

Chapter six The goodness of fit test

4) Pearson chi-square test 4:

4.1) The multi-nominal distribution is the sampling distribution, each category has a probability and the summation of probability is 1.

category	1	2	k
probability	p_1	p_2	p_k

$$p_1 + \dots + p_k = 1,$$

$$f(x_1, \dots, x_k) = \frac{n!}{x_1! \times \dots \times x_k!} \times p_1^{x_1} \times \dots \times p_k^{x_k}, \quad x_1 + \dots + x_k = n \quad \text{that is trial number.}$$

4.2)

$$X_i = np_i + \varepsilon_i, \quad E(\varepsilon_i) = 0, \quad E[(\varepsilon_i)^2] = np_i(1 - p_i), \quad i = 1, 2, \dots, k$$

$$X_i - np_i = \varepsilon_i, \quad E\left[\frac{(\varepsilon_i)^2}{np_i}\right] = E\left[\frac{(X_i - np_i)^2}{np_i}\right] = 1 - p_i,$$

$$\sum_{i=1}^k E\left[\frac{(X_i - np_i)^2}{np_i}\right] = E\left[\sum_{i=1}^k \left(\frac{(X_i - np_i)^2}{np_i}\right)\right] = k - \sum_{i=1}^k p_i = k - 1,$$

$$\sum_{i=1}^k \left(\frac{(X_i - np_i)^2}{np_i}\right) \rightarrow \chi_{k-1}^2$$

$$H_0 : p_1 = p_{01}, p_2 = p_{02}, \dots, p_k = p_{0k} \quad H_1 : \text{against } H_0$$

$p_{01}, p_{02}, \dots, p_{0k}$ are known value and $p_{01} + p_{02} + \dots + p_{0k} = 1,$

$$E_i = np_{i0}, \quad i = 1, 2, \dots, k, \quad O_i = x_i,$$

O_i : The observed sample number of cell i,

E_i : The expected sample number of cell i,

$$\chi_v^2 = \sum_{i=1}^k \frac{(O_i - E_i)^2}{E_i}, \quad v = k - 1 - (\text{number of point estimator}) \text{ is degree of freedom of}$$

pearsson chi square test.

$$\chi_v^2 > \chi_{\alpha, v}^2 \Rightarrow \text{reject } H_0.$$

4.3) The process of test

H_0 : Population distribution is a continuous probability distribution,

H_1 : against H_0

In here, the parameters are unknown and giving a interval of parameters.

The values of parameter interval will put the null hypothesis and computed the p-value. The minimum of p-value which parameter is the best parameter value of population distribution by the sample data.

The sample size is n.

The process: i) The class number of frequency distribution: $k = \log_2(n) + 1.$

ii) The class limit of frequency distribution:

The class limit is found by the a general frequency distribution table.

iii) The class sample number of frequency distribution (O_i):

The frequency distribution is done and getting the O_i of each class.

iv) The class expected number of frequency distribution (E_i):

$$E_i = n \times \text{the probability of each class.}$$

4.4) Example (The simulated sample data and computing the result by the P_S_CCC)

The Pearson chi square goodness of fit test using the best fitting method and frequency distribution, the fitting method is used the parameter value changed to find the best population distribution

1.H0:Uniform distribution	13.H0:Gumbel distribution
2.H0:Normal distribution	14.H0:Triangular 1 distribution
3.H0:Shifted exponential distribution	15.H0:Trapezoid distribution
4.H0:Pareto 1 distribution	16.H0:U-quadractic distribution
5.H0:Pareto 2 distribution	17.H0:Semi-circle distribution
6.H0:Rayleigh distribution	18.H0:Logistic distribution
7.H0:Double expoenoential distribution	19.H0:Weibull distribution
8.H0:Log normal distribution	20.H0:Pareto 3 distribution
9.H0:Gamma distribution	
10.H0:Beta distribution	
11.H0:Cauchy distribution	
12.H0:Arcsin distribution	

The parameter values are the piont estimator value.

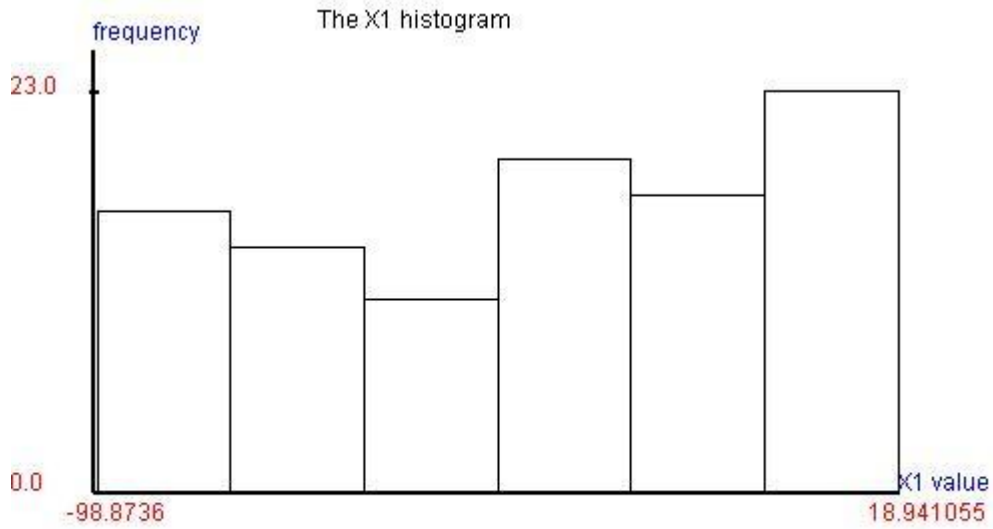
4.4.1)The population distribution is uniform distribution.

X1 is Uniform(alpha=-100.000000,beta=20.000000),

X1
11.7711889122
-93.6058699439
9.0615525757
-84.3012211109
-54.2403444152
13.7452366990
-23.0296143383
-96.1673232952
-23.5509251776
-87.7133799142
-21.4368692235
-72.8383410809
16.5360520613
8.5984029666
3.4825279688
5.3836154591
10.4186250395
-76.3331370276
-86.1032499544
17.8834111257
-30.2033402534
6.4237992578
-5.5517098823
-69.7927452326
-5.4668420226
5.9041420642
-21.9546159707
12.2978213321
-85.0245650690
-76.6061938039
-66.2153452767
-20.2459100274
-67.9806459622
-34.3055888067
-16.7502897772
9.4824034602
-28.8477983205
-59.1634121126
-48.6574603636
-79.2170257686
-33.2609225682
-62.5399491147
-1.9244469451
-46.6847476802
-7.2116195178
-10.8810911099
-22.6067854610
-86.9293866285
-63.5460537944
-35.2476304694
9.4071669657
-2.3543738760
-89.4612904697
-93.4133681895
12.1943700384
-47.9421112510
-98.8341065628
16.6861506386
-40.5054628930
-13.5949797707
-32.9635854752
-86.5506338902
-69.4996599026
-13.5120615983
-33.6713654476
-40.7036236993
-82.0002352054
-31.1837951834
7.6481649105
-39.2991767961
-98.8736612677

-8.9127296079
 1.1717476883
 -38.0070838421
 -91.0014097234
 -50.4828845448
 -13.7169517102
 -5.9947037864
 -18.5651221616
 -5.7943238657
 -28.0274316156
 18.9410558041
 -6.9640657583
 -46.2575761539
 6.1640063724
 -67.3801533224
 -33.1054883823
 -59.5301243198
 -69.6728318690
 -94.5405789533
 4.1438999674
 -72.5653889964
 15.5837067910
 -36.6954117632
 -24.3843066722
 -11.5381140105
 16.4607019909
 -70.5317422079
 -98.1264897761
 -47.5360084010

X1 is Uniform(alpha=-100.000000,beta=20.000000),



alpha point estimated value=-98.873661 (MLE)
 beta point estimated value=18.941056 (MLE)
 alpha value from -101.253757 to -96.493566
 beta value from 16.560961 to 21.321151
 H0: $X1 \sim \text{Uniform}(\alpha=-99.064069, \beta=18.941056)$,

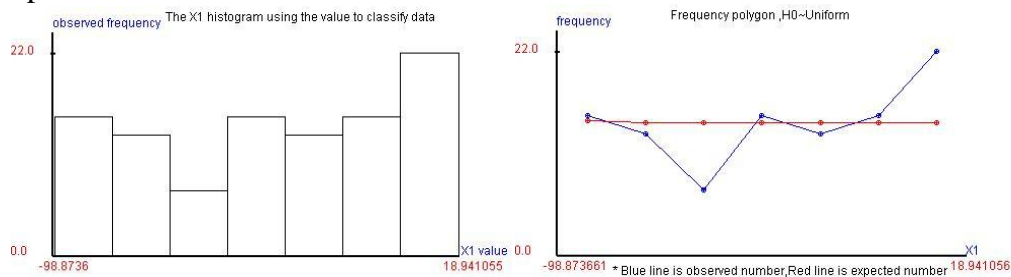
pearson goodness of fit

class	[1]	[2]	[3]	[4]	[5]	[6]	[7]
lower limit	-98.87366	-82.04299	-65.21231	-48.38164	-31.55097	-14.72029	2.11038
upper limit	-82.04299	-65.21231	-48.38164	-31.55097	-14.72029	2.11038	18.94106
observed no	15.00000	13.00000	7.00000	15.00000	13.00000	15.00000	22.00000
probability	0.14424	0.14263	0.14263	0.14263	0.14263	0.14263	0.14263
expected no	14.42402	14.26266	14.26266	14.26266	14.26266	14.26266	14.26266
chi square	0.02300	0.11178	3.69821	0.03812	0.11178	0.03812	4.19742

degree of freedom=4

pearson chi-square test statistic =8.218428

p-value=0.083800

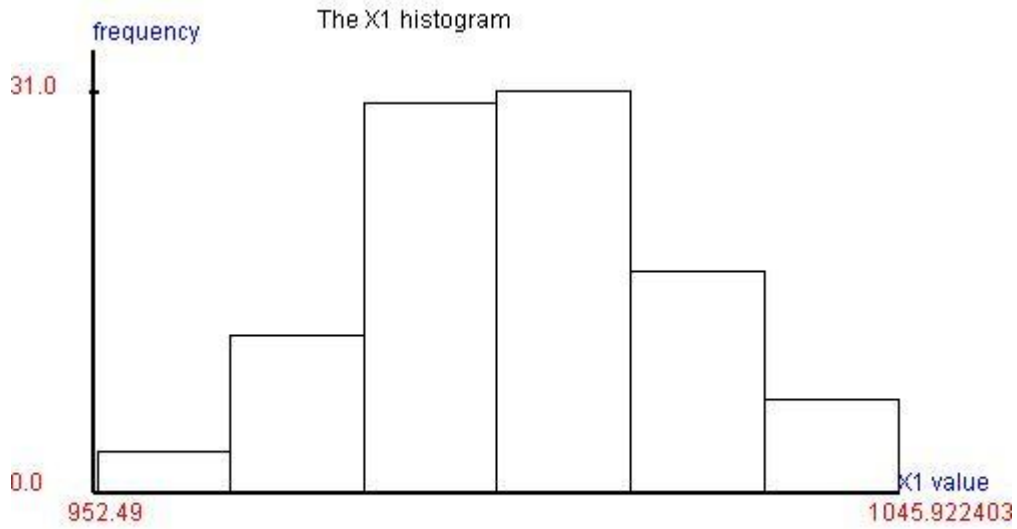


4.4.2)The population distribution is normal distribution.
X1 is Normal($\mu=1000.000000$, $\sigma^2=400.000000$),

X1
998.9380931514
1002.2567003291
985.8535422857
977.4885189854
976.0292875162
1026.2275757385
1000.5286003613
1030.3707025240
985.6556652126
997.3943523748
1003.3792883530
1022.2535169435
1001.6614605258
1001.6915554033
992.5805941413
992.1829711106
990.6710347820
1002.2127865266
1015.5241972808
990.0211288892
1024.1241175313
1006.5513074177
978.6076203275
980.5421760862
982.3959822403
992.9918576817
1014.7286144785
1001.0553790612
991.4060757163
1004.4893063583
999.1258783300
998.9980248327
988.3853529532
999.7335552288
989.4888285637
979.3849040822
1015.9834033166
975.1362972028
1037.0972575887
1000.9637900395
1007.7086678146
964.7625205118
1019.1341076625
1018.6666025476
1024.0105201470
1009.8876631727
986.7896029751
1000.5323226570
986.3955905333
989.4115901388
1014.0184803125
999.0309464808
1000.0589286676
984.5020836262
1009.6569834114
1015.2550270002
975.7852415517
1002.9653085742
978.8854938141
1016.0804733879
998.9657014274
986.9492941198
999.9785520377
1017.1010281777
999.8115554087
1035.9021175070
999.0762201391
1002.1993758498
1010.8455793923
988.0061105643
1024.6945369425

1024.4787593412
 979.8881776797
 952.4942700171
 1034.2801707239
 987.5379731097
 1033.0933709982
 1024.1871914673
 1013.7156051845
 976.3039879490
 998.3996354355
 985.6058570969
 991.8825639259
 1045.9224032856
 1003.1970777091
 992.5465380720
 1027.1266256340
 1031.8866918563
 966.2752205322
 999.5743825409
 1009.5836578328
 979.1209915237
 1017.9252354587
 993.5360822994
 999.2332102824
 1016.8092652937
 1004.0732375156
 996.3814701068
 1008.8586595781
 1001.4146649438

X1 is Normal($\mu=1000.000000$, $\sigma^2=400.000000$),



mu point estimated value=1001.165125 (MLE)
 sigma point estimated value=17.578527 (MLE)
 mu value from 997.649420 to 1004.680830
 sigma value from 14.648772 to 21.973159
 H0: $X1 \sim \text{Normal}(\mu=1000.251042, \sigma^2=298.790286)$,
 pearson goodness of fit

class	[1]	[2]	[3]	[4]
lower limit	952.49427	965.84115	979.18802	992.53490
upper limit	1005.88177	1019.22865	1032.57553	1045.92240
observed no	2.00000	9.00000	21.00000	34.00000
probability	0.02330	0.08830	0.21610	0.30010
expected no	2.33000	8.83000	21.61000	30.01000
chi square	0.04674	0.00327	0.01722	0.53049
	0.90013	0.02767	1.21332	

pearson chi square test statistic=2.738843
 degree of freedom=4, p-value=0.908000

correction:

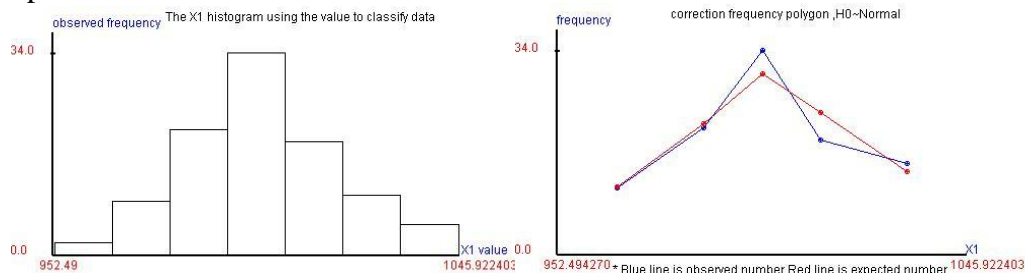
expected number ≥ 5 in each cell, the frequency table is adjusted

class	[1]	[2]	[3]	[4]
lower limit	952.49427	979.18802	992.53490	1005.88177
upper limit	1019.22865	1045.92240	1072.67027	1100.00000
observed no	11.00000	21.00000	34.00000	19.00000
probability	0.11160	0.21610	0.30010	0.23610
expected no	11.16000	21.61000	30.01000	23.61000
chi square	0.00229	0.01722	0.53049	0.90013
	0.14196			

degree of freedom=2

pearson chi-square test statistic =1.592099

p-value=0.451100



4.4.3)The population distribution is shifted exponential distribution.

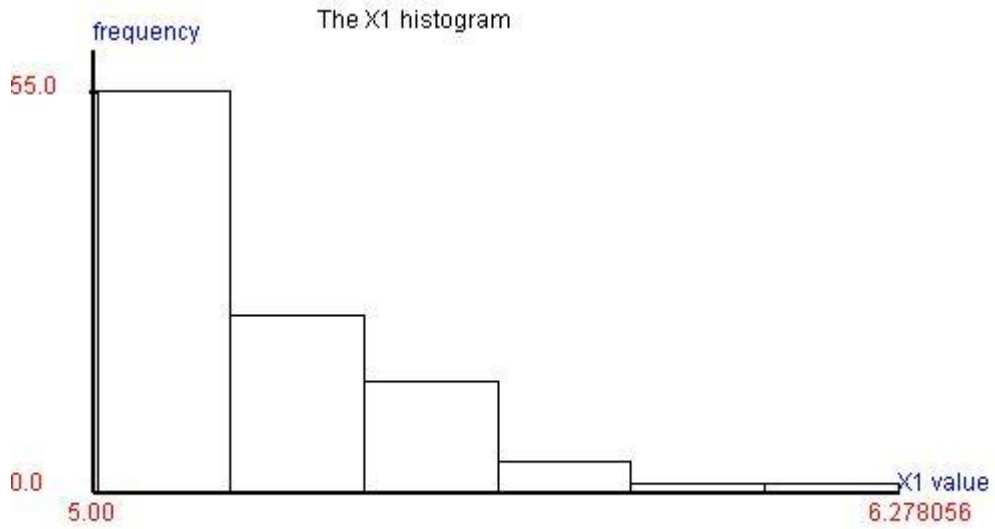
X1 is Exponential($\lambda=4.000000, c=5.000000$),

X1

5.1494341698
5.0687907179
5.0235459574
5.1268205486
5.2777375667
5.1209346420
5.4376037103
5.1708124517
5.2106450656
5.4959763015
5.0565231741
5.1248221327
5.2105638363
5.3927924065
6.2780565734
5.3507008268
5.3435458685
5.6204091896
5.1219107762
5.2785883862
5.4186268070
5.1412666721
5.1236642414
5.1419438519
5.8422475312
5.2902924754
5.0373448551
5.0421179256
5.2451251340
5.3540612692
5.0131889380
5.0877107325
5.0087815246
5.5612309306
5.5278433032
5.0109061384
5.3406713891
5.4705510421
5.2425854257
5.2124193292
5.8314947641
5.0177117842
5.0961827590
5.0523920552
5.3981496522
5.3777881769
5.3229943113
5.1019879749
5.1041437931
5.6516873026
5.4629272324
5.1624725587
5.2793052174
5.0004087947
5.1067538023
5.5147812419
5.3028389685
5.3407562610
5.5253416149
5.0548643435
5.5796404551
5.3114769685
5.0569902090
5.0169297946
5.6087351309
5.3313977607
5.0790716446
5.1335015541
5.0064405773
5.0659806358
5.0312230875

5.3585259447
 5.1921303080
 5.1360677675
 5.2742190203
 5.1075264912
 5.9293158785
 5.1192290596
 5.1252659046
 5.0406867611
 5.0605560845
 5.0021553623
 5.5199689540
 5.3519608335
 5.1481893690
 5.4678499136
 5.0548487984
 5.0369077345
 5.0105248502
 5.3672688501
 5.0737806588
 5.0842382238
 5.0799114045
 5.2122067705
 5.6369223238
 5.3189357259
 5.6765336025
 5.1310440871
 5.0222187976
 5.5225313988

X1 is Exponential(lamda=4.000000,c=5.000000),



lamda point estimated value=3.858054 (MLE)

c point estimated value=5.000409 (MLE)

lamda value from 3.215045 to 4.822567

c value from 4.904627 to 5.096191

H0: $X_1 \sim \text{Shifted exponential}(\lambda=4.069677, c=5.013818)$,

pearson goodness of fit

class	[1]	[2]	[3]	[4]
lower limit	5.00041	5.18293	5.36545	5.54797
5.73049	5.91301	6.09554		
upper limit	5.18293	5.36545	5.54797	5.73049
5.91301	6.09554	6.27806		
observed no	50.00000	24.00000	15.00000	7.00000
2.00000	1.00000	1.00000		
probability	0.49754	0.26340	0.12532	0.05963
0.02837	0.01350	0.01225		
expected no	49.75352	26.34026	12.53214	5.96253
2.83685	1.34971	1.22499		
chi square	0.00122	0.20793	0.48598	0.18052
0.24686	0.09061	0.04132		

pearson chi square test statistic=1.254437

degree of freedom=4,p-value=0.989600

correction:

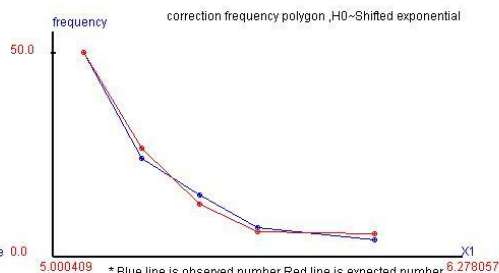
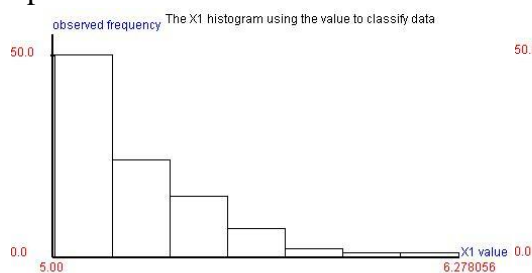
expected number ≥ 5 in each cell, the frequency table is adjusted

class	[1]	[2]	[3]	[4]
lower limit	5.00041	5.18293	5.36545	5.54797
5.73049				
upper limit	5.18293	5.36545	5.54797	5.73049
6.27806				
observed no	50.00000	24.00000	15.00000	7.00000
4.00000				
probability	0.49754	0.26340	0.12532	0.05963
0.05412				
expected no	49.75352	26.34026	12.53214	5.96253
5.41155				
chi square	0.00122	0.20793	0.48598	0.18052
0.36819				

degree of freedom=2

pearson chi-square test statistic =1.243828

p-value=0.536900



4.4.4)The population distribution is pareto1 distribution.

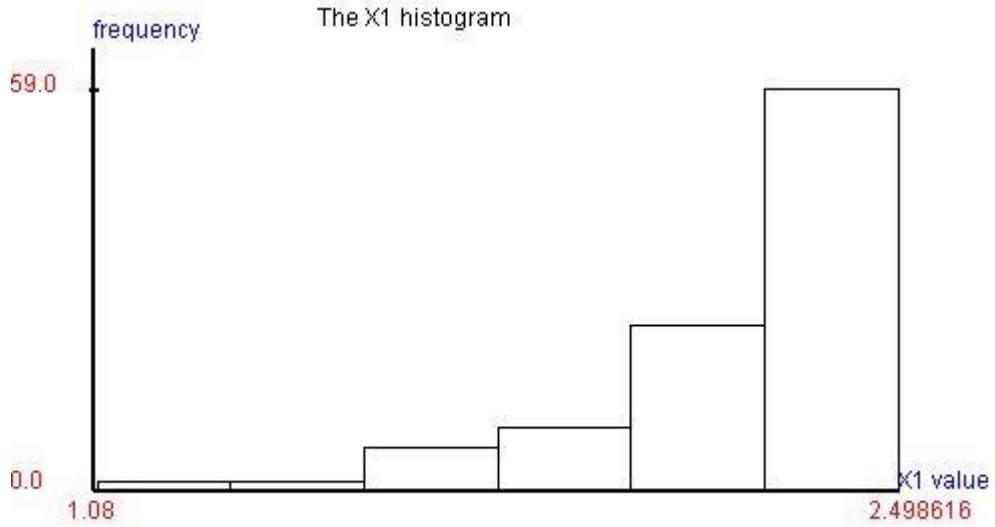
X1 is Pareto1(lamda=7.000000,c=2.500000),

X1

2.1303280042
2.3245672353
2.2151089679
2.4971344397
2.0717373274
2.1484416150
2.4456818218
2.1024612960
2.3357693178
1.9141765144
2.3417265346
2.0464341203
2.4946811020
2.2985202610
2.3853158575
2.0898790925
1.9913713584
2.2817850172
2.1687547637
2.3655621683
2.3890717700
2.4072672283
2.2918850208
2.1446512612
1.8130978622
1.7539950130
2.2422413346
2.4191062648
2.3202175732
2.4691541006
2.1039099831
2.0457046459
2.4604005758
2.4942173624
2.4780902404
2.4219232370
1.4633243696
1.6216519560
2.4834182566
2.4182298298
2.0993360917
2.4274651975
2.3352710199
2.0358573781
2.2178753237
2.4794793230
1.8192325349
2.2088103554
2.4275607888
1.9967518437
2.4119635561
2.3232110976
2.4338375809
2.3977315438
2.3289980273
2.4366357319
2.1682094256
2.2462133124
2.3981095014
2.4846488412
2.2249001011
2.3158724900
1.9621409856
2.3437727173
2.4662100401
2.0093259421
2.2968076417
2.3211460180
2.1499349572
1.9880488480
1.6714319356

2.4777925541
 1.5873166751
 2.4891225683
 2.2844560699
 2.3342104671
 1.7779672210
 2.3074949621
 2.3200872436
 1.0899370322
 2.3546169322
 2.4267028297
 2.4869011122
 2.3205567652
 2.1210020296
 2.2847767178
 2.4587192204
 2.4440550292
 2.2986469258
 2.4986160309
 2.3712222758
 1.9317072194
 2.1799633768
 2.2018098305
 2.2197838332
 2.4664312882
 1.7028913341
 2.3097034104
 2.3965835347
 2.3662104639

X1 is Pareto1(lamda=7.000000,c=2.500000),



lamda point estimated value=8.166541 (MLE)

c point estimated value=2.498616 (MLE)

lamda value from 6.805451 to 10.208176

c value from 2.450613 to 2.546619

H0: $X_1 \sim \text{Pareto 1}(\lambda=6.828211, c=2.545659)$,

pearson goodness of fit

class	[1]	[2]	[3]	[4]	[5]	[6]	[7]
lower limit	1.08994	1.29118	1.49242	1.69366			
1.89490	2.09614	2.29738					
upper limit	1.29118	1.49242	1.69366	1.89490			
2.09614	2.29738	2.49862					
observed no	1.00000	1.00000	3.00000	5.00000			
12.00000	24.00000	54.00000					
probability	0.00970	0.01639	0.03579	0.07132			
0.13215	0.23087	0.50377					
expected no	0.97038	1.63858	3.57943	7.13216			
13.21513	23.08687	50.37744					
chi square	0.00090	0.24887	0.09380	0.63741			
0.11173	0.03612	0.26049					

pearson chi square test statistic=1.389315

degree of freedom=4, p-value=0.985800

correction:

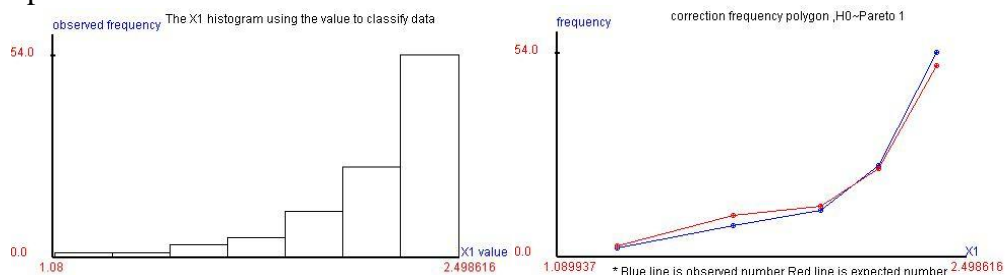
expected number ≥ 5 in each cell, the frequency table is adjusted

class	[1]	[2]	[3]	[4]	[5]
lower limit	1.08994	1.49242	1.89490	2.09614	
2.29738					
upper limit	1.49242	1.89490	2.09614	2.29738	
2.49862					
observed no	2.00000	8.00000	12.00000	24.00000	
54.00000					
probability	0.02609	0.10712	0.13215	0.23087	
0.50377					
expected no	2.60897	10.71159	13.21513	23.08687	
50.37744					
chi square	0.14214	0.68643	0.11173	0.03612	
0.26049					

degree of freedom=2

pearson chi-square test statistic =1.236906

p-value=0.538700



4.4.5)The population distribution is pareto2 distribution.

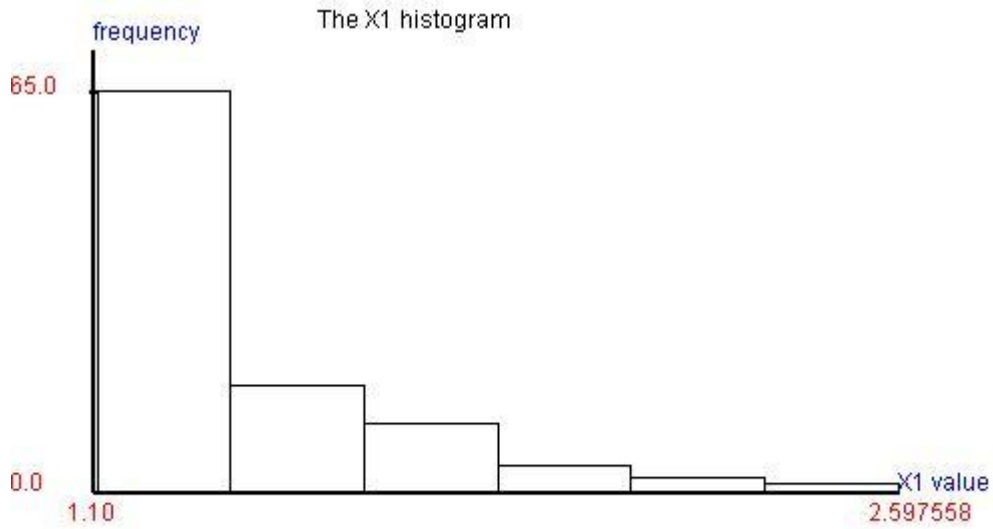
X1 is Pareto2(lamda=5.000000,c=1.100000),

X1

1.2329952607
1.1044348459
2.0954872616
1.2748329287
1.4220422137
1.1165203750
2.3220573120
1.2192247191
1.2378744511
1.1842648179
1.3251895835
1.6212017405
1.1732506441
1.2740668957
1.3486822274
1.1251867754
1.1741254987
1.5123439222
1.1855219059
1.1370390528
1.1617784511
1.2372215654
1.3030174797
1.1286205845
1.6320409678
1.4921137081
1.1522217280
1.2628661346
1.5842507055
1.5276026291
1.6495546227
1.1621409899
1.5308618345
1.4346426913
1.9806100357
1.4459862587
1.1423461061
1.2289827797
1.1961123806
1.4137076318
1.5118367738
1.3716552523
1.1219334938
1.6877923319
1.3119569981
1.2003734671
1.7347709941
1.3341338558
1.8970006762
1.2718115833
1.2577742379
1.2238933584
1.1109411709
1.1652367941
1.7615384655
1.1586388657
1.1349484906
1.2669102524
1.3385028572
1.4302818750
1.2777768384
1.4595777541
1.3068515227
1.2167796769
1.2814224896
1.2840608344
1.1063874714
1.2587327455
1.2036372777
1.6608925600
1.1969239639

1.8450880112
 1.4746766608
 1.2936947554
 1.3399926469
 2.5975584244
 1.3512118446
 1.1913375388
 1.1581805890
 1.9561653081
 1.2411661810
 1.8268676946
 1.1308064624
 1.2777519482
 1.3073458119
 2.1493528549
 1.1542298203
 1.1760075195
 1.2634378508
 1.5067883169
 1.2167005686
 1.5326365275
 1.6792169899
 1.1238783461
 1.3452195492
 1.6124149462
 1.2674598704
 1.2631817667
 1.5494679960
 1.1793663920

X1 is Pareto2(lamda=5.000000,c=1.100000),



lamda point estimated value=4.813978 (MLE)

c point estimated value=1.104435 (MLE)

lamda value from 4.011648 to 6.017472

c value from 1.086320 to 1.122550

H0: $X_1 \sim \text{Pareto } 2(\text{lamda}=4.025065, c=1.086320)$,

pearson goodness of fit

class	[1]	[2]	[3]	[4]	[5]	[6]	[7]
lower limit	1.10443	1.31774	1.53104	1.74434			
1.95765	2.17095	2.38426					
upper limit	1.31774	1.53104	1.74434	1.95765			
2.17095	2.38426	2.59756					
observed no	58.00000	21.00000	11.00000	5.00000			
3.00000	1.00000	1.00000					
probability	0.54037	0.20836	0.10263	0.05521			
0.03181	0.01936	0.04225					
expected no	54.03666	20.83597	10.26303	5.52146			
3.18129	1.93624	4.22535					
chi square	0.29069	0.00129	0.05292	0.04925			
0.01033	0.45271	2.46201					

pearson chi square test statistic=3.319206

degree of freedom=4, p-value=0.853900

correction:

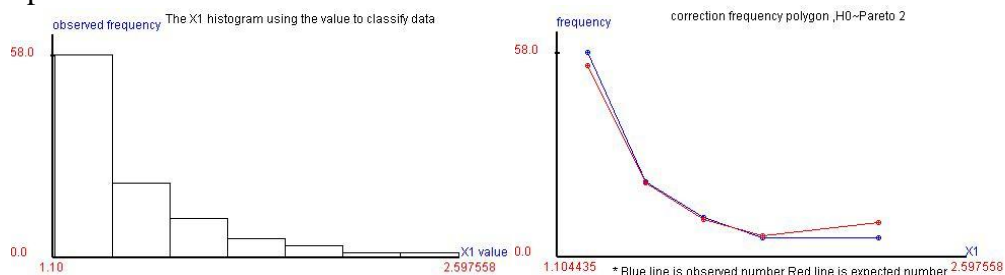
expected number ≥ 5 in each cell, the frequency table is adjusted

class	[1]	[2]	[3]	[4]	[5]
lower limit	1.10443	1.31774	1.53104	1.74434	
1.95765					
upper limit	1.31774	1.53104	1.74434	1.95765	
2.59756					
observed no	58.00000	21.00000	11.00000	5.00000	
5.00000					
probability	0.54037	0.20836	0.10263	0.05521	
0.09343					
expected no	54.03666	20.83597	10.26303	5.52146	
9.34288					
chi square	0.29069	0.00129	0.05292	0.04925	
2.01871					

degree of freedom=2

pearson chi-square test statistic =2.412866

p-value=0.299200



4.4.6)The population distribution is rayleigh distribution.

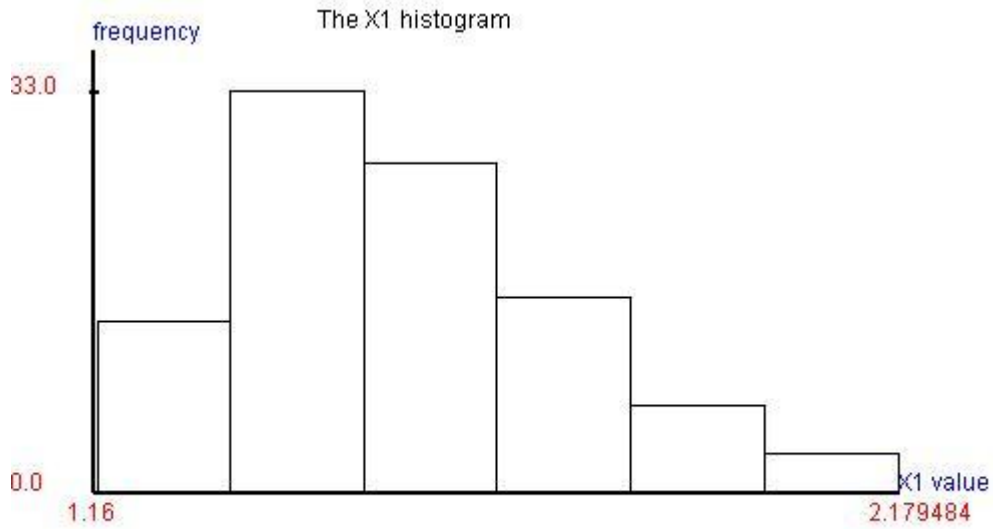
X1 is Rayleigh($\lambda=4.000000, c=1.120000$),

X1

1.3505439176
2.0784382128
1.1623095460
1.4737209113
1.7464202894
1.4241863783
1.4707723122
1.1602501962
1.6306474010
1.6629216204
1.5082224972
1.5312609245
1.4933804770
1.5608945899
1.5579075921
1.3215113023
1.4176981333
1.5398541532
1.8028833878
1.8413419779
1.5459751132
1.7405197450
1.3806425674
1.3219059467
1.4707899085
1.7584729794
1.4609098438
1.4885609117
1.6103797367
1.4021222814
1.3774900799
1.7868245380
1.1761325977
1.6134295965
1.2191216867
1.4561051346
1.5855551843
1.7677084569
1.5445878135
1.4290085690
2.1794846502
1.5468158450
1.5634044888
1.3484856491
1.4838832237
1.6523603422
1.3591179872
1.9013674187
1.6968829803
1.5023453145
1.3268811338
1.5299418515
1.3276877351
1.5195708981
1.4976369630
2.0751482534
1.3191933519
1.6138957459
1.5907607450
1.8111650514
1.7352895676
1.1662901073
1.3662724971
1.7391164917
1.5149560923
1.4947255594
1.8281207399
1.6898519238
1.1935850014
1.3464384490
1.4263917373

1.4950148249
1.3254333404
1.3915940737
1.3644234004
1.4222134304
1.4128732258
1.8203233270
1.3027150351
1.6512617219
1.7419614722
1.4713593280
1.9152803087
1.4743944600
1.5555416029
1.6849128608
1.5697224394
1.7973809738
1.8804787385
1.8676211840
1.3711547480
1.4128904705
1.2842612776
1.3357155998
1.8570826688
1.6438885306
1.8600328920
1.5200839314
1.3381757177
1.6354806596

X1 is Rayleigh(lamda=4.000000,c=1.120000),



lamda point estimated value=5.180196 (MLE)

c point estimated value=1.160250 (MLE)

lamda value from 4.316830 to 6.475245

c value from 1.140711 to 1.179790

H0: $X_1 \sim \text{Rayleigh}(\text{lamda}=4.331267, c=1.140711)$,

pearson goodness of fit

class	[1]	[2]	[3]	[4]
lower limit	1.16025	1.30586	1.45146	1.59706
1.74267	1.88827	2.03388		
upper limit	1.30586	1.45146	1.59706	1.74267
1.88827	2.03388	2.17948		
observed no	8.00000	26.00000	31.00000	16.00000
14.00000	2.00000	3.00000		
probability	0.11142	0.23039	0.25245	0.19759
0.11928	0.05730	0.03158		
expected no	11.14153	23.03868	25.24513	19.75882
11.92846	5.72951	3.15787		
chi square	0.88580	0.38064	1.31188	0.71506
0.35975	2.42765	0.00789		

pearson chi square test statistic=6.088672

degree of freedom=4, p-value=0.529400

correction:

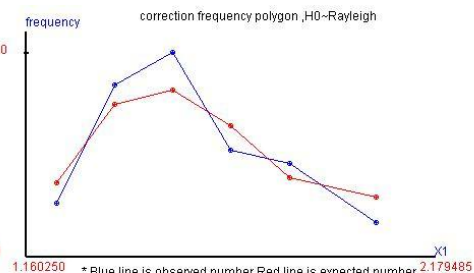
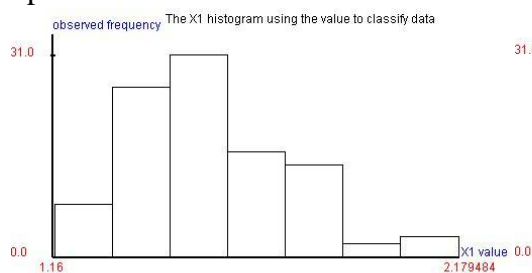
expected number ≥ 5 in each cell, the frequency table is adjusted

class	[1]	[2]	[3]	[4]
lower limit	1.16025	1.30586	1.45146	1.59706
1.74267	1.88827			
upper limit	1.30586	1.45146	1.59706	1.74267
1.88827	2.17948			
observed no	8.00000	26.00000	31.00000	16.00000
14.00000	5.00000			
probability	0.11142	0.23039	0.25245	0.19759
0.11928	0.08887			
expected no	11.14153	23.03868	25.24513	19.75882
11.92846	8.88738			
chi square	0.88580	0.38064	1.31188	0.71506
0.35975	1.70036			

degree of freedom=3

pearson chi-square test statistic =5.353487

p-value=0.147600



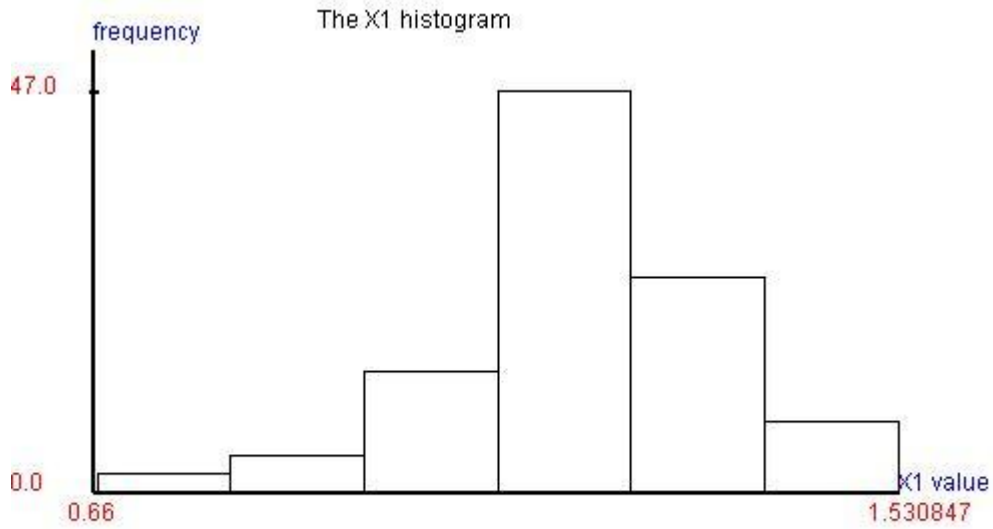
4.4.7)The population distribution is double exponential distribution.

X1 is Double Exponential($\lambda=10.000000,\mu=1.200000$),

X1
1.2401443801
1.2444561696
1.3434937801
1.3005800187
1.0254892543
1.1707618892
1.1894138979
1.2466167896
1.2195032959
1.0829519290
1.3012765671
1.4209634799
0.7771885025
1.0859409881
1.1508282252
1.1808227201
1.2312019750
0.9467679629
1.5308471093
1.0602045745
1.3374788659
1.1499486014
0.8951499181
1.2860766062
1.2799861189
1.1021786905
1.3419085094
1.1339141672
1.1821218174
1.1843258843
0.9642190151
1.2383919294
1.1443283108
1.0222711450
1.1623716866
1.3949231889
1.1869621791
1.3827200009
1.0946952084
1.0877655216
1.0973480862
0.6660182680
1.1787533859
1.0842033066
1.2454577862
1.1617857009
1.2723665932
1.1967381972
1.2324811684
1.1659118782
1.2687764245
1.2091580141
1.2255962225
1.2866959764
1.3264488895
1.2061828865
1.1318868164
1.1346336221
1.1199860464
1.1742764432
1.2003460483
1.1487582161
1.2486300270
1.3137763947
1.2566184134
1.4699825509
1.3526635621
1.1769744658
1.0826240204
1.2432312685
1.4191716360

1.2396063835
 1.0998201180
 1.1300816630
 1.4522344314
 1.0401384159
 1.0562935204
 0.8117002698
 1.2810894341
 1.2917773457
 1.1982074081
 1.1650590150
 1.2294831247
 1.2539077288
 1.3951373029
 0.9101935686
 1.3021517329
 1.2413627808
 1.0492621374
 1.1793517488
 1.4189134984
 1.2039114754
 1.1907243165
 1.2986245528
 1.1429510574
 1.1592113053
 1.1597686372
 1.1473476364
 1.1898050189
 1.2105389649

X1 is Double Exponential(lamda=10.000000,mu=1.200000),



lamda point estimated value=10.022255 (MLE)

mu point estimated value=1.190265 (MLE)

lamda value from 8.351879 to 12.527818

mu value from 1.168235 to 1.212295

H0: $X_1 \sim \text{Double exponential}(\lambda=8.379811, \mu=1.168235)$,

pearson goodness of fit

class	[1]	[2]	[3]	[4]
lower limit	0.66602	0.78957	0.91311	1.03666
1.16021	1.28375	1.40730		
upper limit	0.78957	0.91311	1.03666	1.16021
1.28375	1.40730	1.53085		
observed no	2.00000	3.00000	4.00000	26.00000
43.00000	16.00000	6.00000		
probability	0.02094	0.03802	0.10705	0.30146
0.34262	0.12247	0.06744		
expected no	2.09352	3.80172	10.70547	30.14610
34.26153	12.24734	6.74431		
chi square	0.00418	0.16907	4.20004	0.57023
2.22876	1.14984	0.08214		

pearson chi square test statistic=8.404251

degree of freedom=4,p-value=0.298300

correction:

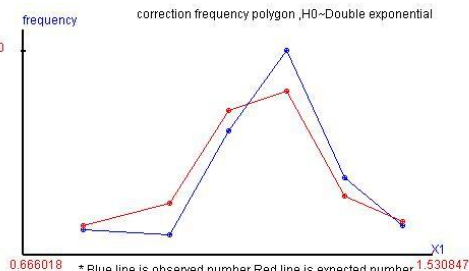
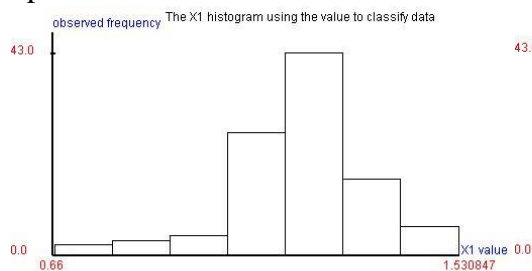
expected number ≥ 5 in each cell, the frequency table is adjusted

class	[1]	[2]	[3]	[4]
lower limit	0.66602	0.91311	1.03666	1.16021
1.28375	1.40730			
upper limit	0.91311	1.03666	1.16021	1.28375
1.40730	1.53085			
observed no	5.00000	4.00000	26.00000	43.00000
16.00000	6.00000			
probability	0.05895	0.10705	0.30146	0.34262
0.12247	0.06744			
expected no	5.89524	10.70547	30.14610	34.26153
12.24734	6.74431			
chi square	0.13595	4.20004	0.57023	2.22876
1.14984	0.08214			

degree of freedom=3

pearson chi-square test statistic =8.366953

p-value=0.039000



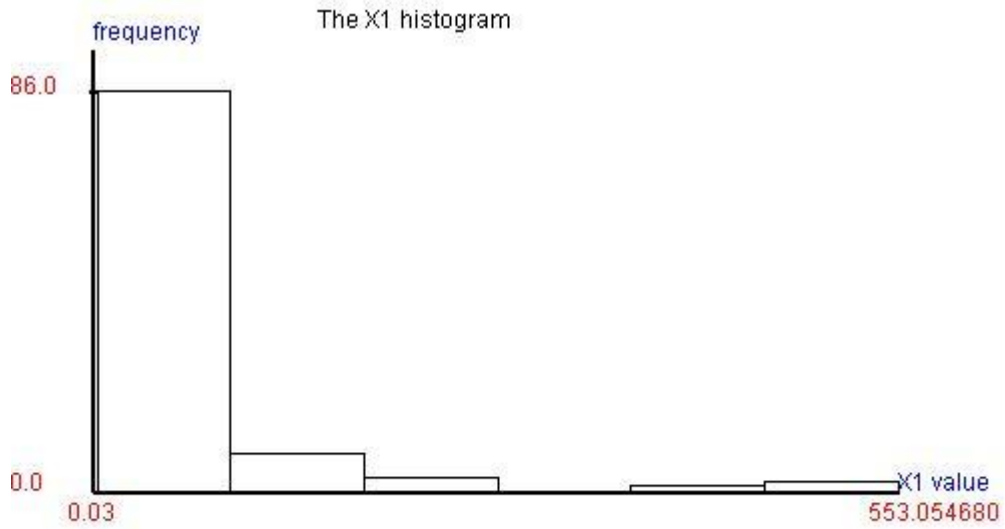
4.4.8)The population distribution is lognormal distribution.

X1 is Log normal($\mu=2.000000$, $\sigma=2.000000$),

X1
1.8348075384
23.5375316087
1.5628779961
16.0908082222
13.7943608351
6.6512477593
43.8292845694
0.5570746636
1.9893769969
104.7149143712
5.3501487169
6.2264287712
14.4073757415
9.1928546344
22.3240040793
102.4104927569
86.4155711255
1.4118222107
13.4439826088
1.5615408442
0.1209632706
10.5081524986
0.1809463782
26.0495578477
0.1956840085
9.4034869153
2.5750226488
54.3811815416
0.0319313945
5.2848392293
17.7376986380
1.9486183716
251.4346738230
443.3556503040
13.5822884937
3.2617382110
5.1960513710
8.4221265374
67.0668590013
1.2458439010
10.5146213463
7.1552816656
0.4815283130
4.8130462233
1.7175322459
160.7592026248
0.1824696522
0.7726212578
5.0272367657
4.8259178557
2.7984455158
17.9718214111
3.5519846634
15.2272860140
4.9545828904
12.4623417058
156.0587344416
0.2705725162
12.5926351582
0.1030141847
2.6734112544
10.5422733603
37.4694764278
10.0628857949
0.3200835690
0.5003569155
0.9496658317
4.8448571599
231.8168205159
12.1768887220
101.6170181224

0.5002802917
 483.4816261351
 1.8080991071
 10.6623086552
 1.2603899689
 50.8981700892
 79.0918898620
 17.3468272172
 0.9777850063
 553.0546808218
 11.2419960818
 99.8284802048
 0.9066098965
 1.4049291798
 13.7039750495
 144.2373730083
 0.7838379252
 179.7235736881
 0.4392009116
 12.6638413344
 0.0592621474
 3.6576407904
 32.1858946946
 8.0533977253
 0.1516064393
 197.6122311982
 2.4995254201
 47.4726768781
 4.9215043829

X1 is Log normal(mu=2.000000,sigma=2.000000),



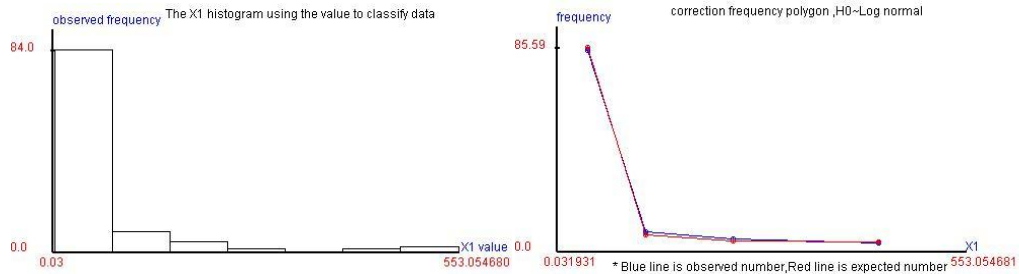
mu point estimated value=1.877620 (MLE)
 sigma point estimated value=2.147776 (MLE)
 mu value from 1.448065 to 2.307175
 sigma value from 1.789813 to 2.684720
 H0: $X_1 \sim \text{Log_Normal}(\mu=2.298584, \sigma=3.805996)$,
 pearson goodness of fit

class	[1]	[2]	[3]	[4]
lower limit	0.03193	79.03518	158.03843	237.04168
316.04493	395.04818	474.05143		
upper limit	79.03518	158.03843	237.04168	316.04493
395.04818	474.05143	553.05468		
observed no	84.00000	8.00000	4.00000	1.00000
0.00000	1.00000	2.00000		
probability	0.85590	0.06590	0.02610	0.01400
0.00850	0.00580	0.02380		
expected no	85.59000	6.59000	2.61000	1.40000
0.85000	0.58000	2.38000		
chi square	0.02954	0.30168	0.74027	0.11429
0.85000	0.30414	0.06067		

pearson chi square test statistic=2.400586
 degree of freedom=4, p-value=0.934300
 correction:
 expected number ≥ 5 in each cell, the frequency table is adjusted

class	[1]	[2]	[3]	[4]
lower limit	0.03193	79.03518	158.03843	316.04493
upper limit	79.03518	158.03843	316.04493	553.05468
observed no	84.00000	8.00000	5.00000	3.00000
probability	0.85590	0.06590	0.04010	0.03810
expected no	85.59000	6.59000	4.01000	3.81000
chi square	0.02954	0.30168	0.24441	0.17220

degree of freedom=1
 pearson chi-square test statistic =0.747840
 p-value=0.387100



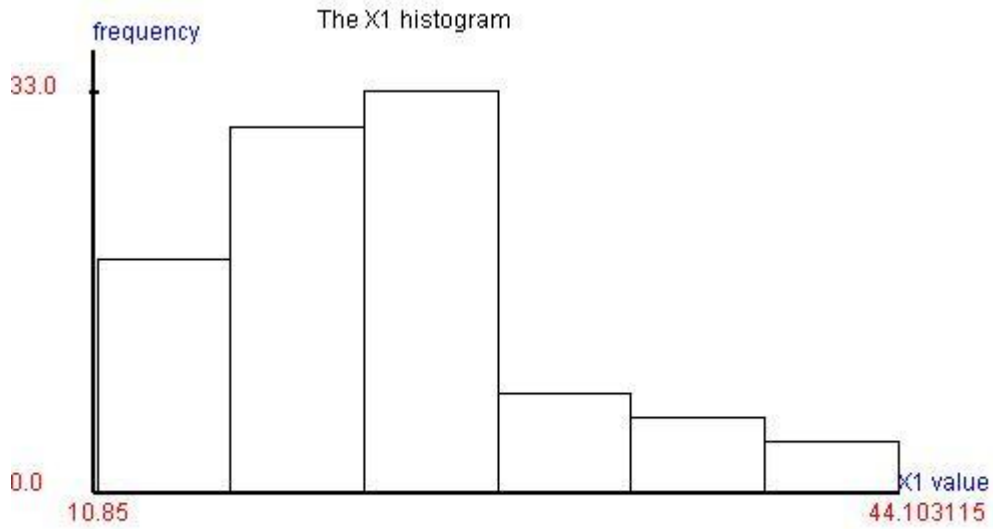
4.4.9)The population distribution is gamma distribution.

X1 is Gamma(alpha=10.000000,beta=2.200000),

X1
27.3843641511
26.9624359674
17.2506023586
17.2856588104
13.1738523507
22.9945481551
30.1666621501
17.2687558628
24.1087625031
22.3112271498
22.6291792811
25.9952120720
20.8239057204
27.5782651763
15.0382854449
15.6638474977
26.2363333678
24.2769914070
20.6976961721
24.1899055892
20.7346398911
23.8325067474
16.3452159302
38.9791880661
10.8543775501
19.7928861777
19.6252037335
14.4523647418
23.0576600225
14.8125299800
29.6136535614
22.1187134846
24.6774314263
23.6448699707
29.5165274224
19.2425083139
19.9583741774
26.8362098751
26.9437294901
15.6335855219
19.4996876701
33.9999481786
25.5606767946
21.3113971693
18.0631178490
22.2055757123
27.3632907987
19.9388967121
13.2574202511
14.7145243902
17.9244306582
15.9713356992
26.3139806885
34.2459632902
21.6409105105
17.1709871089
34.8222461955
39.1535731617
13.9275810655
19.3309932970
36.6343023339
11.5539346583
16.5049336983
25.9988798693
13.3566096273
36.1579765975
19.1964484810
18.6452530689
29.2062388013
17.0308922687
26.4889788164

15.1899583516
 24.8885104864
 22.1371863316
 17.2704918618
 24.2368598815
 21.9611893867
 11.7716297209
 19.5944878913
 32.8407850955
 19.4485670324
 23.2920111638
 27.0840065800
 11.0720958714
 20.8244902458
 44.1031150985
 27.6078435364
 18.4766454034
 15.9523484021
 22.5744579261
 21.4798214478
 27.1375571643
 20.1189326783
 22.8720615015
 41.4477053706
 35.8936051957
 19.9146218598
 12.7507135078
 23.2566439258
 28.0845403520

X1 is Gamma(alpha=10.000000,beta=2.200000),



alpha point estimated value=10.500000 (MME)
 beta point estimated value=2.157984 (MME)
 alpha values are 10.000000, 10.500000 and 11.000000
 beta value from 1.726387 to 2.589581
 H0: $X1 \sim \text{Gamma}(\alpha=11.000000, \beta=2.028505)$,

pearson goodness of fit

class	[1]	[2]	[3]	[4]
lower limit	10.85438	15.60420	20.35402	25.10384
29.85366	34.60348	39.35330		
upper limit	15.60420	20.35402	25.10384	29.85366
34.60348	39.35330	44.10312		
observed no	14.00000	28.00000	27.00000	19.00000
4.00000	6.00000	2.00000		
probability	0.15490	0.26640	0.26960	0.17630
0.08510	0.03290	0.01480		
expected no	15.49000	26.64000	26.96000	17.63000
8.51000	3.29000	1.48000		
chi square	0.14332	0.06943	0.00006	0.10646
2.39014	2.23225	0.18270		

pearson chi square test statistic=5.124367
 degree of freedom=4,p-value=0.644800

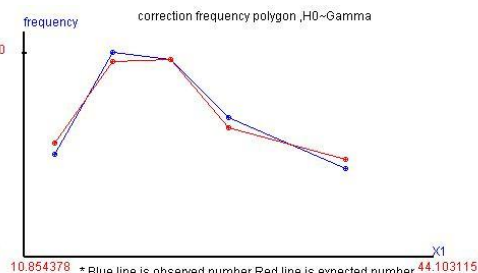
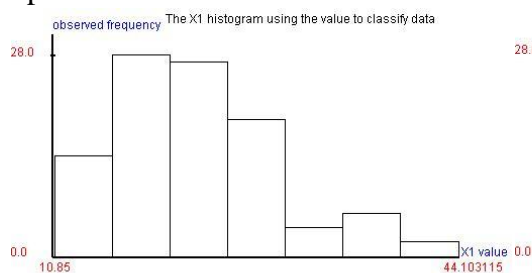
correction:

expected number ≥ 5 in each cell, the frequency table is adjusted

class	[1]	[2]	[3]	[4]
lower limit	10.85438	15.60420	20.35402	25.10384
29.85366				
upper limit	15.60420	20.35402	25.10384	29.85366
44.10312				
observed no	14.00000	28.00000	27.00000	19.00000
12.00000				
probability	0.15490	0.26640	0.26960	0.17630
0.13280				
expected no	15.49000	26.64000	26.96000	17.63000
13.28000				
chi square	0.14332	0.06943	0.00006	0.10646
0.12337				

degree of freedom=2

pearson chi-square test statistic =0.442648
 p-value=0.801400



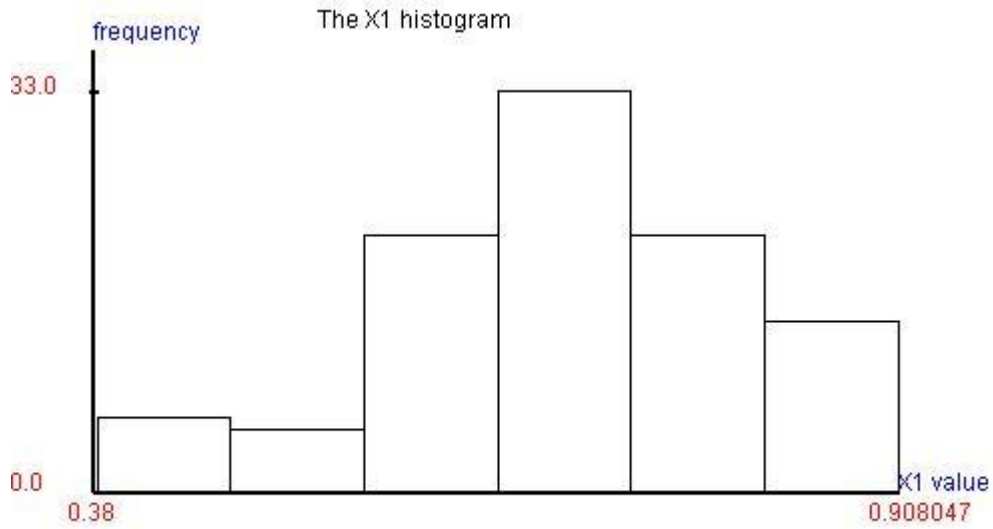
4.4.10)The population distribution is beta distribution.

X1 is Beta(alpha=12.000000,beta=5.000000),

X1
0.6269365050
0.6903735086
0.6212958870
0.6365547395
0.6821277833
0.7338114862
0.8050751094
0.4235553326
0.8857541487
0.6721301324
0.6060510149
0.7541109181
0.7813019361
0.6510566328
0.4109965347
0.6570331838
0.6828828849
0.7073135057
0.8887303724
0.8968035374
0.9080475816
0.5984939146
0.5876415397
0.8426491747
0.7129799505
0.8027309100
0.8197821658
0.7797352491
0.5591482776
0.7350980349
0.7841701920
0.8520645536
0.7025420257
0.8325114679
0.7040708274
0.7137058998
0.6853007032
0.6858685757
0.5894474112
0.6786534602
0.5912284100
0.6446151264
0.8372343677
0.7948080144
0.6588832598
0.7405808805
0.7091942302
0.5556282295
0.8120348781
0.7273702406
0.4525703740
0.6921969461
0.7983411461
0.6744223465
0.6006821019
0.6733969148
0.4486912232
0.7191111060
0.7211617693
0.6022554837
0.8032462132
0.7844501528
0.6656163741
0.7746683702
0.6481573309
0.7253628223
0.8003851997
0.6605370907
0.6331124073
0.5978543724
0.4616527742

0.6313962313
0.8312303139
0.7878964873
0.6864567966
0.7354173405
0.5009075952
0.7726443269
0.8657668225
0.8933967835
0.6841102836
0.5244635180
0.3834666803
0.6143270669
0.8424366156
0.6440851816
0.6991597592
0.7991286839
0.5300710624
0.8373015151
0.6223186081
0.6969586354
0.5716897062
0.5238652923
0.7323589179
0.7080228671
0.7303566887
0.8218500434
0.6237555396
0.5827470544

X1 is Beta(alpha=12.000000,beta=5.000000),



alpha point estimated value=10.000000 (MME)

beta point estimated value=4.500000 (MME)

alpha values are 9.500000 ,10.000000 and 10.500000,

beya values are 4.000000 ,4.500000 and 5.000000,

H0: $X1 \sim \text{Beta}(\alpha=9.500000, \beta=4.000000)$,

pearson goodness of fit

class	[1]	[2]	[3]	[4]
lower limit	0.38347	0.45841	0.53335	0.60829
0.68323	0.75817	0.83311		
upper limit	0.45841	0.53335	0.60829	0.68323
0.75817	0.83311	0.90805		
observed no	5.00000	5.00000	12.00000	22.00000
26.00000	19.00000	11.00000		
probability	0.03097	0.06009	0.12120	0.19256
0.23802	0.21535	0.14181		
expected no	3.09694	6.00890	12.11978	19.25635
23.80233	21.53492	14.18078		
chi square	1.16943	0.16940	0.00118	0.39092
0.20291	0.29839	0.71346		

pearson chi square test statistic=2.945682

degree of freedom=4, p-value=0.889900

correction:

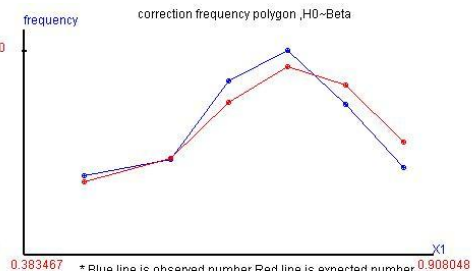
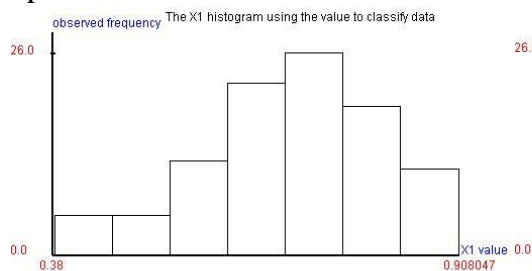
expected number ≥ 5 in each cell, the frequency table is adjusted

class	[1]	[2]	[3]	[4]
lower limit	0.38347	0.53335	0.60829	0.68323
0.75817	0.83311			
upper limit	0.53335	0.60829	0.68323	0.75817
0.83311	0.90805			
observed no	10.00000	12.00000	22.00000	26.00000
19.00000	11.00000			
probability	0.09106	0.12120	0.19256	0.23802
0.21535	0.14181			
expected no	9.10584	12.11978	19.25635	23.80233
21.53492	14.18078			
chi square	0.08780	0.00118	0.39092	0.20291
0.29839	0.71346			

degree of freedom=3

pearson chi-square test statistic =1.694659

p-value=0.638100



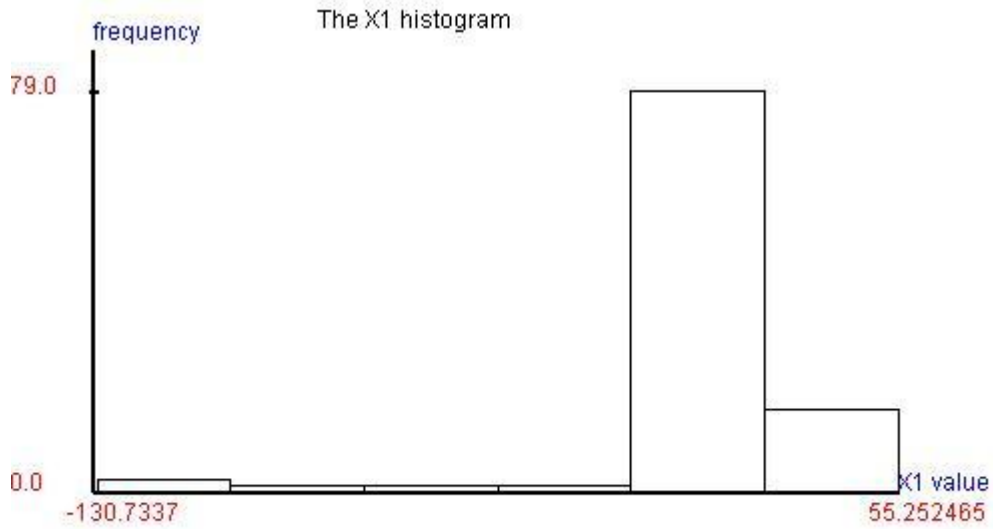
4.4.11)The population distribution is cauchy distribution.

X1 is Cauchy($\mu=20.000000$, $\sigma=3.000000$),

X1
15.3117695711
22.3603920309
20.2473167813
3.2334490288
-8.6684828007
37.1188259879
25.8363734666
13.4330590782
20.8963120597
20.2641493972
16.1964939584
13.1959427112
23.6502542925
16.9348343272
20.6846428366
23.0828294718
14.0249765042
18.2242417510
23.4194157205
42.4351344302
21.2759603560
19.3591116492
20.9109934010
25.4742808676
29.6670040082
21.0676510525
-68.9137765169
19.8339769424
21.5277372853
21.8204991907
21.6071973990
19.3325078753
18.3640138269
15.1085583718
19.2862060303
16.2836895996
19.0285191334
22.1479271577
1.0693125800
28.4851265895
20.4541238916
22.6428686511
4.0495809605
20.3176324228
19.2931071225
15.6688568959
14.4891144540
14.5562973001
21.1400946189
18.6568002047
7.2919473372
20.0535205836
23.7291829958
29.9825170137
35.4301438713
55.2524657912
25.4259603411
25.7534960246
13.4081102983
20.0994267852
16.7241945042
-2.5592751057
23.5864626593
23.5281370203
27.4061134147
21.0325458188
6.1144658052
22.4456190914
8.6587890773
22.2201794110
24.5401648765

15.7064410174
 14.8859499534
 -130.7337144098
 19.2172332277
 -114.3766529112
 19.0846860193
 19.8056230164
 22.4250699281
 18.3788152929
 24.5452829443
 14.6996116490
 22.5165345566
 19.3979269943
 11.1994864105
 17.8101545174
 29.3518251667
 17.6753222585
 -68.6066690925
 22.5781924984
 19.9709965795
 20.2057494524
 22.0374300734
 3.8053514661
 18.4209400801
 23.3384081876
 30.3137042580
 19.0553649379
 16.0566253442
 21.6311091948

X1 is Cauchy(mu=20.000000,sigma=3.000000),



mu point estimated value=20.012259
 sigma point estimated value=3.445272
 mu value from 19.323204 to 20.701313
 sigma value from 2.871060 to 4.306590
 H0: $X_1 \sim \text{Cauchy}(\mu=19.323204, \sigma=3.244298)$,

pearson goodness of fit

class	[1]	[2]	[3]	[4]	[5]	[6]	[7]
lower limit	-130.73371	-104.16426	-77.59481	-51.02535	-24.45590		
upper limit	2.11356	28.68301					
observed no	2.00000	0.00000	2.00000	0.00000			
probability	0.00836	0.00229	0.00402	0.00888			
expected no	0.83608	0.22905	0.40179	0.88764			
chi square	1.62031	0.22905	6.35720	0.88764			
	0.09292	0.02886	0.64669				

pearson chi square test statistic=9.862675
 degree of freedom=4, p-value=0.196500

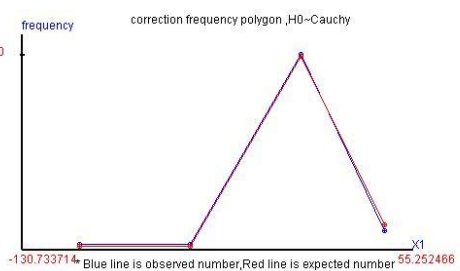
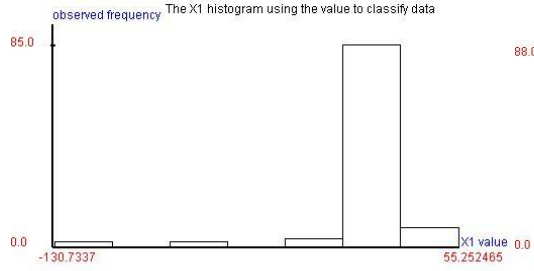
correction:

expected number ≥ 5 in each cell, the frequency table is adjusted

class	[1]	[2]	[3]	[4]
lower limit	-130.73371	-77.59481	-24.45590	28.68301
upper limit	-77.59481	-24.45590	28.68301	55.25247
observed no	2.00000	2.00000	88.00000	8.00000
probability	0.01065	0.01289	0.87025	0.10621
expected no	1.06513	1.28943	87.02469	10.62075
chi square	0.82053	0.39157	0.01093	0.64669

degree of freedom=1

pearson chi-square test statistic =1.869723
 p-value=0.171500



4.4.12) The population distribution is arcsin distribution.

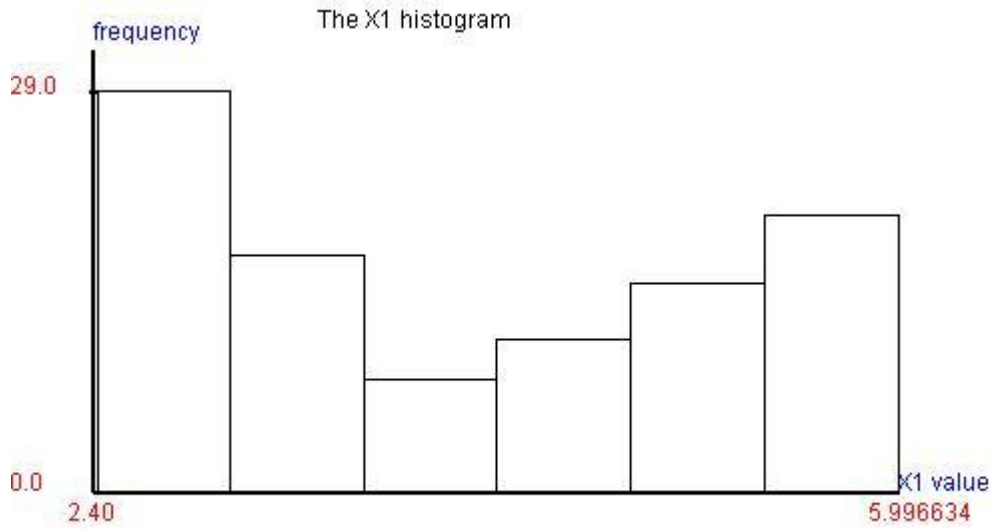
X1 is $\text{Arcsin}(\mu=4.200000, c=1.800000)$,

X1

2.8834688302
4.2083475873
5.9871425738
5.7374140234
5.3031331349
3.1438932829
2.9665797763
2.6616006612
2.6771999315
4.3085978609
5.6039182510
2.4941767543
2.7833675265
2.9553341758
4.1934108908
5.9178158227
2.4885393499
3.6589286147
5.3695371979
3.4615151209
5.9357492819
2.4766020366
2.9742324080
5.6683018464
5.1550022788
3.1134866439
4.8579837436
2.7414621185
5.1035104894
4.2356570616
3.5636526615
4.0329604469
5.7765536871
4.7620548772
5.3077357412
5.8431263777
3.5504239581
5.5633304386
5.9526758365
4.1221351217
3.5884646049
4.8964455162
3.0453624103
5.9513770767
5.0782399560
3.5224820522
5.6244178390
2.4177945184
4.9365896714
2.7087510449
2.9542910199
5.8341170940
3.7700035071
4.9132337563
3.3853171258
4.6271914268
2.4003818286
2.9315517302
5.8409194521
5.2946222686
4.3221760734
3.1351105658
2.4515357579
3.4954595488
5.2209496968
4.4340674765
2.8522712612
5.1145132578
3.5266961277
2.8544339073
2.6192505613

2.7767589598
 2.9375523630
 2.6876467607
 2.4016467212
 5.4819225283
 4.7682269340
 5.9966348178
 2.5571230521
 5.9959030909
 2.9373313850
 3.2045187193
 2.4093864036
 3.5641986789
 4.8573258842
 3.8021754216
 4.2513052162
 3.1595688863
 5.8795046852
 5.3331356672
 5.8918007711
 2.4188006818
 3.1059885629
 3.7232828686
 3.5726776912
 2.6574279082
 4.6839870693
 3.8733773206
 4.7194071724
 5.8875564310

X1 is Arcsin(mu=4.200000,c=1.800000),



mu point estimated value=4.198508
 c point estimated value=1.798126
 mu value from 3.838883 to 4.558134
 c value from 1.498439 to 2.247658
 H0: $X1 \sim \text{Arcsin}(\mu=4.191316, c=1.603329)$,

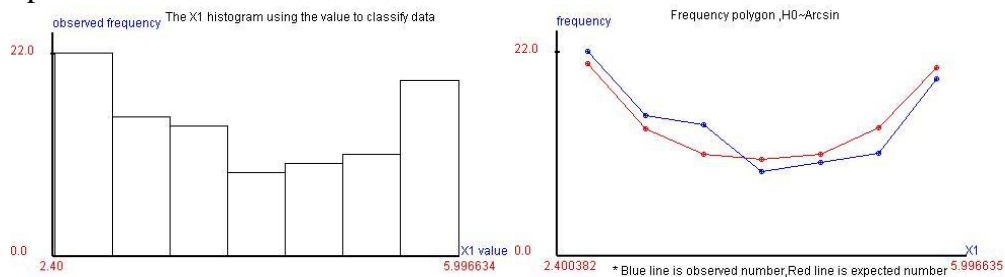
pearson goodness of fit

class	[1]	[2]	[3]	[4]
[5]	[6]	[7]		
lower limit	2.40038	2.91413	3.42788	3.94163
4.45538	4.96913	5.48288		
upper limit	2.91413	3.42788	3.94163	4.45538
4.96913	5.48288	5.99663		
observed no	22.00000	15.00000	14.00000	9.00000
10.00000	11.00000	19.00000		
probability	0.20664	0.13539	0.10820	0.10244
0.10856	0.13691	0.20187		
expected no	20.66392	13.53910	10.81976	10.24377
10.85611	13.69056	20.18678		
chi square	0.08639	0.15763	0.93476	0.15101
0.06751	0.52877	0.06977		

degree of freedom=4

pearson chi-square test statistic =1.995853

p-value=0.736500



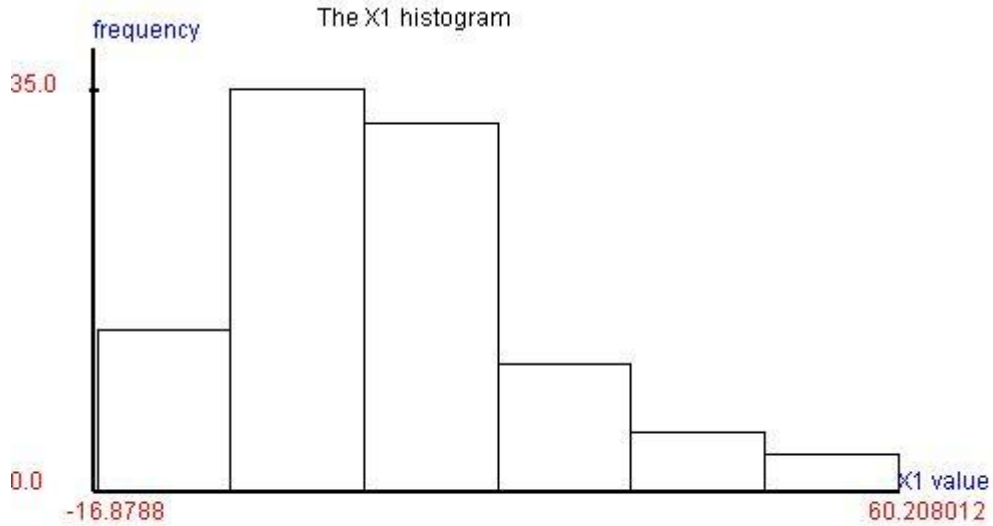
4.4.13)The population distribution is gumbel distribution.

X1 is Gumbel($\mu=4.500000$, $\sigma=12.200000$),

X1
2.0879798083
12.6377575650
47.1553372443
27.7654719300
7.6377968691
23.7894002843
23.0532347170
36.2571792776
21.6644782419
45.3525208544
5.4747201584
4.2643793722
13.6924335426
4.8265748636
-11.3142408787
22.6688845077
-0.5719792304
6.7650703606
20.0524687664
0.6799344581
6.3275558708
4.5727815120
9.4305556886
17.9363625220
-6.2765505909
15.4986984383
20.8916094830
4.4711770149
-6.9159258710
-5.6158941413
28.3822180865
6.5091905957
29.9561803329
5.6253174643
41.8307401510
14.3480570650
30.1371724128
10.0952180686
10.7321190292
4.0715775597
-0.2092078696
12.6775478006
19.0412495462
0.8856621284
3.3904137987
5.4594089261
60.2080128097
18.4983612623
-12.9694708595
10.9701786354
12.4440202614
21.7493472683
0.1842743919
3.3571407016
18.5917547893
17.5432546572
9.3821854638
-11.1662865775
-2.9628330647
0.8835125631
28.0056802545
-16.8788612193
10.2808387201
9.6107639499
8.9869194780
4.2790624799
4.2530290552
4.1997958766
-9.8416286134
-0.6393874534
10.0715369411

10.9319003218
 5.4994210777
 13.3801416513
 10.5793242789
 8.6943055791
 2.0781477786
 -7.7659102263
 2.6374375825
 9.8624363815
 4.9111986917
 16.6532729508
 12.5586089145
 39.0046411573
 56.3063931805
 -10.5260761005
 -6.6290310583
 11.4996128869
 9.2597988707
 7.0375915122
 -7.5971058164
 57.1949536456
 5.9183969794
 -4.0405692115
 21.1856332854
 4.9725403508
 26.8636705806
 24.4352664594
 2.2440490782
 -12.1333944208

X1 is Gumbel($\mu=4.500000$, $\sigma=12.200000$),



mu point estimated value=4.275879
 sigma point estimated value=11.842415
 mu value from 1.907395 to 6.644362
 sigma value from 9.868680 to 14.803019
 H0: $X_1 \sim \text{Gumbel}(\mu=5.507490, \sigma=10.362114)$,

pearson goodness of fit

class	[1]	[2]	[3]	[4]
lower limit	-16.87886	-5.86645	5.14596	16.15837
upper limit	27.17078	38.18319	49.19560	60.20801
observed no	6.00000	4.00000	3.00000	3.00000
probability	0.04993	0.30512	0.34419	0.18449
expected no	7.44671	2.71612	1.46472	0.33200
chi square	0.28106	0.60688	1.60925	0.32509

pearson chi square test statistic=13.380555
 degree of freedom=4,p-value=0.063300

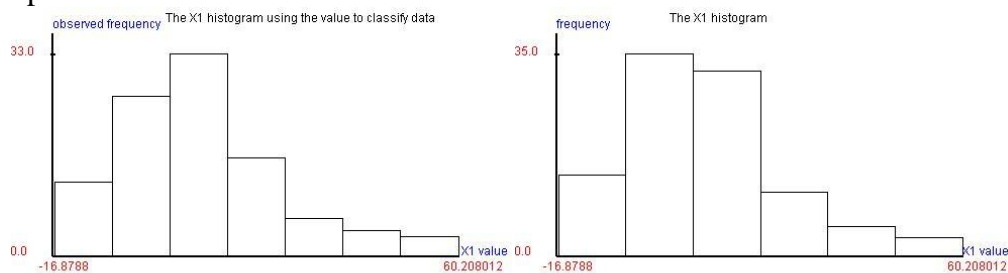
correction:

expected number ≥ 5 in each cell, the frequency table is adjusted

class	[1]	[2]	[3]	[4]
lower limit	-16.87886	5.14596	16.15837	27.17078
upper limit	5.14596	16.15837	27.17078	60.20801
observed no	38.00000	33.00000	16.00000	13.00000
probability	0.35505	0.34419	0.18449	0.11628
expected no	35.50469	34.41876	18.44900	11.62755
chi square	0.17537	0.05848	0.32509	0.16200

degree of freedom=1

pearson chi-square test statistic =0.720943
 p-value=0.395800



4.4.14) The population distribution is triangular 1 distribution.

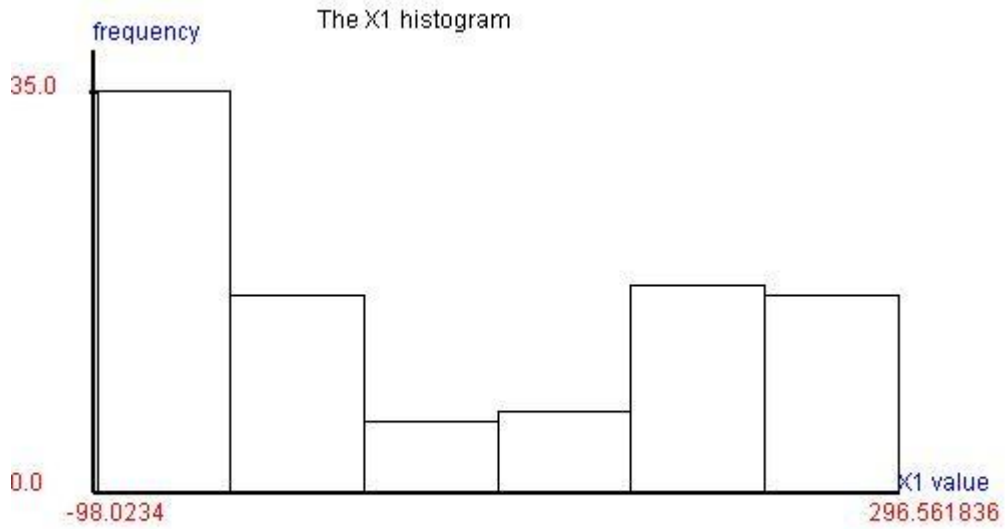
X1 is $\text{Triangular}(\mu=100.000000, c=200.000000)$,

X1

-86.2578480905
-13.9799593168
-44.9109460889
-5.5344607266
238.1287186711
-98.0234304407
16.6583853657
230.4136785380
193.1500832405
-86.6367384395
289.2776448816
-91.5240504288
71.1680122872
217.8703171576
-89.4148981348
-50.8755821248
296.3932257848
-13.3169925057
-64.7394228679
5.3710673640
125.1866010107
-51.8639170597
251.1751372080
201.0767599103
294.1376454081
-94.2428104387
296.5618365894
-62.1613401061
-91.2227971857
220.7964866057
38.9551410412
-25.4756104402
-85.0313128660
16.6403103770
-37.1598386489
269.5199447371
197.8215479362
199.0398900542
-55.0766875057
-7.5493413250
197.7415019908
-21.0177071243
234.5668270552
239.1793389439
-64.1445736249
-61.8049862798
189.3539357689
250.3941323024
281.2510929021
-47.3265185674
-83.0312157449
-88.1288338028
-73.3172717315
-54.4653383156
251.6424958711
186.3316561254
-16.2858836382
24.4062153350
-64.2750401779
-75.3787648056
191.4648942545
26.2637531193
178.3969234715
285.4027393857
58.6332074590
-76.0533930480
-90.2142100619
21.5360460858
59.9482955508
171.8712287366
184.4195815808

-56.2654604780
 -29.7353252602
 137.7553069280
 -59.2994956305
 257.3339490395
 146.9129904271
 12.3072137869
 0.7881081458
 -61.0201963405
 -64.2190731668
 -90.2733405150
 125.7406569424
 164.0962186260
 162.9013210452
 296.5484000556
 163.1687101333
 -62.4318523357
 -82.1285909745
 8.5624823379
 181.8801490183
 86.6475511647
 89.2349782979
 222.6726469716
 278.6339534283
 -76.7361935162
 243.3282046636
 217.3806977208
 230.5685004985
 -56.2537083529

X1 is Triangular1(mu=100.000000,c=200.000000),



mu point estimated value=99.269203
 c point estimated value=197.292634
 mu value from 59.810676 to 138.727730
 c value from 164.410528 to 246.615792
 H0: $X_1 \sim \text{Triangular } 1(\mu=80.329110, c=194.004423)$,
 pearson goodness of fit

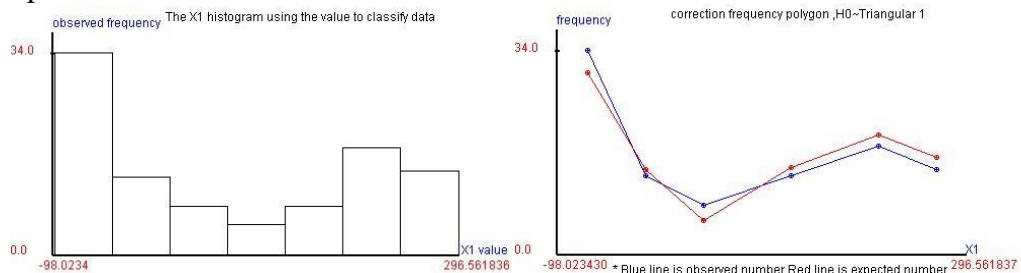
class	[1]	[2]	[3]	[4]
lower limit	-98.02343	-41.65411	14.71522	71.08454
127.45387	183.82319	240.19251		
upper limit	-41.65411	14.71522	71.08454	127.45387
183.82319	240.19251	296.56184		
observed no	34.00000	13.00000	8.00000	5.00000
8.00000	18.00000	14.00000		
probability	0.30233	0.14048	0.05606	0.03064
0.11279	0.19721	0.16050		
expected no	30.23272	14.04804	5.60571	3.06369
11.27895	19.72129	16.04960		
chi square	0.46944	0.07819	1.02264	1.22379
0.95324	0.15023	0.26174		

pearson chi square test statistic=4.159273
 degree of freedom=4,p-value=0.761200

correction:
 expected number ≥ 5 in each cell, the frequency table is adjusted

class	[1]	[2]	[3]	[4]
lower limit	-98.02343	-41.65411	14.71522	71.08454
183.82319	240.19251			
upper limit	-41.65411	14.71522	71.08454	183.82319
240.19251	296.56184			
observed no	34.00000	13.00000	8.00000	13.00000
18.00000	14.00000			
probability	0.30233	0.14048	0.05606	0.14343
0.19721	0.16050			
expected no	30.23272	14.04804	5.60571	14.34264
19.72129	16.04960			
chi square	0.46944	0.07819	1.02264	0.12569
0.15023	0.26174			

degree of freedom=3
 pearson chi-square test statistic =2.107934
 p-value=0.550300



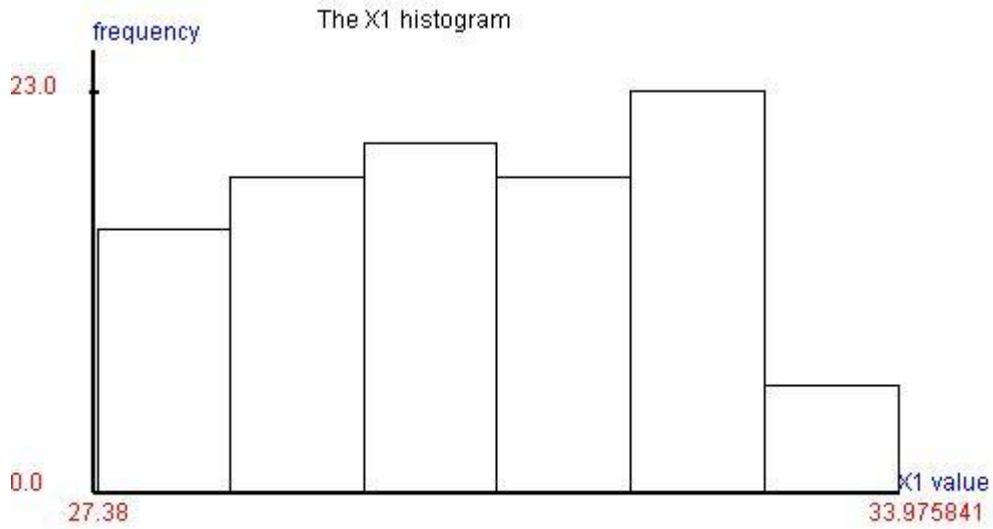
4.4.15) The population distribution is trapezoid distribution.

X_1 is Trapezoid($\mu=30.400000, c=2.400000$),

X_1
32.9005631731
27.9835544310
30.2615384209
29.3368618578
32.9585049492
28.3410208308
32.2977913519
31.9293916105
29.2824790352
29.6729946432
29.9877365477
32.1472366245
27.6842141341
30.8667099009
32.2096833401
30.1771457218
27.5098430771
30.5207346092
28.4590395570
28.6545400882
30.0427316398
32.4194011873
28.2646601122
31.3571147715
31.8073841012
32.2747576620
31.2225153375
33.6025428265
28.0504691907
30.4451427304
32.1014930729
28.0457050619
31.7602810741
29.4793933273
32.7962047046
29.2050711184
29.6340138840
32.7231889622
33.3322886086
30.0872590584
30.1757448750
27.5998071764
28.9047837863
30.2343508879
31.2985715262
28.9970201163
30.8474650702
31.1944162939
28.5959660723
28.4091009871
31.5788806255
30.2925100219
29.1596590972
30.9464649116
30.9573671429
29.1235651778
31.2641088394
28.7352432304
30.7075292255
27.8994262285
30.1682830371
29.1309629149
30.1826989844
32.1577997045
27.7330467498
32.6541211939
32.4653578808
30.4192076776
31.3826617967
30.9966431955
30.9291228042

28.2657750008
 32.7287562270
 32.4778598742
 32.5213456209
 29.5351889735
 32.9535720278
 29.9582732896
 29.4611716921
 27.3875754331
 30.1576124735
 33.9758413289
 29.3323474558
 31.7822188448
 29.6139731218
 32.6495855403
 28.3537496688
 31.7910135530
 31.3225900057
 28.6200756899
 32.2368298714
 30.9491864068
 31.9177819917
 32.0022581240
 30.6383668331
 29.5338335750
 29.8250567260
 32.4399572479
 29.4418838049
 30.7009926006

X1 is Trapezoid($\mu=30.400000, c=2.400000$),



mu point estimated value=30.681708
 c point estimated value=2.196089
 mu value from 30.242491 to 31.120926
 c value from 1.689299 to 3.137269
 H0: X1~Trapezoid(mu=30.444531,c=2.456723),
 mu point estimated value=30.444531 (MLE)
 c point estimated value=2.456723 (MLE)
 pearson goodness of fit

class	[1]	[2]	[3]	[4]
[5]	[6]	[7]		
lower limit	27.38758	28.32876	29.26994	30.21112
31.15230	32.09348	33.03466		
upper limit	28.32876	29.26994	30.21112	31.15230
32.09348	33.03466	33.97584		
observed no	11.00000	14.00000	21.00000	16.00000
15.00000	20.00000	3.00000		
probability	0.10201	0.15893	0.19155	0.19155
0.18422	0.12207	0.04966		
expected no	10.20106	15.89324	19.15521	19.15521
18.42248	12.20665	4.96616		
chi square	0.06257	0.22553	0.17767	0.51972
0.63582	4.97568	0.77842		

pearson chi square test statistic=7.375411,degree of freedom=4,p-value=0.390800
 correction:
 expected number>=5 in each cell, the frequency table is adjusted

class	[1]	[2]	[3]	[4]
[5]	[6]			
lower limit	27.38758	28.32876	29.26994	30.21112
31.15230	32.09348			
upper limit	28.32876	29.26994	30.21112	31.15230
32.09348	33.97584			
observed no	11.00000	14.00000	21.00000	16.00000
15.00000	23.00000			
probability	0.10201	0.15893	0.19155	0.19155
0.18422	0.17173			
expected no	10.20106	15.89324	19.15521	19.15521
18.42248	17.17280			
chi square	0.06257	0.22553	0.17767	0.51972
0.63582	1.97733			

degree of freedom=3
 pearson chi-square test statistic =3.598635,p-value=0.308100

The X1 histogram using the value to classify data

correction frequency polygon ,H0~Trapezoid

* Blue line is observed number,Red line is expected number

4.4.16)The population distribution is U quadratic distribution.

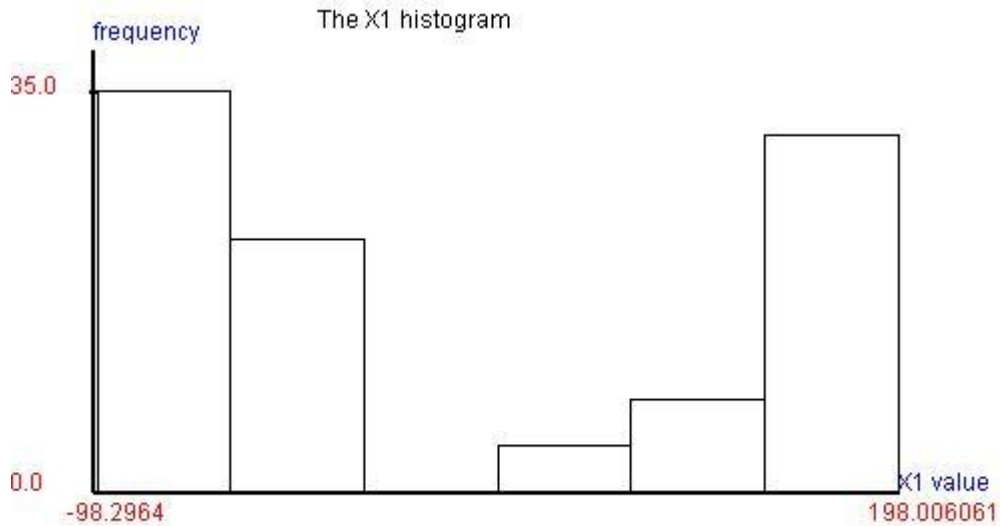
X1 is U-quadratic(a=-100.000000,b=200.000000),

X1

-70.8268247884
-35.9476288323
171.5366841429
92.2494812033
-41.0733500181
180.5153991451
-76.5731351957
-38.8230258490
177.9483470097
184.7789034430
-66.6370624370
-22.0323683221
-59.1250076022
-68.8325885845
147.8728961952
-64.6872023705
196.6129032617
-47.2910986231
-61.3302685347
-90.1250936584
118.6976034934
-98.2964599305
162.4987617652
-61.9922749919
190.1057929498
92.9853721556
-25.1664124903
149.3254631981
198.0060614199
143.5929323792
-44.3278865881
-78.3228072574
184.3018686591
149.1642324113
145.0048825967
169.5334592205
-89.0501697870
102.0690851302
-85.5833679196
-97.0964578908
194.0075786513
159.5471840876
191.4223391153
184.8959351389
195.3690975450
-32.6291442722
196.2646898754
-39.1913266815
-95.8030417194
167.1464279630
134.5339336931
-34.8525580171
180.4426027425
-54.9784812373
-9.8386451453
-10.8242137105
-94.1137932645
-48.7102429467
192.2074661428
-83.4494218851
-59.9578984269
162.6853668991
-91.4888816773
-26.8732970234
-9.5486209661
-73.8090369528
-85.0576417820
-95.3574345704
149.7933780159
-42.9358505418
187.1264953703

-81.7700617337
 -44.2842780820
 -43.1803575714
 -64.2057292470
 -56.9916280658
 -46.4910960407
 186.7655809566
 -70.6371449264
 167.3345708915
 -87.9300188839
 -87.2857707382
 193.5138723656
 -31.1872823186
 181.2202681790
 97.4744741077
 -73.7749214060
 -80.0772124092
 171.6771488657
 194.4164721258
 115.1893233305
 -44.8004578674
 178.8354979401
 -6.8067709075
 129.1507145132
 -51.3111454305
 97.0733497270
 -85.1763912856
 -91.4399182965
 -93.2960357072

X1 is U-quadratic(a=-100.000000,b=200.000000,)



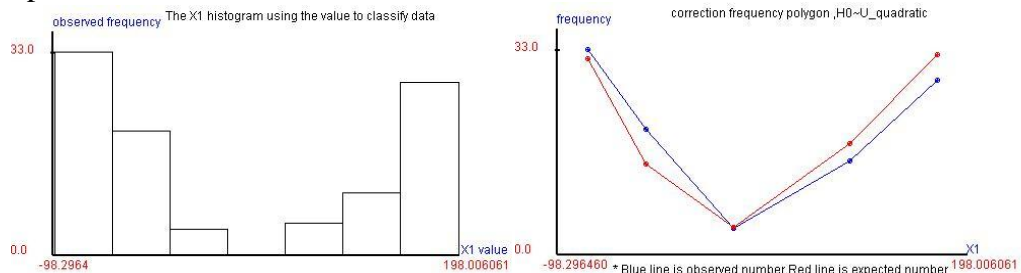
a point estimated value=-98.296460
 b point estimated value=198.006061
 a value from -98.889065 to -97.703855
 b value from 197.413456 to 198.598666
 H0: $X1 \sim U_quadratic(a=-97.727559, b=198.586814)$,
 pearson goodness of fit

class	[1]	[2]	[3]	[4]
lower limit	-98.29646	-55.96753	-13.63860	28.69033
71.01927	113.34820	155.67713		
upper limit	-55.96753	-13.63860	28.69033	71.01927
113.34820	155.67713	198.00606		
observed no	33.00000	20.00000	4.00000	0.00000
5.00000	10.00000	28.00000		
probability	0.31480	0.14480	0.03880	0.00300
0.03690	0.14100	0.32070		
expected no	31.48000	14.48000	3.88000	0.30000
3.69000	14.10000	32.07000		
chi square	0.07339	2.10431	0.00371	0.30000
0.46507	1.19220	0.51652		

pearson chi square test statistic=4.655203, degree of freedom=4, p-value=0.701900
 correction:
 expected number ≥ 5 in each cell, the frequency table is adjusted

class	[1]	[2]	[3]	[4]
lower limit	-98.29646	-55.96753	-13.63860	71.01927
155.67713				
upper limit	-55.96753	-13.63860	71.01927	155.67713
198.00606				
observed no	33.00000	20.00000	4.00000	15.00000
28.00000				
probability	0.31480	0.14480	0.04180	0.17790
0.32070				
expected no	31.48000	14.48000	4.18000	17.79000
32.07000				
chi square	0.07339	2.10431	0.00775	0.43755
0.51652				

degree of freedom=2
 pearson chi-square test statistic =3.139531
 p-value=0.208000



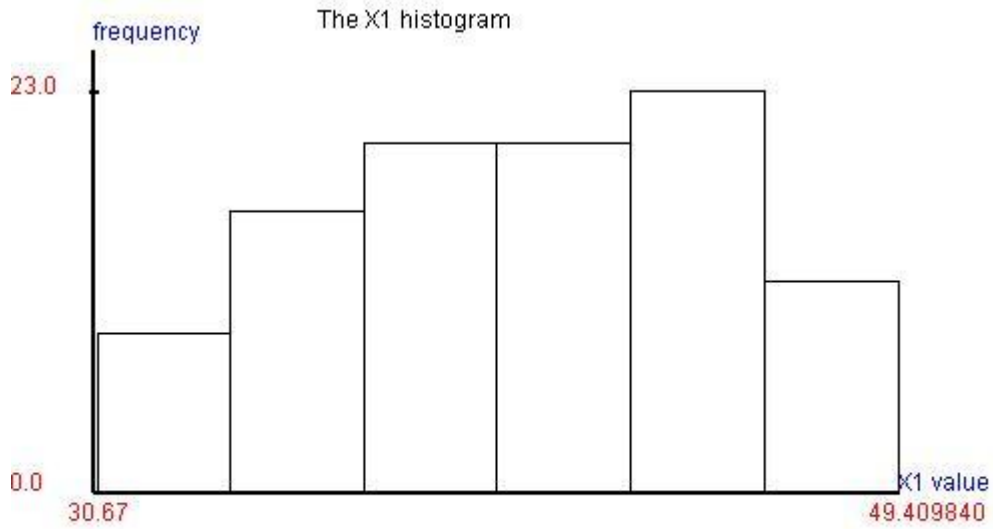
4.4.17)The population distribution is semi circle distribution.

X1 is Semi-circle($\mu=40.000000,R=10.000000$),

X1
37.2377685248
43.7985040159
38.1220835319
37.0376294478
33.4410154175
36.3318887816
43.8796014987
49.2098648971
34.8460616032
40.6845255580
35.8936716065
31.2987394896
42.9387375697
38.7960924440
44.8849389058
43.1747883936
42.0291947910
43.1813646709
37.5652063375
45.7028010804
36.8682890133
44.3401936827
38.2861273814
41.7981870872
42.5885049203
44.9375328517
33.4931051803
44.7478539193
44.7166053846
38.5288404382
46.1695570801
42.1248654716
42.2260315836
36.1513817513
41.4510376686
47.0015591487
34.2868545598
42.2254278672
41.6160027146
45.3409804539
37.5522173511
38.4044872148
43.6506493348
39.1093568671
36.4372545680
35.2833289598
40.2115863125
41.3055479057
37.5383267668
34.3991320924
34.3167869107
32.6912138743
46.2913581057
46.8933633728
43.8793266899
47.4998644926
42.2058917346
39.3646908371
33.1910574137
45.5400015490
46.6032655139
32.2370041838
36.9091804886
36.8103162293
42.8454790992
36.5365637306
38.6080198142
49.0748956137
47.6309092392
49.4098408140
45.1339699472

43.4434140019
 40.3345033805
 41.8518654702
 38.0215805880
 44.8107209977
 38.8732325050
 48.1813221640
 43.1100037206
 37.0333337776
 40.5090874298
 37.0266950081
 44.4916205752
 47.3621071212
 42.4390552357
 43.3979955390
 44.5612760699
 45.0392301699
 38.1371284430
 45.7504761745
 32.7791485790
 43.0409223502
 36.1830954557
 35.6547772565
 40.0173280519
 31.7329414334
 48.7649628854
 39.3874245749
 34.5354962535
 30.6742213771

X1 is Semi-circle(mu=40.000000,R=10.000000),|



mu point estimated value=40.042031
 R point estimated value=9.367810
 mu value from 38.168469 to 41.915593
 R value from 7.806508 to 11.709762
 H0: X1~Semi-circle(mu=40.753985,R=8.626191),

pearson goodness of fit

class	[1]	[2]	[3]	[4]
lower limit	30.67422	33.35074	36.02726	38.70377
41.38029	44.05681	46.73332		
upper limit	33.35074	36.02726	38.70377	41.38029
44.05681	46.73332	49.40984		
observed no	7.00000	10.00000	22.00000	11.00000
23.00000	17.00000	10.00000		
probability	0.03140	0.13820	0.18060	0.19600
0.19150	0.16520	0.09710		
expected no	3.14000	13.82000	18.06000	19.60000
19.15000	16.52000	9.71000		
chi square	4.74510	1.05589	0.85956	3.77347
0.77402	0.01395	0.00866		

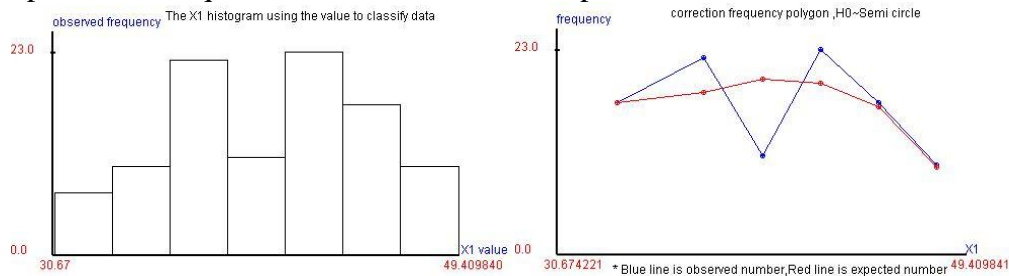
pearson chi square test statistic=11.230641
 degree of freedom=4,p-value=0.128800

correction:

expected number >=5 in each cell, the frequency table is adjusted

class	[1]	[2]	[3]	[4]
lower limit	30.67422	36.02726	38.70377	41.38029
44.05681	46.73332			
upper limit	36.02726	38.70377	41.38029	44.05681
46.73332	49.40984			
observed no	17.00000	22.00000	11.00000	23.00000
17.00000	10.00000			
probability	0.16960	0.18060	0.19600	0.19150
0.16520	0.09710			
expected no	16.96000	18.06000	19.60000	19.15000
16.52000	9.71000			
chi square	0.00009	0.85956	3.77347	0.77402
0.01395	0.00866			

degree of freedom=3
 pearson chi-square test statistic =5.429750,p-value=0.142800



4.4.18)The population distribution is logistic distribution.

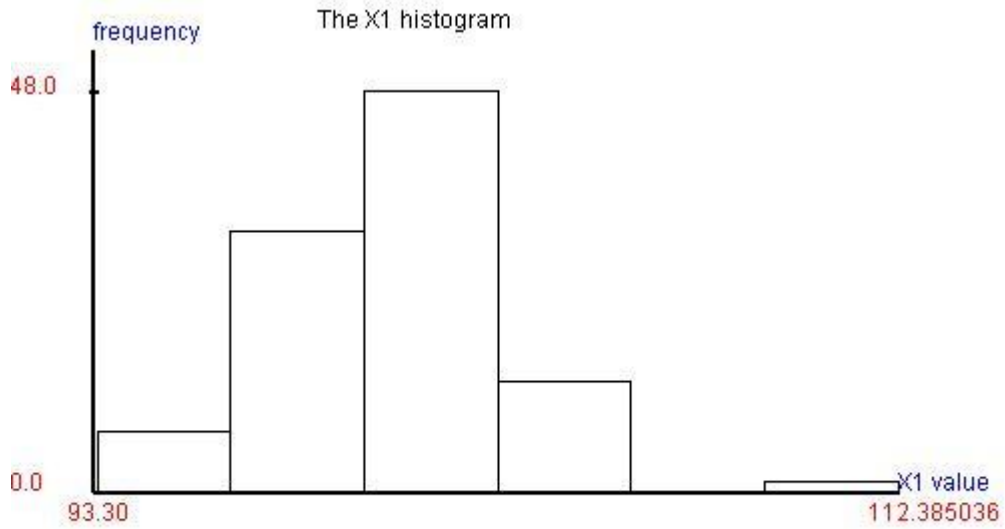
X1 is Logistic($\mu=100.000000$, $\sigma=2.000000$),

X1

99.7327958445
98.6001617432
99.9651474552
100.1821164103
97.0282702691
103.9059663799
99.7145936837
103.5829144730
98.4271747885
98.3705603160
101.2547634364
102.3400030354
100.0911045198
97.1468890780
103.7780066759
102.6818319635
94.7126673323
112.3850360648
97.4461526278
99.0651298521
100.0395903984
100.7704483016
102.4469458756
105.9049538832
98.6071645963
104.8346821731
98.9740985739
94.9591856740
104.3257360405
101.9605316633
100.4908112074
103.5124534577
98.9816444570
100.0345460606
100.9449155027
98.7299309167
104.1695341156
99.7830222897
102.1444571438
96.7511778269
102.1106252401
102.1156051748
98.9929059358
96.8510322087
102.5383390398
100.6204753788
100.7115396682
98.4792653896
104.1219582275
102.6683470652
101.2856396975
100.8178053243
100.7206664641
98.0114164990
101.9921581356
96.3650989514
102.7670365715
100.8996127209
101.1522859496
99.1035489001
101.5021613173
104.2770712535
102.9537794618
99.7655750941
101.0302942036
100.6589816623
98.6164828169
101.0165057039
99.0764278938
100.2315981434
95.7894456874

102.7333362736
 98.2690397783
 96.2968594951
 96.9378124550
 98.7220549815
 101.1632112076
 100.9197287364
 96.2852466256
 103.8180207170
 96.5020842861
 99.0018185965
 101.5012995534
 96.8862716536
 100.7733067889
 100.5871758330
 98.2224771487
 98.6589611895
 100.4275050988
 101.8046854789
 93.3018673300
 105.3783673626
 97.7590741846
 99.5675737535
 99.8218857618
 98.8680357601
 100.3266644326
 99.7139207335
 102.2209041785
 98.1932014797

X1 is Logistic(mu=100.000000,sigma=2.000000),



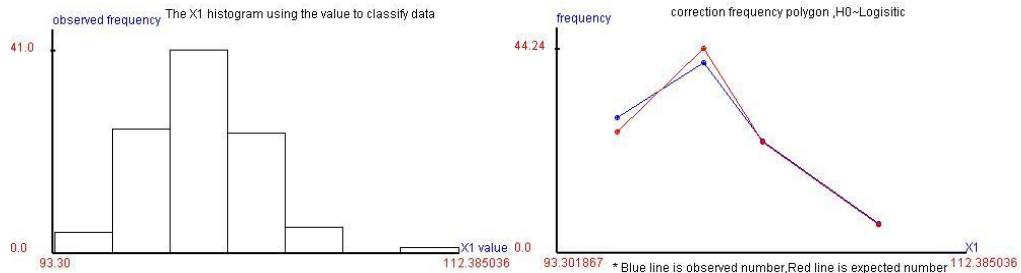
mu point estimated value=100.306832
 sigma point estimated value=1.521766
 mu value from 100.002479 to 100.611185
 sigma value from 1.170589 to 2.173951
 H0: $X1 \sim \text{Logistic}(\mu=100.245961, \sigma=1.431463)$,
 pearson goodness of fit

class	[1]	[2]	[3]	[4]
lower limit	93.30187	96.02803	98.75420	101.48037
upper limit	104.20654	106.93270	109.65887	112.38504
observed no	4.00000	25.00000	41.00000	24.00000
probability	0.04990	0.21084	0.44241	0.23771
expected no	4.98982	21.08425	44.24084	23.77061
chi square	0.19635	0.72723	0.23741	0.00221

pearson chi square test statistic=7.275244
 degree of freedom=4,p-value=0.400700
 correction:
 expected number ≥ 5 in each cell, the frequency table is adjusted

class	[1]	[2]	[3]	[4]
lower limit	93.30187	98.75420	101.48037	104.20654
upper limit	98.75420	101.48037	104.20654	112.38504
observed no	29.00000	41.00000	24.00000	6.00000
probability	0.26074	0.44241	0.23771	0.05914
expected no	26.07407	44.24084	23.77061	5.91448
chi square	0.32834	0.23741	0.00221	0.00124

degree of freedom=1
 pearson chi-square test statistic =0.569194,p-value=0.450500



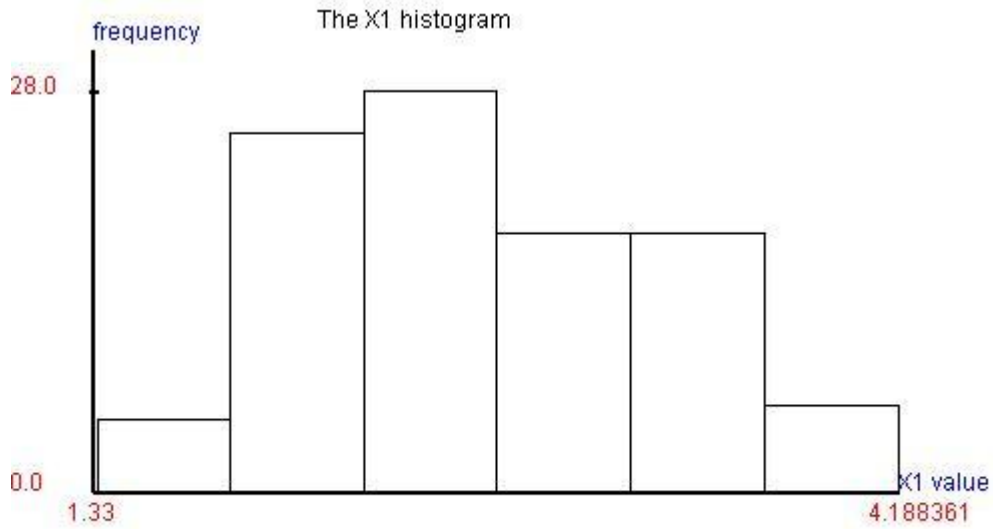
4.4.19) The population distribution is Weibull distribution.

X_1 is Weibull($\alpha=1.000000, \beta=2.000000, \gamma=3.000000$),

X_1
2.7670085275
3.3208764674
2.4390481779
4.0196111397
2.1333451586
2.6166633078
3.1498366684
2.1942700637
3.6082259141
2.4397652621
1.8837428696
2.9925379903
2.3435889761
2.0368381096
2.1610191951
2.3355273383
1.3313056983
2.7623922503
2.6531492864
3.9397547091
3.0677874159
2.1706487889
2.4802997089
3.4602664723
3.0954322591
3.0049961707
2.1488605558
4.1883618433
3.3220171945
3.5553003017
2.1178139124
2.7354082413
3.4952667329
3.2901173129
3.7424794282
3.8800850375
1.7513528081
3.1596847781
2.5650957064
2.5137223351
2.6196527921
2.9445773640
4.1695582268
2.2292088769
3.2778405327
2.3700442622
2.4787335538
2.1831299382
1.8410829342
2.4017887602
2.1965215631
1.8672818845
1.7063649374
1.8251809017
3.4569058320
2.0176488826
2.3531519299
2.6214361023
3.4323890210
3.1987669474
2.9501562212
2.6485694884
1.9655334892
2.9185806345
1.4443616499
3.3144635121
2.7451232103
1.8535671351
3.1578012053
2.5729759333
2.8640175471

2.1252709714
3.2949595176
2.5413871701
3.6485532848
2.9314128215
1.6406397233
2.8256328865
3.2036641673
2.7330197797
2.5623840286
1.9764492843
3.6167097153
2.3465370329
2.7422349060
3.6813166685
2.1971525784
1.8272924812
3.4135463666
3.4882186572
2.2063326748
2.6049457527
2.0744521670
3.5530517892
2.4655651211
2.3344112581
2.2680994924
2.2017513051
2.5147482950
2.8738777455

X1 is Weibull(alpha=1.000000,beta=2.000000,gamma=3.000000),



alpha point estimated value=1.331306 (MLE)
 beta point estimated value=1.514373
 gamma point estimated value=1.514373
 beta value from 0.528272 to 0.792407
 gamma value from 0.200000 to 10.000000
 H0: $X1 \sim \text{Weibull}(\alpha=1.331306, \beta=1.533302, \gamma=2.356000)$,
 pearson goodness of fit

class	[1]	[2]	[3]	[4]
lower limit	1.33131	1.73946	2.14761	2.55576
2.96391	3.37206	3.78021		
upper limit	1.73946	2.14761	2.55576	2.96391
3.37206	3.78021	4.18836		
observed no	4.00000	15.00000	26.00000	22.00000
15.00000	13.00000	5.00000		
probability	0.04327	0.15937	0.24228	0.24138
0.17301	0.09157	0.04912		
expected no	4.32698	15.93747	24.22757	24.13842
17.30099	9.15688	4.91168		
chi square	0.02471	0.05514	0.12967	0.18944
0.30603	1.61295	0.00159		

pearson chi square test statistic=2.319524
 degree of freedom=4,p-value=0.940000

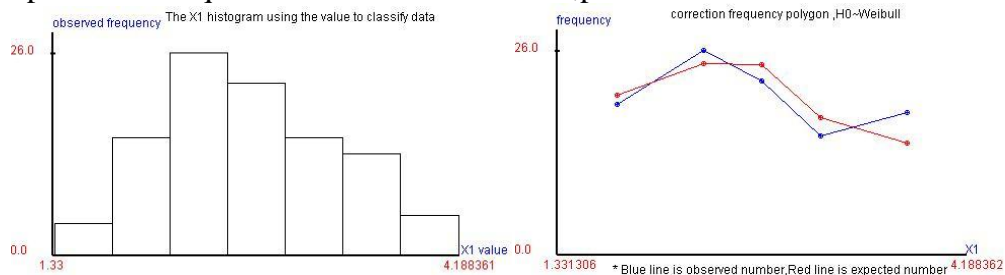
correction:

expected number ≥ 5 in each cell, the frequency table is adjusted

class	[1]	[2]	[3]	[4]
lower limit	1.33131	2.14761	2.55576	2.96391
3.37206				
upper limit	2.14761	2.55576	2.96391	3.37206
4.18836				
observed no	19.00000	26.00000	22.00000	15.00000
18.00000				
probability	0.20264	0.24228	0.24138	0.17301
0.14069				
expected no	20.26445	24.22757	24.13842	17.30099
14.06856				
chi square	0.07890	0.12967	0.18944	0.30603
1.09864				

degree of freedom=2

pearson chi-square test statistic =1.802670,p-value=0.406000



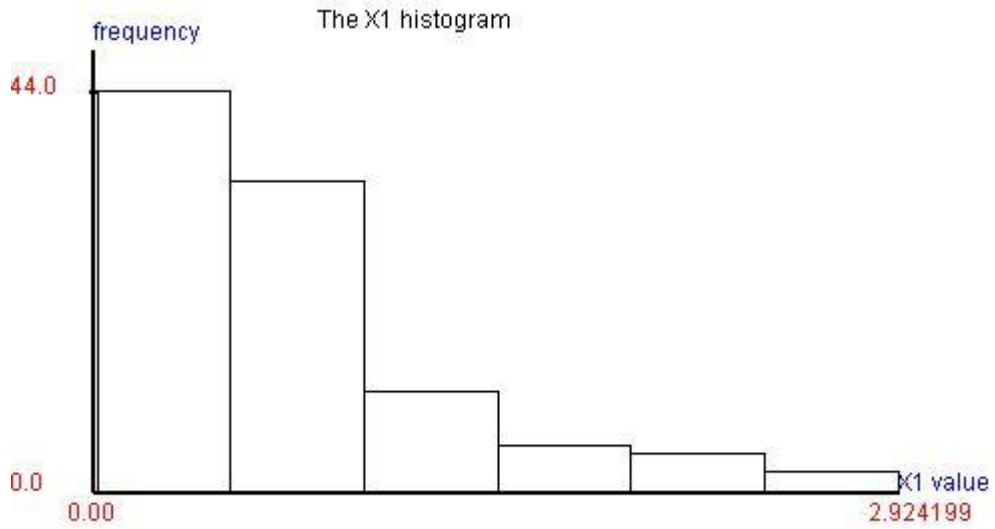
4.4.20)The population distribution is pareto 3 distribution
X1 is Pareto3(lamda=4.000000,c=4.000000),

X1

0.9319199358
0.5412229081
0.2645128662
0.4559896532
0.0543358416
0.1323903208
0.9016415400
1.7739256866
0.1506753297
0.1419182083
0.6297699609
1.8494439204
1.0401226988
0.2396905364
0.2883730803
0.8169220896
0.4566972092
1.2949345541
0.0620811375
0.0136401959
0.7337573390
0.3925939656
0.6000354378
2.2229296207
2.4344038091
1.0915688351
0.1781715749
1.0695227114
0.4605201946
0.3854725938
0.4421673324
0.8427024118
2.1247629427
0.1638796306
0.3762670346
0.7456357141
0.7907739611
0.9441194167
0.6192564112
0.4423228380
0.1625099673
0.8534323129
1.0568921349
0.0024207972
0.7069425899
1.1867825797
0.2283054316
0.9172018148
0.8364999938
0.3476131400
0.6435063481
1.1338969970
0.0139345940
0.5051679575
0.1422121492
1.2636296213
1.8003636734
0.5675941072
0.3660171749
0.1095335739
0.6628287190
0.1860931309
0.5693705129
0.5583022008
0.7740785434
1.1331035330
0.6886584349
0.5769257527
0.9884438003
0.9453577013
0.3971647816

0.0380921984
 0.5542230558
 0.8089573127
 0.9749720598
 0.7345738974
 0.8900152151
 2.9241993352
 0.3221310519
 2.7899000586
 0.0472800136
 0.9443428951
 1.3583092165
 1.6912770666
 0.4612965312
 0.1634151799
 0.6497566791
 0.2396038386
 2.1063204065
 0.1035468158
 1.8006608363
 0.1140532208
 0.2047524429
 0.4012488159
 0.3677205542
 0.5562539398
 0.1891671548
 0.0053550841
 0.2364491149
 0.4453728781

X1 is Pareto3(lamda=4.000000,c=4.000000),



lamda point estimated value=0.327790 (MLE)

c point estimated value=2.924199 (MLE)

lamda value from 2.542276 to 3.813414

c value from 2.871747 to 2.976652

H0: $X_1 \sim \text{Pareto } 3(\text{lamda}=2.550778, c=2.871747)$,

pearson goodness of fit

class	[1]	[2]	[3]	[4]	[5]	[6]	[7]
lower limit	0.00242	0.41982	0.83721	1.25461			
1.67201	2.08941	2.50680					
upper limit	0.41982	0.83721	1.25461	1.67201			
2.08941	2.50680	2.92420					
observed no	37.00000	31.00000	18.00000	3.00000			
5.00000	4.00000	2.00000					
probability	0.33178	0.25308	0.18402	0.12320			
0.07166	0.03108	0.00518					
expected no	33.17807	25.30786	18.40214	12.31991			
7.16585	3.10772	0.51845					
chi square	0.44027	1.28026	0.00879	7.05044			
0.65462	0.25619	4.23374					

pearson chi square test statistic=13.924301

degree of freedom=4, p-value=0.052500

correction:

expected number ≥ 5 in each cell, the frequency table is adjusted

class	[1]	[2]	[3]	[4]	[5]
lower limit	0.00242	0.41982	0.83721	1.25461	
1.67201					
upper limit	0.41982	0.83721	1.25461	1.67201	
2.92420					
observed no	37.00000	31.00000	18.00000	3.00000	
11.00000					
probability	0.33178	0.25308	0.18402	0.12320	
0.10792					
expected no	33.17807	25.30786	18.40214	12.31991	
10.79202					
chi square	0.44027	1.28026	0.00879	7.05044	
0.00401					

degree of freedom=2

pearson chi-square test statistic =8.783755 , p-value=0.012300

