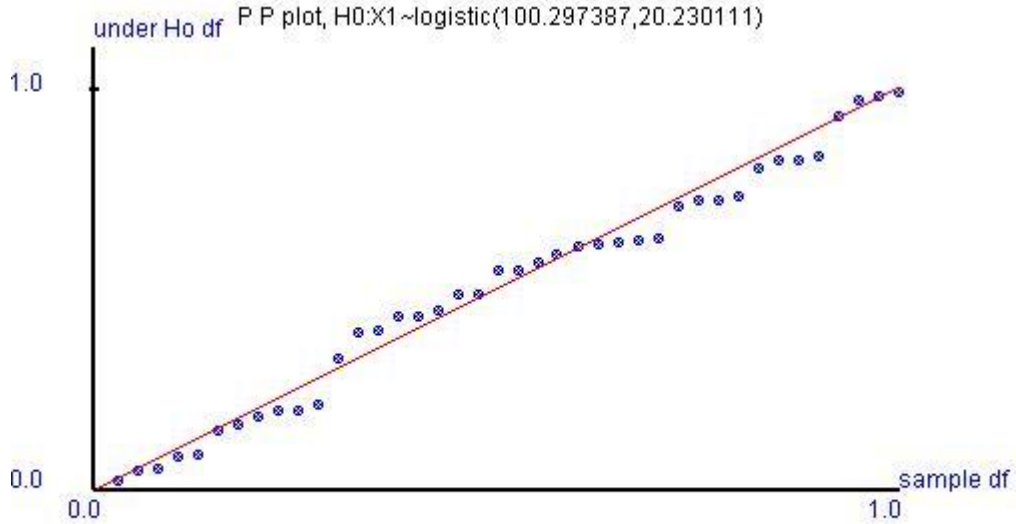


6.2.18) The population distribution is logistic distribution.

X1 is Logistic( $\mu=100.000000, \sigma=20.000000$ ),



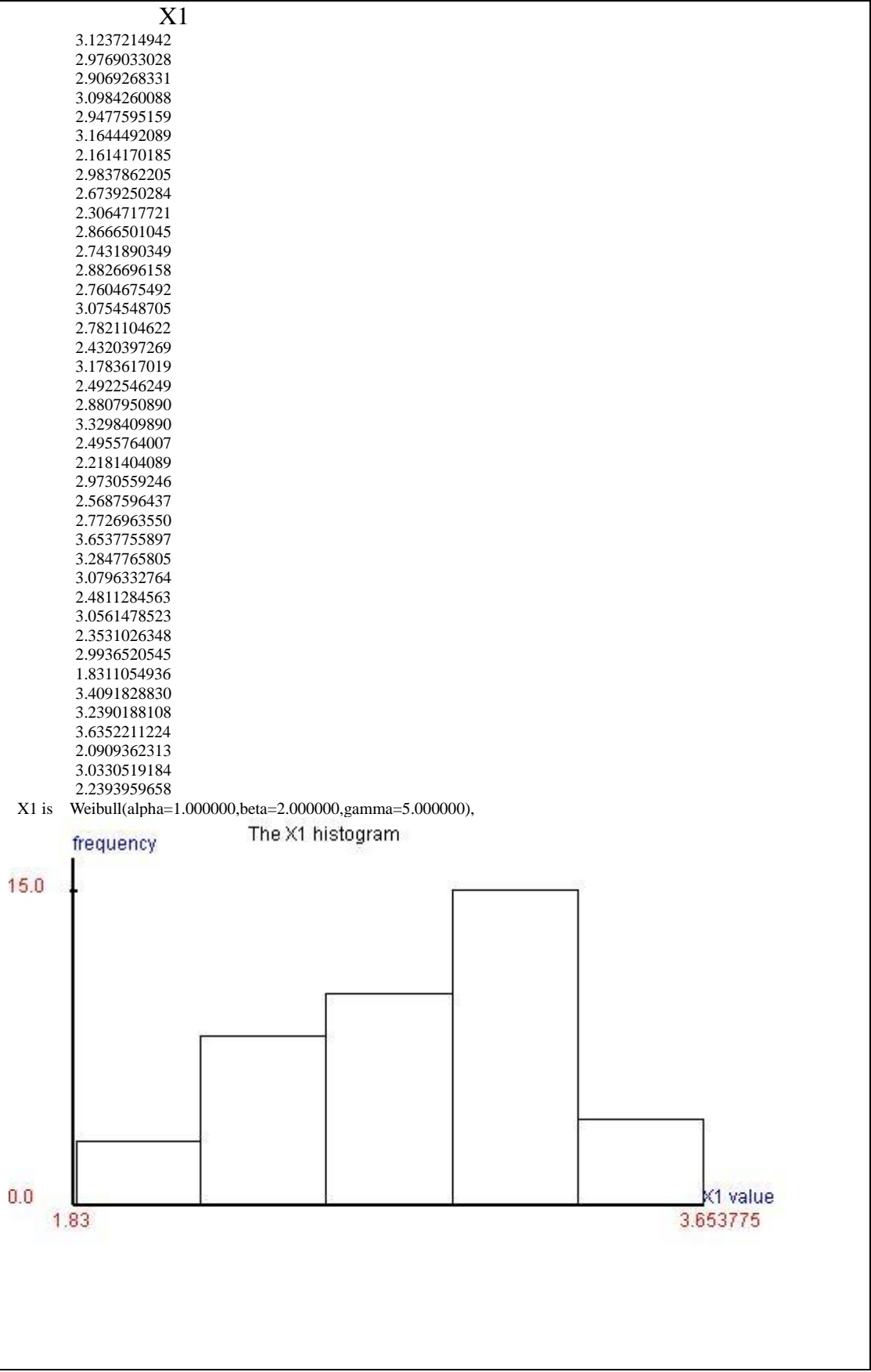
H0:  $X_1 \sim \text{Logistic}(\mu, \sigma)$ ,  $\mu, \sigma$  are unknown  
mu point estimated value=100.297387 (MME)  
sigma point estimated value=20.230111 (MME)  
horizon axis is samples cumulative relative frequency,  
vertical axis is cumulative probability under H0 from sample values.  
The p p plot images is stored in c:\book\_01\pp\_plot18\_image.jpg



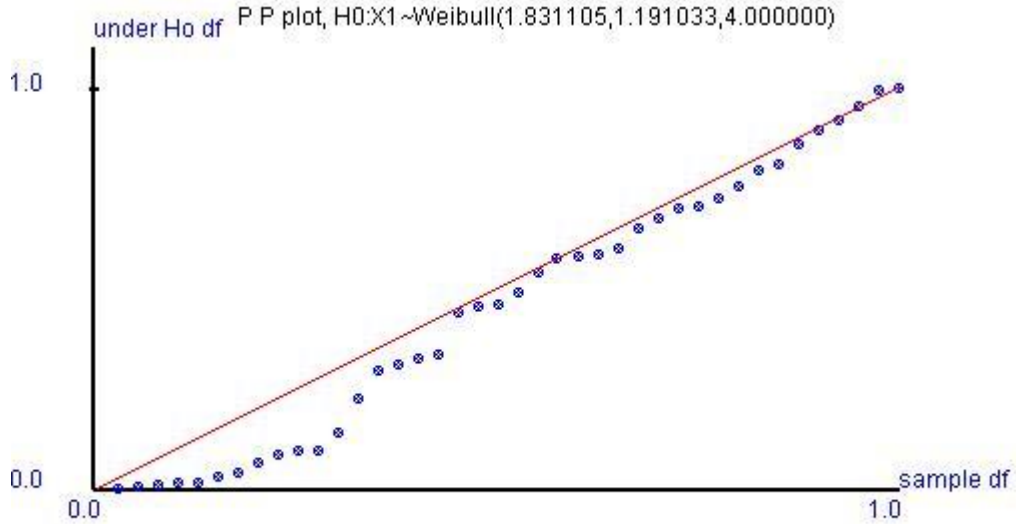
6.2.19) The population distribution is weibull distribution.

The gamma values is supposed to 4.

X1 is Weibull(alpha=1.000000,beta=2.000000,gamma=5.000000),

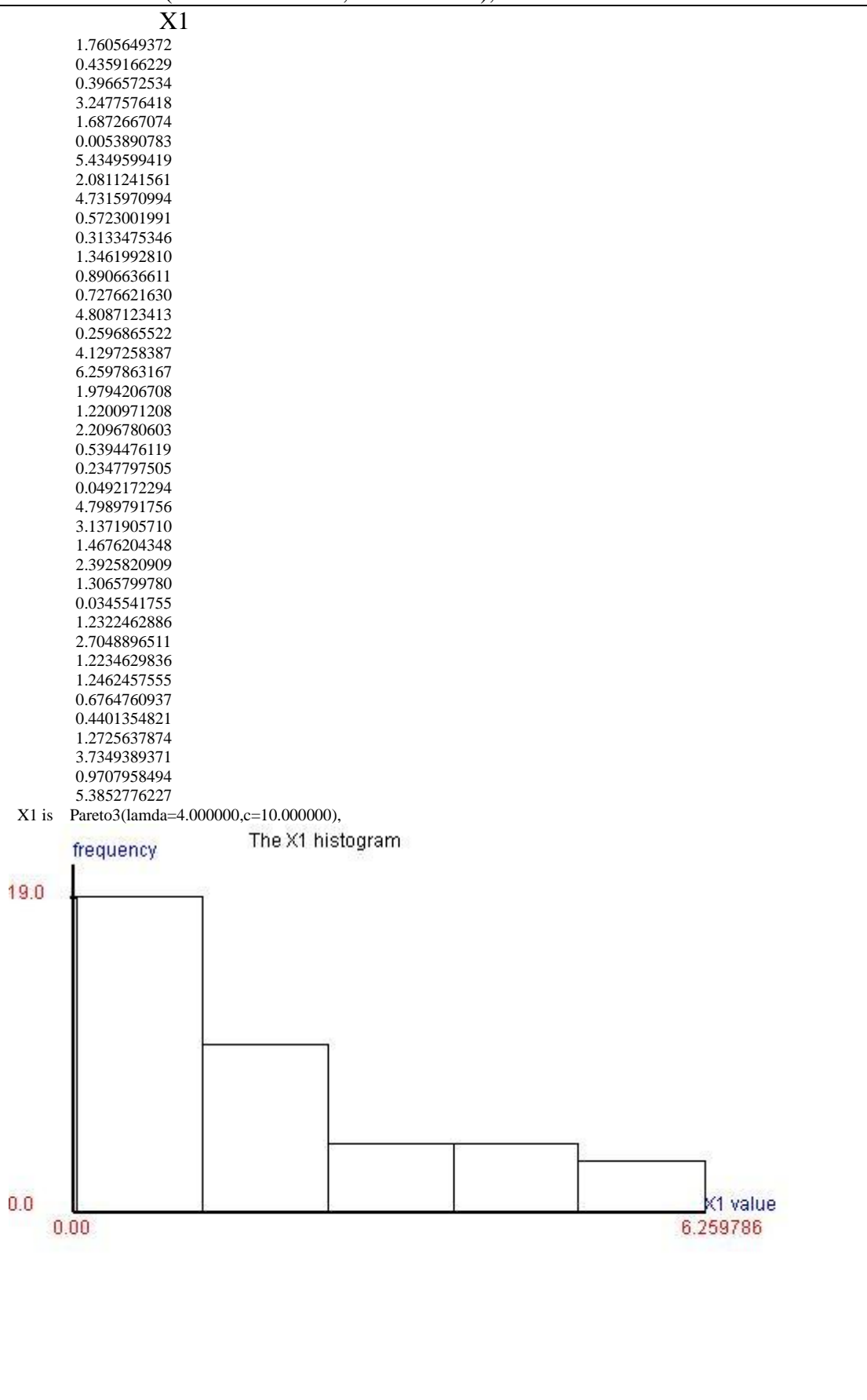


H0:  $X_1 \sim \text{Weibull}(\alpha, \beta, \gamma = 4.000000)$ ,  $\alpha, \beta$  are unknown  
alpha point estimated value=1.831105 (MLE)  
beta point estimated value=1.191033 (MLE)  
horizon axis is samples cumulative relative frequency,  
vertical axis is cumulative probability under H0 from sample values.  
The p p plot images is stored in c:\book\_01\pp\_plot19\_image.jpg



6.2.20)The population distribution is pareto 3 distribution

X1 is Pareto3(lamda=4.000000,c=10.000000),





H0:  $X_1 \sim \text{Pareto } 3(\lambda, c)$ ,  $\lambda, c$  are unknown

$\lambda$  point estimated value = 2.273331 (MLE)

$c$  point estimated value = 6.259786 (MLE)

horizontal axis is samples cumulative relative frequency,

vertical axis is cumulative probability under H0 from sample values.

The p p plot images is stored in c:\book\_01\pp\_plot22\_image.jpg

