

Chapter six The goodness of fit test

3) Pearson chi-square test 3:

3.1) The multi-nominal distribution is the sampling distribution, each category has a probability and the summation of probability is 1.

category	1	2	k
probability	p_1	p_2	p_k

$$p_1 + \dots + p_k = 1,$$

$$f(x_1, \dots, x_k) = \frac{n!}{x_1! \times \dots \times x_k!} \times p_1^{x_1} \times \dots \times p_k^{x_k}, \quad x_1 + \dots + x_k = n \quad \text{that is trial number.}$$

3.2)

$$X_i = np_i + \varepsilon_i, \quad E(\varepsilon_i) = 0, \quad E[(\varepsilon_i)^2] = np_i(1 - p_i), \quad i = 1, 2, \dots, k$$

$$X_i - np_i = \varepsilon_i, \quad E\left[\frac{(\varepsilon_i)^2}{np_i}\right] = E\left[\frac{(X_i - np_i)^2}{np_i}\right] = 1 - p_i,$$

$$\sum_{i=1}^k E\left[\frac{(X_i - np_i)^2}{np_i}\right] = E\left[\sum_{i=1}^k \left(\frac{(X_i - np_i)^2}{np_i}\right)\right] = k - \sum_{i=1}^k p_i = k - 1,$$

$$\sum_{i=1}^k \left(\frac{(X_i - np_i)^2}{np_i}\right) \rightarrow \chi_{k-1}^2$$

$$H_0 : p_1 = p_{01}, p_2 = p_{02}, \dots, p_k = p_{0k} \quad H_1 : \text{against } H_0$$

$p_{01}, p_{02}, \dots, p_{0k}$ are known value and $p_{01} + p_{02} + \dots + p_{0k} = 1,$

$$E_i = np_{i0}, \quad i = 1, 2, \dots, k, \quad O_i = x_i,$$

O_i : The observed sample number of cell i,

E_i : The expected sample number of cell i,

$$\chi_v^2 = \sum_{i=1}^k \frac{(O_i - E_i)^2}{E_i}, \quad v = k - 1 - (\text{number of point estimator}) \text{ is degree of freedom of}$$

pearsson chi square test.

$$\chi_v^2 > \chi_{\alpha, v}^2 \Rightarrow \text{reject } H_0.$$

3.3) The process of test

H_0 : Population distribution is a continuous probability distribution,

H_1 : against H_0

The sample size is n.

The process: i) The class number of frequency distribution: $k = \log_2(n) + 1.$

ii) The class limit of frequency distribution:

The class limit will be found by the a general frequency distribution table.

iii) The class sample number of frequency distribution (O_i):

The frequency distribution is done and getting the O_i of each class.

iv) The class expected number of frequency distribution (E_i):

$$E_i = n \times \text{the probability of each class.}$$

Note: There are 20 kinds of continuous probability distribution that is can be assigned to null hypothesis.

3.4)Example (The simulated sample data and computing the result by the P_S_CCC)

The Pearson chi square test (goodness of fit) ,the frequency distribution method,
please select the population distribution

1.H0:Uniform distribution	13.H0:Gumbel distribution
2.H0:Normal distribution	14.H0:Triangular 1 distribution
3.H0:Shifted exponential distribution	15.H0:Trapezoid distribution
4.H0:Pareto 1 distribution	16.H0:U-quadractic distribution
5.H0:Pareto 2 distribution	17.H0:Semi-circle distribution
6.H0:Rayleigh distribution	18.H0:Logistic distribution
7.H0:Double expoenoential distribution	19.H0:Weibull distribution
8.H0:Log normal distribution	20.H0:Pareto 3 distribution
9.H0:Gamma distribution	** Above H0 population all do once
10.H0:Beta distribution	
11.H0:Cauchy distribution	
12.H0:Arcsin distribution	

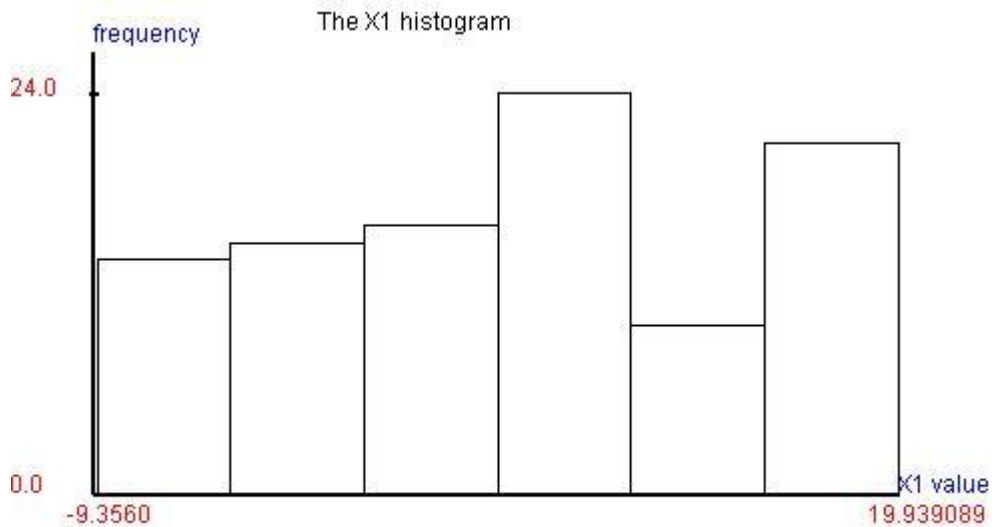
3.4.1)The population distribution is uniform distribution.

X1 is Uniform(alpha=-10.000000,beta=20.000000),

X1
2.6129488150
-4.3573278116
-8.4087583055
2.8653957319
-7.7642599331
-4.5312691499
18.9451919456
-7.5661366219
-0.3928282174
-7.7173966606
-1.7694033374
15.7138338682
17.6132863932
14.6519894977
18.1990567797
0.0723027235
6.4417814978
5.3358818510
18.4903584532
6.9010383508
16.2129599599
8.9492367735
6.6430406713
1.4078921401
16.2368634101
0.8424747527
1.8651070767
9.4282500808
14.9490745930
-2.4593595577
-2.8537909622
19.0524731876
19.5959860518
-5.9761202624
6.2816394474
19.8999677004
-0.2752320735
8.1830776447
7.3500937031
17.8302383753
3.5515783537
17.7552030031
-0.7725596216
-2.0683631449
-8.7468208031
10.0520623452
12.6857828433
3.5699193061
14.2637394765
19.7925011086
-7.3591733797
4.3263373039
4.6247362565
15.3526709359
-6.8720361832
-9.3560004326
13.4596648080
18.7508099397
1.5380441459
-2.0234787456
12.6287912995
19.6674210873
13.7060617909
18.2622762497
8.6319983101
8.8291980545
6.3974613450
11.5374960990
2.8222809150
-8.9451628818
8.7578249202

5.4977971037
 6.9860405822
 3.2883375495
 9.2926174045
 15.8305358730
 8.3563976884
 -2.0893954764
 19.9390894866
 15.0927821818
 3.1303188624
 6.6778463915
 3.7777887385
 13.0186530614
 -7.4625320639
 -4.4425624442
 17.9814852014
 -4.0158969644
 0.5288308828
 5.8115704369
 8.5169174390
 -1.4852544873
 7.1549964871
 -8.3942241233
 -1.7982573612
 -5.3994930969
 6.6884102731
 9.4917211114
 14.6668887763
 1.6477550134

X1 is Uniform(alpha=-10.000000,beta=20.000000),
 H0: $X1 \sim \text{Uniform}(\alpha, \beta)$, alpha,beta are unknown
 alpha point estimated value=-9.356000 (MLE)
 beta point estimated value=19.939089 (MLE)



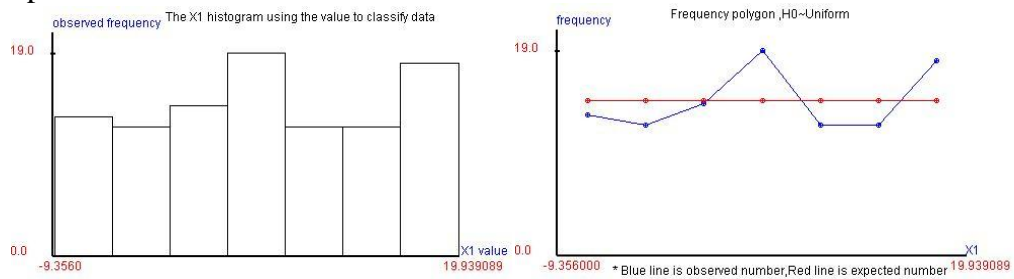
pearson goodness of fit

class	[1]	[2]	[3]	[4]
[5]	[6]	[7]		
lower limit	-9.35600	-5.17099	-0.98597	3.19904
7.38405	11.56906	15.75408		
upper limit	-5.17099	-0.98597	3.19904	7.38405
11.56906	15.75408	19.93909		
observed no	13.00000	12.00000	14.00000	19.00000
12.00000	12.00000	18.00000		
probability	0.14286	0.14286	0.14286	0.14286
0.14286	0.14286	0.14286		
expected no	14.28571	14.28571	14.28571	14.28571
14.28571	14.28571	14.28571		
chi square	0.11571	0.36571	0.00571	1.55571
0.36571	0.36571	0.96571		

degree of freedom=4

pearson chi-square test statistic =3.740000

p-value=0.442300

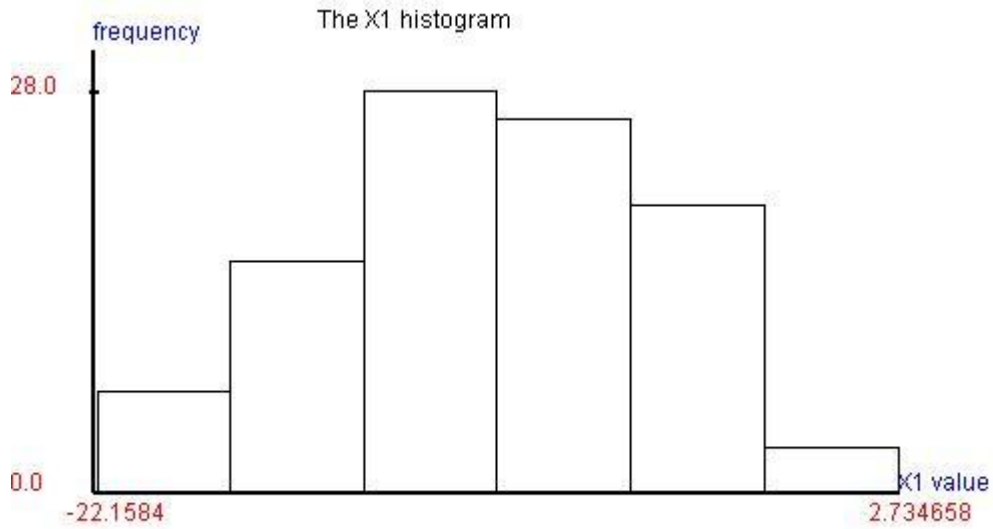


3.4.2)The population distribution is normal distribution.
X1 is Normal($\mu=-10.000000$, $\sigma^2=25.000000$),

X1
-11.4054253373
-18.0494803561
-5.3523763380
-15.1298195759
-3.8860933277
-5.3406086141
-10.7444396560
2.6753089930
-2.7569259443
-12.0904267901
-10.0408987610
-14.5713183548
-3.4715721150
-19.2283462628
-12.3824538403
2.7346588541
-3.8885071336
-16.4724293394
-9.9541837444
-4.6726093279
-14.6861974992
-10.0386586897
-4.6591054983
-6.5078797070
-9.4445858718
-2.5311302500
-12.3841854762
-8.1812897842
-4.2017581541
-10.9208881313
-8.5527375653
-2.6835308764
-15.5052755327
-10.1908700666
-14.5873604519
-7.0266258732
-6.4899987533
-3.9529644671
-10.4505367217
-17.8939902575
-13.3427424835
-12.2396153191
-21.1673800524
-7.7742940421
-10.3987252217
-3.8077848379
-15.1344285415
-13.0213596903
-11.9447692573
-7.6679577393
-7.1750052584
-20.4236422921
-15.0304137516
-15.6166213124
-7.8690111685
-14.8986169865
-5.3924941008
-6.2991518062
-17.5700100275
-5.5404757223
-8.2678590580
-14.9511253972
-7.1841497180
-8.6609850455
-13.6369301712
-8.4994751831
-7.5040746678
-16.5074361856
-20.1757499940
-5.6121615413
-7.5876754903

-7.6402646217
 -12.6699192823
 -13.5170981597
 -8.4809969331
 -16.4281443478
 -0.5632378764
 -7.8255768339
 -12.9054820035
 -19.5608931237
 -15.1717574534
 -5.9311277512
 -2.1861430018
 -11.9679890478
 -7.3555469686
 -11.2945512711
 -2.4134795929
 -5.0841356352
 -9.2014817150
 -12.1986441107
 -22.1584588319
 -12.3497253726
 -12.4415279543
 -13.6106707721
 -1.4536994810
 -6.5704267949
 -6.6334507786
 -10.4694782210
 -11.7481183006
 -2.8494306617

X1 is Normal(mu=-10.000000,sigma*sigma=25.000000),



H0: $X_1 \sim \text{Normal}(\mu, \sigma^2)$, μ, σ are unknown
 population mean(μ) point estimated value=-9.885011 (MLE,UMVUE)
 population variance(σ^2) which point estimated value=27.945242 (UMVUE)
 pearson goodness of fit

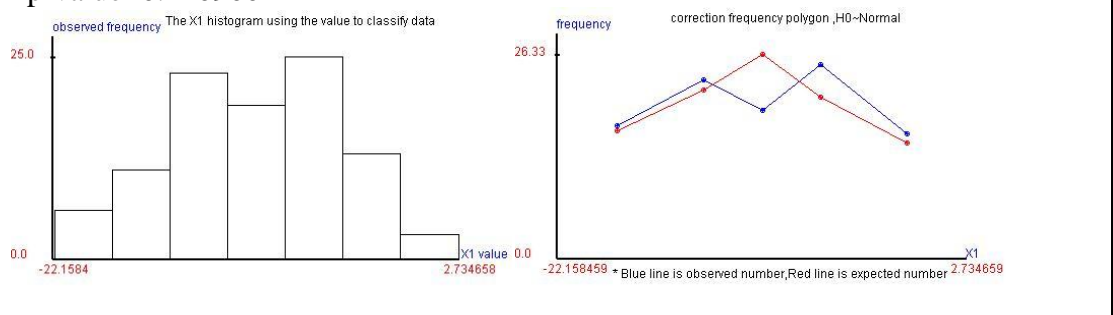
class	[1]	[2]	[3]	[4]	[5]
lower limit	-22.15846	-18.60230	-15.04614	-11.48998	-7.93382
upper limit	-4.37766	-0.82150			
observed no	6.00000	11.00000	23.00000	19.00000	25.00000
probability	0.04960	0.11490	0.21620	0.26330	0.20730
expected no	4.96000	11.49000	21.62000	26.33000	20.73000
chi square	0.21806	0.02090	0.08809	2.04060	0.87954

pearson chi square test statistic=4.219475
 degree of freedom=4
 p-value=0.754200

correction:
 expected number ≥ 5 in each cell, the frequency table is adjusted

class	[1]	[2]	[3]	[4]	[5]
lower limit	-22.15846	-15.04614	-11.48998	-7.93382	-4.37766
upper limit	-15.04614	-11.48998	-7.93382	-4.37766	2.73466
observed no	17.00000	23.00000	19.00000	25.00000	16.00000
probability	0.16450	0.21620	0.26330	0.20730	0.14870
expected no	16.45000	21.62000	26.33000	20.73000	14.87000
chi square	0.01839	0.08809	2.04060	0.87954	0.08587

degree of freedom=2
 pearson chi-square test statistic =3.112483
 p-value=0.210900



3.4.3)The population distribution is shifted exponential distribution.

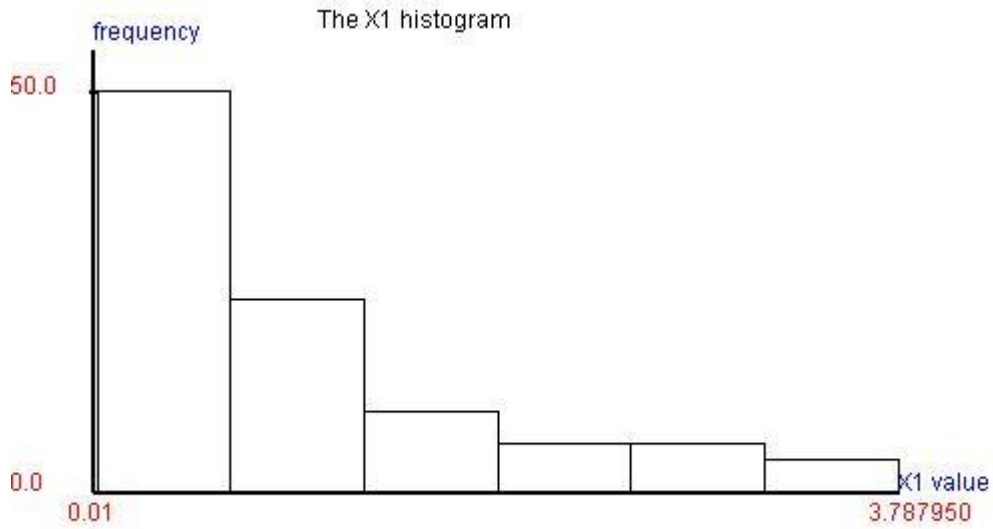
X1 is Exponential($\lambda=1.000000, c=0.000000$),

X1

1.1121841893
3.6042344572
0.4731735030
0.6269050446
0.7756278590
0.9390087019
0.3317167312
0.5145958527
0.9435907728
3.1621586175
0.0910805723
0.3814508513
2.8042415606
0.2109996401
0.6002265911
0.2299897909
3.6328240768
1.4656836933
0.5160048052
0.1333250435
0.6398262406
1.2347647812
1.1844366706
0.5011287665
0.3573739480
1.2535874319
0.0302016713
1.1040269186
0.0980168048
0.2115388386
0.1926392291
2.1859132638
1.6232874084
0.1400788860
2.7394863605
0.1062138235
0.1202627492
2.0966722048
0.0106264481
0.1810119603
1.3323534838
1.4218472540
1.2982793724
0.3892268615
0.8590740705
0.9219237111
1.0275659668
1.0315062386
1.4989785486
0.1266418782
0.0256054257
0.4419521452
1.0059283688
0.1822624109
0.2395665332
0.0484270823
0.5025209045
2.1719879557
1.8305412423
1.2983294837
2.5336146332
1.2473512407
2.8584818673
1.5531748611
0.2638801002
0.3063653512
2.8672219106
1.7523641328
2.6149451614
0.1051173560
0.6959422325

0.4138592512
0.6406882647
0.4474773874
0.2842368246
0.3551916550
0.4725552726
0.3268215764
0.4878471950
0.9321150161
0.2536116505
0.8307476709
0.1958567133
0.5276622317
0.9189597415
1.1014669933
0.3486963894
0.3690910330
2.1650024183
2.4787697207
0.0497771014
0.5512874341
2.4519746521
0.7460303838
0.6940428262
0.4303639044
0.7488639222
0.6685827342
0.5750543715
3.7879507691

X1 is Exponential(lamda=1.000000,c=0.000000),



H0: $X_1 \sim \text{Shifted exponential}(\lambda, c)$, λ, c are unknown

λ point estimated value = 1.039447 (MLE)

c point estimated value = 0.010626 (MLE)

pearson goodness of fit

class	[1]	[2]	[3]	[4]
lower limit	0.01063	0.55024	1.08986	1.62948
2.16910	2.70872	3.24833		
upper limit	0.55024	1.08986	1.62948	2.16910
2.70872	3.24833	3.78795		
observed no	45.00000	22.00000	15.00000	4.00000
6.00000	5.00000	3.00000		
probability	0.40497	0.24097	0.14338	0.08532
0.05077	0.03021	0.04439		
expected no	40.49676	24.09688	14.33843	8.53183
5.07671	3.02081	4.43858		
chi square	0.50076	0.18247	0.03052	2.40716
0.16792	1.29674	0.46625		

pearson chi square test statistic = 5.051818

degree of freedom = 4

p-value = 0.653600

correction:

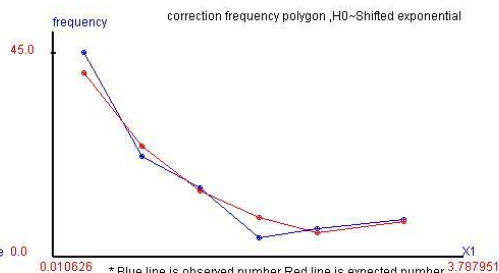
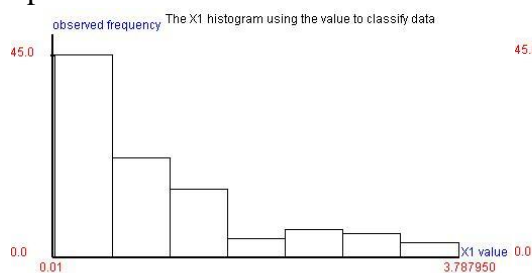
expected number ≥ 5 in each cell, the frequency table is adjusted

class	[1]	[2]	[3]	[4]
lower limit	0.01063	0.55024	1.08986	1.62948
2.16910	2.70872			
upper limit	0.55024	1.08986	1.62948	2.16910
2.70872	3.78795			
observed no	45.00000	22.00000	15.00000	4.00000
6.00000	8.00000			
probability	0.40497	0.24097	0.14338	0.08532
0.05077	0.07459			
expected no	40.49676	24.09688	14.33843	8.53183
5.07671	7.45938			
chi square	0.50076	0.18247	0.03052	2.40716
0.16792	0.03918			

degree of freedom = 3

pearson chi-square test statistic = 3.328008

p-value = 0.343700



3.4.4)The population distribution is pareto1 distribution.

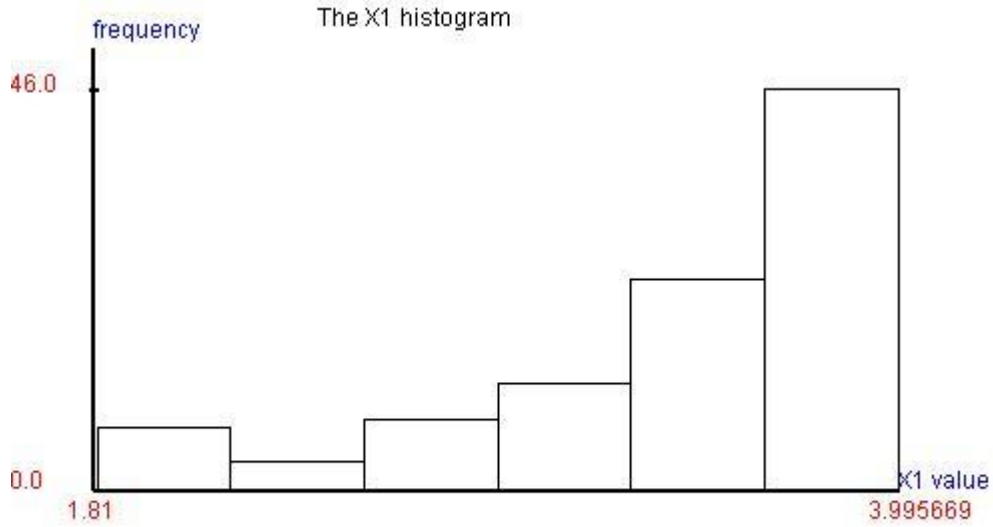
X1 is Pareto1(lamda=5.000000,c=4.000000),

X1

1.8570247233
3.8134290799
3.5719866873
3.0949691616
2.9122095370
3.7244749813
3.9462498637
3.8881559083
2.7198506352
3.9546682334
3.3301212896
3.8398385385
1.8140997441
3.5812615158
3.7623519399
3.6932638996
2.9108572406
3.7461973982
3.6573051717
3.5466909550
3.2935813400
3.4284061411
3.6557855684
3.1676659082
3.0161365301
3.2259845799
3.6472637225
3.3228124031
3.8921167453
3.4582252330
2.1396331258
3.0189202731
3.7024205556
3.2512834375
3.6345733285
3.2978878207
1.8637280644
2.7103958726
3.8465421299
2.4886035654
3.3632381992
3.7830313268
3.8967067622
3.7823667022
3.3399364646
3.4215476695
3.3769064427
3.7956225577
3.9956699904
2.5927288362
3.8496780510
2.6425619635
3.8097289147
2.4072445990
2.0065348698
3.6139628079
3.6976074307
3.7172139476
2.7762185579
3.5801653939
2.9601775226
3.3569599857
3.6556744768
3.4265212519
3.6151787833
3.5868553992
3.9844857011
3.8147589485
2.9513800338
3.7915680771
2.9426722786

3.9016992612
 3.9291418782
 3.8339450676
 3.9229820435
 2.8545910192
 3.8523796713
 3.3797369792
 3.8468513635
 2.8817942826
 3.8850563470
 3.4643625744
 2.9380879176
 2.0145883840
 3.8138981460
 3.8741932974
 3.7551289943
 3.6556392928
 3.9604953009
 3.9329456427
 3.9579134494
 3.5853375123
 3.6584789310
 3.7556920867
 3.5400270669
 3.4784596341
 2.1451548101
 3.7559998828
 2.2799837122
 2.8225256670

X1 is Pareto1(lamda=5.000000,c=4.000000),



H0: $X_1 \sim \text{Pareto}(1(\lambda, c))$, λ, c are unknown

λ point estimated value = 5.410135 (MLE)

c point estimated value = 3.995670 (MLE)

pearson goodness of fit

class	[1]	[2]	[3]	[4]
lower limit	1.81410	2.12575	2.43741	2.74906
3.06071	3.37236	3.68402		
upper limit	2.12575	2.43741	2.74906	3.06071
3.37236	3.68402	3.99567		
observed no	5.00000	4.00000	5.00000	12.00000
11.00000	24.00000	39.00000		
probability	0.03290	0.03607	0.06327	0.10418
0.16308	0.24496	0.35554		
expected no	3.29008	3.60676	6.32718	10.41794
16.30794	24.49603	35.55407		
chi square	0.88867	0.04287	0.27839	0.24025
1.72764	0.01004	0.33398		

pearson chi square test statistic = 3.521846

degree of freedom = 4

p-value = 0.832900

correction:

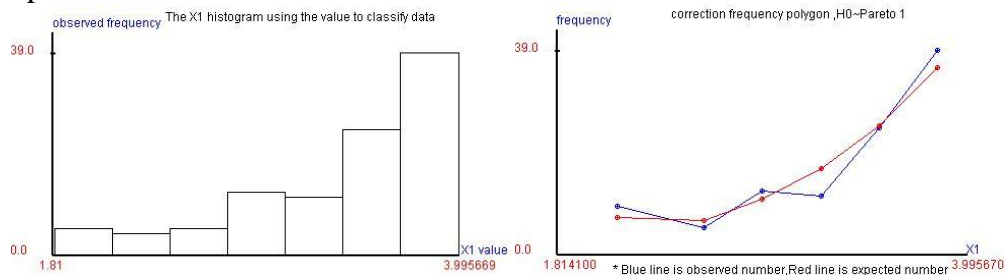
expected number ≥ 5 in each cell, the frequency table is adjusted

class	[1]	[2]	[3]	[4]
lower limit	1.81410	2.43741	2.74906	3.06071
3.37236	3.68402			
upper limit	2.43741	2.74906	3.06071	3.37236
3.68402	3.99567			
observed no	9.00000	5.00000	12.00000	11.00000
24.00000	39.00000			
probability	0.06897	0.06327	0.10418	0.16308
0.24496	0.35554			
expected no	6.89685	6.32718	10.41794	16.30794
24.49603	35.55407			
chi square	0.64134	0.27839	0.24025	1.72764
0.01004	0.33398			

degree of freedom = 3

pearson chi-square test statistic = 3.231643

p-value = 0.357200



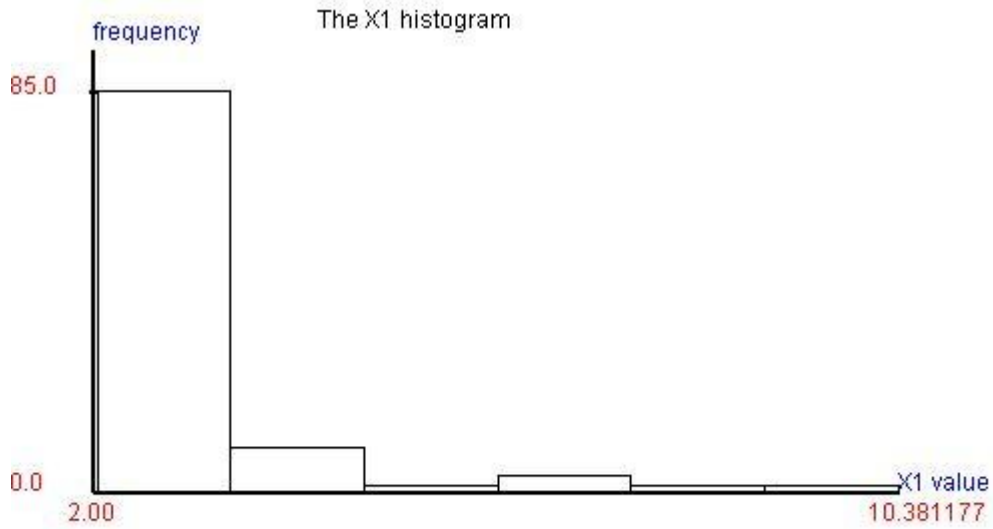
3.4.5)The population distribution is pareto2 distribution.

X1 is Pareto2(lamda=4.000000,c=2.000000),

X1
2.0707286859
2.4936569908
6.8681223013
2.2579292849
3.4440419963
2.6531522352
2.4999369821
3.4556834797
2.4151119252
2.2258798557
2.1762328778
2.6393600827
2.7003290587
2.2482490653
2.1566622805
2.1411424885
2.0213280717
2.2287328674
3.2290505591
2.1962238642
3.2986063632
2.4756226492
2.2794415310
2.2546250409
2.0918669137
2.1670960796
2.3605649211
2.5798525193
2.0132519158
2.1479316779
2.3251327502
2.1815988062
2.2753882195
3.2057749306
2.3228266835
2.1488527345
2.1627717656
2.2337536139
2.7732802092
2.0928553965
2.0788704986
2.7804940544
2.2426936919
3.4814920644
8.5905881193
2.7668940243
2.2349412884
2.1640817983
2.4112696826
2.2607653028
2.0411965877
2.2113993208
3.0238879591
3.8013817521
2.0666422004
3.1565201554
2.2070094832
4.3093899657
2.4087141374
2.1375176203
2.2444497647
2.0695589651
2.2205015671
3.9856368173
3.0635702729
2.0664453726
2.5524653504
2.0176831157
2.1779942150
2.0060487144
2.8213612560

2.2176120296
 2.6271065208
 2.0072271973
 2.0765554011
 3.0552885588
 7.4425434485
 2.5352858846
 2.2350103611
 2.2752603793
 2.9131478473
 2.0265796527
 2.4624121794
 2.1033748989
 2.5275798067
 2.1194263121
 2.2334027854
 4.1852986338
 2.1183207579
 3.1345435885
 6.2030222596
 2.0728902192
 3.0618877467
 2.1064150824
 4.4748687722
 2.0676091331
 10.3811771712
 2.0079662081
 5.1691232829
 4.0514628907

X1 is Pareto2(lamda=4.000000,c=2.000000),



H0: $X_1 \sim \text{Pareto } 2(\lambda, c)$, λ, c are unknown

λ point estimated value = 3.713141 (MLE)

c point estimated value = 2.006049 (MLE)

pearson goodness of fit

class	[1]	[2]	[3]	[4]
lower limit	2.00605	3.20250	4.39894	5.59539
6.79184	7.98828	9.18473		
upper limit	3.20250	4.39894	5.59539	6.79184
7.98828	9.18473	10.38118		
observed no	82.00000	11.00000	2.00000	1.00000
2.00000	1.00000	1.00000		
probability	0.82393	0.12190	0.03200	0.01138
0.00489	0.00239	0.00352		
expected no	82.39297	12.18960	3.20007	1.13755
0.48867	0.23908	0.35207		
chi square	0.00187	0.11609	0.45004	0.01663
4.67422	2.42182	1.19238		

pearson chi square test statistic = 8.873069

degree of freedom = 4

p-value = 0.261800

correction:

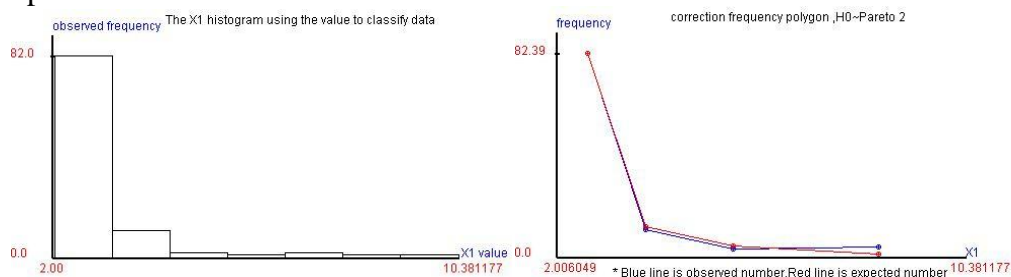
expected number ≥ 5 in each cell, the frequency table is adjusted

class	[1]	[2]	[3]	[4]
lower limit	2.00605	3.20250	4.39894	6.79184
upper limit	3.20250	4.39894	6.79184	10.38118
observed no	82.00000	11.00000	3.00000	4.00000
probability	0.82393	0.12190	0.04338	0.01080
expected no	82.39297	12.18960	4.33761	1.07982
chi square	0.00187	0.11609	0.41249	7.89713

degree of freedom = 1

pearson chi-square test statistic = 8.427591

p-value = 0.003600



3.4.6)The population distribution is rayleigh distribution.

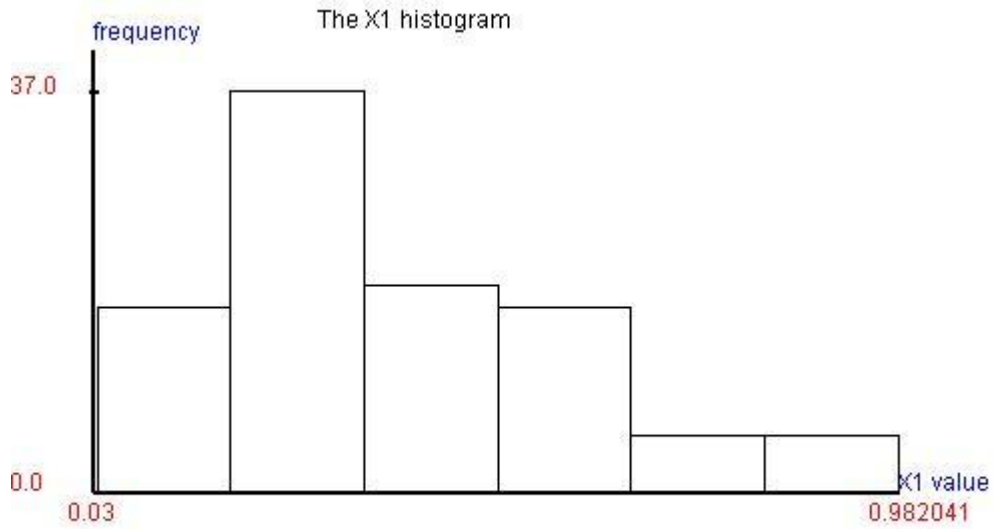
X1 is Rayleigh($\lambda=5.000000, c=0.000000$),

X1

0.9321651590
0.1906360066
0.3004833223
0.3015489863
0.1567503626
0.4341579031
0.1538931923
0.3223854958
0.1720725808
0.1169726346
0.2157794276
0.2454373082
0.1926820730
0.2107832370
0.2094343223
0.5978649690
0.2492008848
0.6502921312
0.2176519263
0.0991843744
0.3389682639
0.2591919228
0.7039136131
0.3180498178
0.3854762709
0.6595307874
0.2872767688
0.2952586628
0.8495978805
0.2654299498
0.4059360597
0.4689821573
0.3527401476
0.0375910448
0.2684377714
0.3899342661
0.3318590173
0.9447965587
0.2990672506
0.4465692296
0.3282454443
0.1958065436
0.7510939299
0.1006799965
0.3205426480
0.5815591813
0.6185301084
0.1225392897
0.6404981362
0.4978813172
0.4200583077
0.6166876052
0.6302029395
0.2726165548
0.2456872041
0.7596793240
0.6098838569
0.4911102180
0.3442461999
0.5189362690
0.3606451292
0.3486803936
0.6383798519
0.5224819868
0.2431715457
0.3035002157
0.8768094059
0.5290780402
0.0479283395
0.1115500106
0.3195385917

0.6667939515
0.2705339559
0.3477804794
0.2449356847
0.3911248078
0.4859980695
0.1703867690
0.3100169171
0.1440305100
0.6160635434
0.4172987929
0.3015296367
0.4975040100
0.4251632535
0.2489125294
0.3892016971
0.5249918439
0.5425511887
0.4531697595
0.5239573336
0.2468277424
0.0351947874
0.9820414896
0.1634892766
0.2496192484
0.2964448519
0.1894667833
0.4283888351
0.6918879423

X1 is Rayleigh(lamda=5.000000,c=0.000000),



H0: $X1 \sim \text{Rayleigh}(\lambda, c)$, λ, c are unknown

λ point estimated value = 5.902225 (MLE)

c point estimated value = 0.035195 (MLE)

pearson goodness of fit

class	[1]	[2]	[3]	[4]
lower limit	0.03519	0.17046	0.30572	0.44099
0.57625	0.71151	0.84678		
upper limit	0.17046	0.30572	0.44099	0.57625
0.71151	0.84678	0.98204		
observed no	13.00000	30.00000	23.00000	13.00000
14.00000	2.00000	5.00000		
probability	0.10236	0.24840	0.27087	0.20069
0.11045	0.04673	0.02049		
expected no	10.23624	24.83993	27.08748	20.06926
11.04468	4.67297	2.04945		
chi square	0.74621	1.07192	0.61680	2.49010
0.79078	1.52896	4.24785		

pearson chi square test statistic = 11.492609

degree of freedom = 4

p-value = 0.118500

correction:

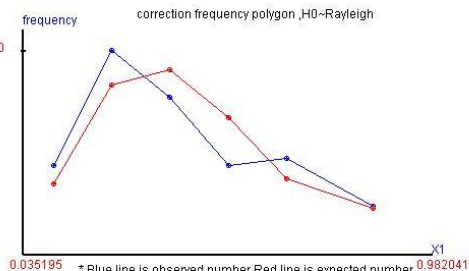
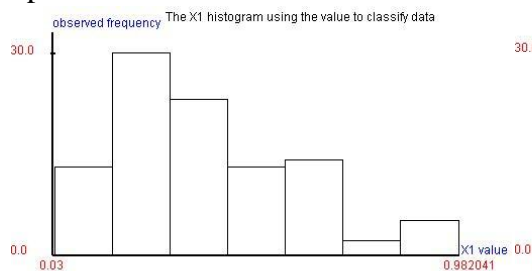
expected number ≥ 5 in each cell, the frequency table is adjusted

class	[1]	[2]	[3]	[4]
lower limit	0.03519	0.17046	0.30572	0.44099
0.57625	0.71151			
upper limit	0.17046	0.30572	0.44099	0.57625
0.71151	0.98204			
observed no	13.00000	30.00000	23.00000	13.00000
14.00000	7.00000			
probability	0.10236	0.24840	0.27087	0.20069
0.11045	0.06722			
expected no	10.23624	24.83993	27.08748	20.06926
11.04468	6.72242			
chi square	0.74621	1.07192	0.61680	2.49010
0.79078	0.01146			

degree of freedom = 3

pearson chi-square test statistic = 5.727261

p-value = 0.125600



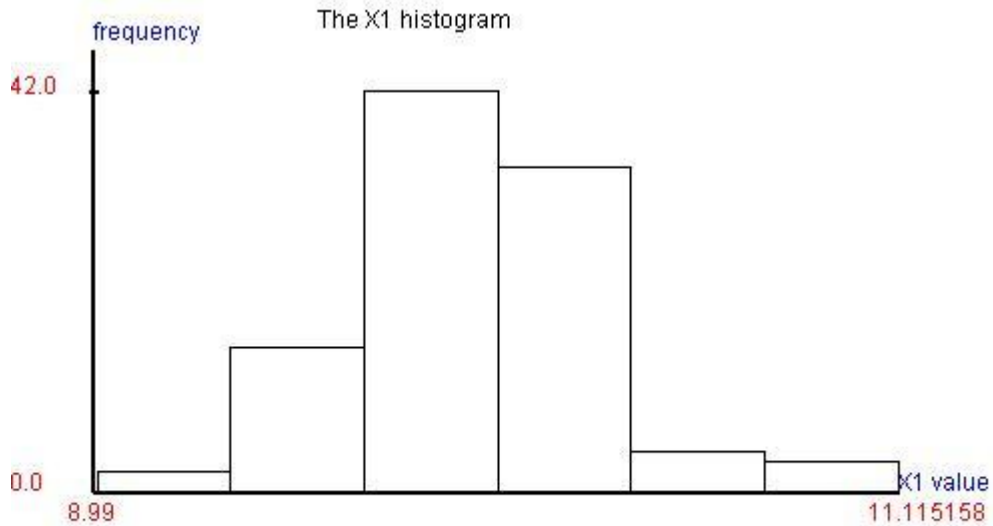
3.4.7)The population distribution is double exponential distribution.

X1 is Double Exponential($\lambda=4.000000,\mu=10.000000$),

X1
10.1811022583
10.2502881046
9.7537220781
9.6976647938
9.8506130934
10.5331708281
9.3882683505
10.0227193776
10.0377078554
9.6249977021
10.0966131146
10.0872799466
10.3203302202
10.5046445901
9.9302712939
9.8433133228
9.5622446339
10.0941011480
9.4769444711
10.1540298002
9.8663174575
9.4559981683
9.7279441130
10.2400725868
9.9894027778
8.9977837185
10.3778921878
10.0544621464
9.9023562751
10.3181752093
10.2178503741
9.6245754016
10.0392860947
10.1094682757
9.6856798086
10.1460562143
9.7080418823
10.2719251095
9.7715571451
10.0953482762
9.7486828283
10.1609394755
9.9886741941
9.8560414398
9.9237130200
9.7620671920
10.1246134738
10.0804800058
10.2588701262
10.7018275512
9.6743927591
10.3465813579
10.1148941716
10.3161680404
10.0846399716
9.9509838056
11.1151589721
10.1496756441
9.8272838048
9.9331901912
9.9817878886
9.9858840778
9.9734589924
9.5859545171
10.0225451938
9.9949260709
10.4304984012
10.4029118235
9.1859433706
10.3769375120
9.9495656045

10.1043337466
 9.8093399455
 10.7951664402
 9.9994982295
 10.2601893562
 10.4016457971
 9.7728245255
 9.4878932789
 9.7941250415
 9.8323358705
 9.5298556979
 10.2606951074
 10.0253586056
 10.3365851641
 9.9844191519
 10.7679078755
 9.5047556656
 10.2609692920
 9.4683929932
 10.0201909813
 10.0501002516
 9.3822503080
 10.2938576051
 9.9765349010
 9.8335130166
 10.1535751374
 9.9220795596
 10.0387651874
 9.8027039712

X1 is Double Exponential(lamda=4.000000,mu=10.000000),



H0: $X_1 \sim \text{Double exponential}(\lambda, \mu)$, λ, μ are unknown

λ point estimated value = 3.948225 (MLE)

μ point estimated value = 10.009845 (MLE)

pearson goodness of fit

class	[1]	[2]	[3]	[4]
lower limit	8.99778	9.30027	9.60275	9.90523
10.20771	10.51019	10.81268		
upper limit	9.30027	9.60275	9.90523	10.20771
10.51019	10.81268	11.11516		
observed no	2.00000	10.00000	23.00000	40.00000
20.00000	4.00000	1.00000		
probability	0.03036	0.06986	0.23060	0.44026
0.15958	0.04834	0.02101		
expected no	3.03571	6.98560	23.06044	44.02604
15.95757	4.83396	2.10068		
chi square	0.35336	1.30076	0.00016	0.36817
1.02405	0.14387	0.57672		

pearson chi square test statistic = 3.767086

degree of freedom = 4

p-value = 0.806200

correction:

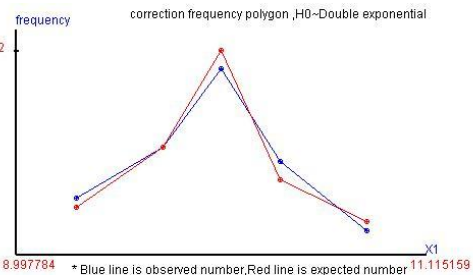
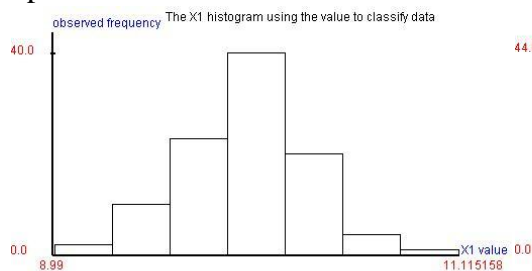
expected number ≥ 5 in each cell, the frequency table is adjusted

class	[1]	[2]	[3]	[4]
lower limit	8.99778	9.60275	9.90523	10.20771
10.51019				
upper limit	9.60275	9.90523	10.20771	10.51019
11.11516				
observed no	12.00000	23.00000	40.00000	20.00000
5.00000				
probability	0.10021	0.23060	0.44026	0.15958
0.06935				
expected no	10.02132	23.06044	44.02604	15.95757
6.93464				
chi square	0.39069	0.00016	0.36817	1.02405
0.53973				

degree of freedom = 2

pearson chi-square test statistic = 2.322788

p-value = 0.313000



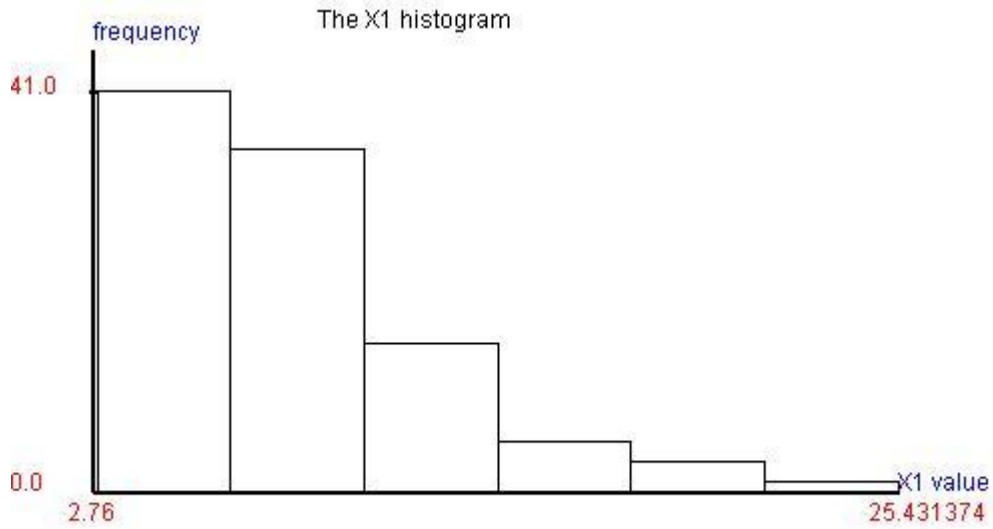
3.4.8)The population distribution is lognormal distribution.

X1 is Log normal($\mu=2.000000$, $\sigma=0.500000$),

X1
9.1118664398
9.4309948865
8.4679209633
6.4010951314
8.9785276999
3.5509063396
12.5963111888
10.4892030282
8.3095039903
6.1168006504
7.7819592822
2.8727597089
4.9235934588
9.6141773745
8.6913545379
12.6008240507
9.5750768831
4.6454223950
8.4301162632
3.9114887759
5.9548153714
5.3446214116
3.8698833092
13.0447454362
4.8095449906
5.7316614520
3.2707750255
13.2613735841
7.9627494404
9.6950017956
16.1193806645
18.0667519683
2.7658013769
8.3330486334
13.2820931780
4.6958787369
12.7852787873
3.5496548171
7.4714312160
3.6484480839
14.6961035271
5.5717114659
6.4938278177
6.7623967168
4.3110894497
11.8891507872
7.2872646864
8.2863340155
8.6231093596
9.5678036251
6.0535579306
5.7652991897
8.3104171676
3.5436071904
5.9204222733
6.5106957792
5.3698278695
11.5526825035
7.6038123746
4.2582620607
11.0051616963
17.1054721604
12.5683241476
7.7016898054
7.3227519293
5.2959880514
12.8728324904
16.6027906480
8.4629853846
4.1979100324
10.5568085341

6.6369781471
 8.8329215510
 3.9666386350
 4.9246348625
 5.8975359826
 8.0321899651
 7.6271886066
 6.8997828375
 11.8150617824
 4.9193738412
 6.3322440334
 16.5834927202
 5.7532207370
 8.1299162815
 7.5702933209
 10.7258003075
 5.2767438979
 18.2393109636
 18.1464801095
 4.2604391312
 6.7560078406
 7.7587930200
 8.4024502715
 25.4313746344
 3.2501649429
 9.0390722939
 5.0874131483
 5.3124204883
 3.3360162039

X1 is Log normal($\mu=2.000000$, $\sigma=0.500000$),



H0: $X1 \sim \text{Log_Normal}(\mu, \sigma^2)$, μ, σ are unknown
 population mean(μ) point estimated value=2.000858 (MLE,UMVUE)
 population variance(σ^2) which point estimated value=0.223785 (UMVUE)

pearson goodness of fit

class	[1]	[2]	[3]	[4]	[5]	[6]	[7]
lower limit	2.76580	6.00374	9.24168	12.47962	15.71756	18.95550	22.19344
upper limit	6.00374	9.24168	12.47962	15.71756	18.95550	22.19344	25.43137
observed no	35.00000	36.00000	12.00000	9.00000	7.00000	0.00000	1.00000
probability	0.32980	0.35150	0.18440	0.07890	0.03210	0.01330	0.01000
expected no	32.98000	35.15000	18.44000	7.89000	3.21000	1.33000	1.00000
chi square	0.12372	0.02055	2.24911	0.15616	4.47480	1.33000	0.00000

pearson chi square test statistic=8.354346
 degree of freedom=4, p-value=0.302300

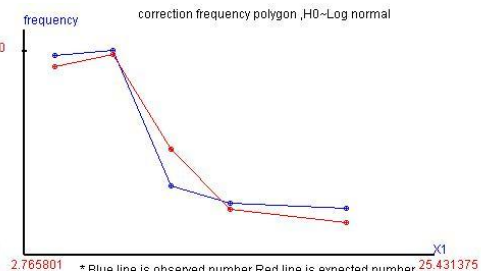
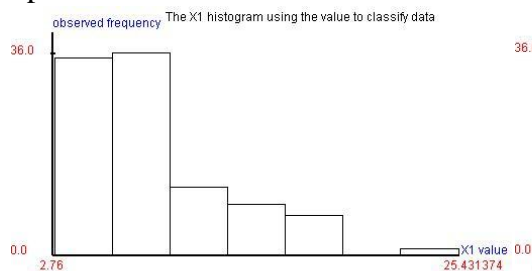
correction:

expected number ≥ 5 in each cell, the frequency table is adjusted

class	[1]	[2]	[3]	[4]	[5]
lower limit	2.76580	6.00374	9.24168	12.47962	15.71756
upper limit	6.00374	9.24168	12.47962	15.71756	25.43137
observed no	35.00000	36.00000	12.00000	9.00000	8.00000
probability	0.32980	0.35150	0.18440	0.07890	0.05540
expected no	32.98000	35.15000	18.44000	7.89000	5.54000
chi square	0.12372	0.02055	2.24911	0.15616	1.09235

degree of freedom=2

pearson chi-square test statistic =3.641895
 p-value=0.161800



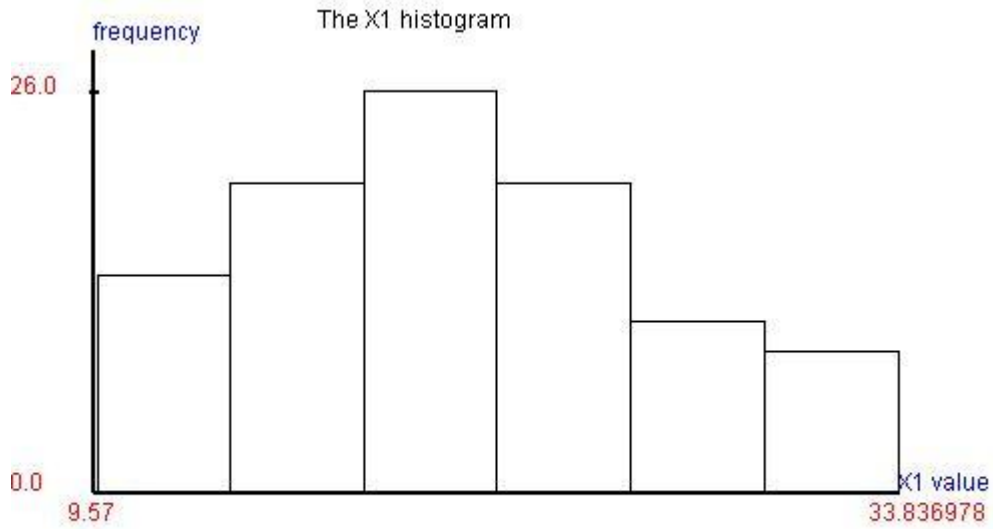
3.4.9)The population distribution is gamma distribution.

X1 is Gamma(alpha=10.000000,beta=2.000000),

X1
30.5334057066
15.0132298515
22.4483809146
24.1530442144
22.1788431007
18.7909071760
14.4031492025
31.6019988940
13.1310098133
23.7630415572
17.7435738783
19.6665844371
23.9397800772
20.1575092461
17.0555573235
32.5870274937
25.9357111829
14.0166024152
33.4691187130
23.9541048105
11.4131693146
20.6848278487
25.3609372182
11.3111539540
24.1130422035
19.0236582744
20.9316484649
25.8232255571
11.0631561389
15.6500019430
30.2129155622
20.1054663539
14.8885040382
19.7671365861
17.4369931224
14.0822029681
29.7225169174
28.5557066520
13.7098884922
15.5810463446
23.4128941293
19.3336486729
16.1113570424
22.8049601175
18.0378582999
22.4809884866
21.4115352083
18.7933587634
24.2567461320
15.7555726141
31.5814652104
9.5729147402
24.0358904782
25.3025629151
28.0139350289
19.2102156380
19.8516725700
13.1736380532
16.3387746346
18.5597172071
13.7537208126
18.1121305203
9.9214590051
31.8691129797
29.8268721714
33.8369785225
27.2724681870
27.9959190078
16.5495348955
12.3392713978
15.4174728401

23.8681550503
 12.8839833000
 28.6539289776
 21.5589809209
 19.3551880752
 13.1060206559
 20.3064710684
 20.2772086063
 21.3320068001
 18.6224653932
 14.6783968301
 19.3427256603
 22.5219965608
 22.4291712439
 19.4896607044
 25.3660044137
 16.6924787717
 9.6610251718
 11.7550473843
 27.0678429700
 29.7856438545
 20.8469627095
 10.2673760965
 15.6025526421
 28.9445337406
 24.2269301541
 15.1009397094
 22.4801535872
 12.6606759282

X1 is Gamma(alpha=10.000000,beta=2.000000),



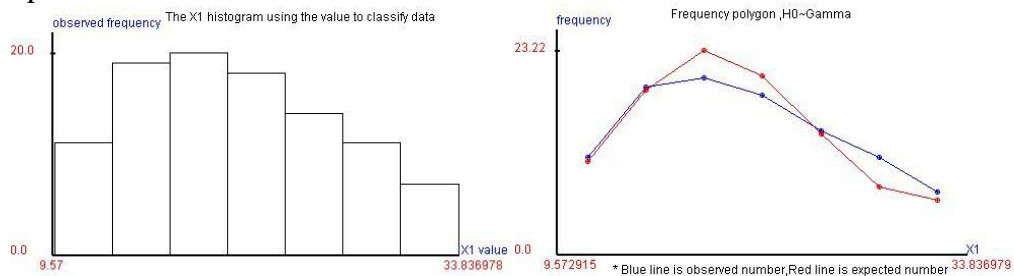
H0: $X_1 \sim \text{Gamma}(\alpha, \beta)$, α, β are unknown
 alpha point estimated value=11.000000 (MME)
 beta point estimated value=1.838753 (MME)

pearson goodness of fit

class	[1]	[2]	[3]	[4]
lower limit	9.57291	13.03921	16.50550	19.97180
23.43809	26.90439	30.37068		
upper limit	13.03921	16.50550	19.97180	23.43809
26.90439	30.37068	33.83698		
observed no	11.00000	19.00000	20.00000	18.00000
14.00000	11.00000	7.00000		
probability	0.10520	0.18610	0.23220	0.20250
0.13660	0.07600	0.06140		
expected no	10.52000	18.61000	23.22000	20.25000
13.66000	7.60000	6.14000		
chi square	0.02190	0.00817	0.44653	0.25000
0.00846	1.52105	0.12046		

degree of freedom=4

pearson chi-square test statistic =2.376574
 p-value=0.666800



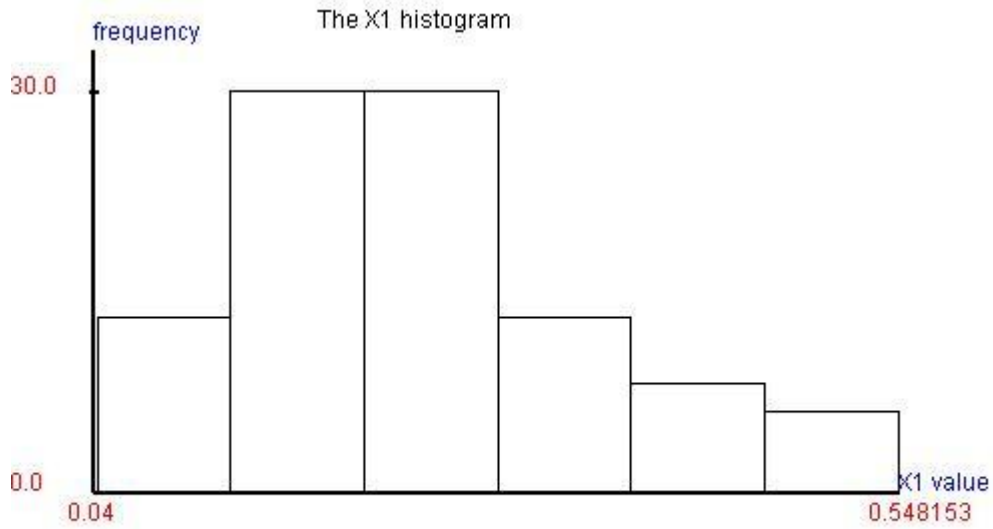
3.4.10)The population distribution is beta distribution.

X1 is Beta(alpha=4.000000,beta=12.000000),

X1
0.0803229485
0.1166599860
0.2675689478
0.1124666578
0.4270526554
0.2080614840
0.2882552092
0.2310039113
0.3266886566
0.1129357859
0.1605342714
0.3734898722
0.4699172354
0.1489746924
0.1835015401
0.1874479275
0.5097023531
0.1906009629
0.2368971159
0.2378588691
0.0924971084
0.2812837370
0.1918117133
0.2221961730
0.1835595457
0.2470307603
0.2655064378
0.0460673870
0.2243924341
0.3111628625
0.4436820179
0.1850791673
0.2167664467
0.3416776516
0.1590891356
0.1497017715
0.2200752620
0.2565496788
0.1904080012
0.1897491421
0.3307570393
0.3175992655
0.1348416868
0.3432435529
0.1975477876
0.1628179147
0.1562778483
0.1163226808
0.2966592675
0.2421333401
0.3411304378
0.4877567436
0.3766637045
0.5481539756
0.5348152867
0.3824770640
0.3211845280
0.1997830719
0.1555360825
0.2057757876
0.1491063136
0.3913726095
0.2148320947
0.1561038068
0.5145559707
0.2318118294
0.1412734675
0.3676561775
0.2478303059
0.2448349859
0.0539938206

0.3409981656
0.1411279529
0.2645753624
0.2638412271
0.1540416267
0.1691265362
0.2868207343
0.2373467197
0.1596683307
0.1251896970
0.2367936211
0.1177536631
0.1973583959
0.4008895888
0.2813357572
0.2246044930
0.2211756121
0.3930283580
0.1574650324
0.0540318562
0.1302036996
0.3929327669
0.3995543401
0.2373346248
0.1065641027
0.3506263558
0.2430284851
0.0649396708
0.2914259738

X1 is Beta(alpha=4.000000,beta=12.000000),



H0: $X_1 \sim \text{Beta}(\alpha, \beta)$, α, β are unknown

alpha point estimated value=3.000000 (MME)

beta point estimated value=10.000000 (MME)

pearson goodness of fit

class	[1]	[2]	[3]	[4]
lower limit	0.04607	0.11779	0.18952	0.26125
0.33297	0.40470	0.47643		
upper limit	0.11779	0.18952	0.26125	0.33297
0.40470	0.47643	0.54815		
observed no	12.00000	22.00000	29.00000	15.00000
14.00000	3.00000	5.00000		
probability	0.16018	0.24415	0.23893	0.17490
0.10324	0.05035	0.02825		
expected no	16.01763	24.41467	23.89313	17.48975
10.32422	5.03511	2.82549		
chi square	1.00772	0.23882	1.09153	0.35443
1.30871	0.82256	1.67351		

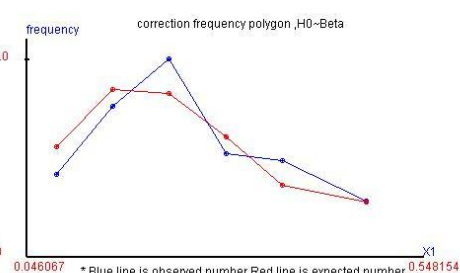
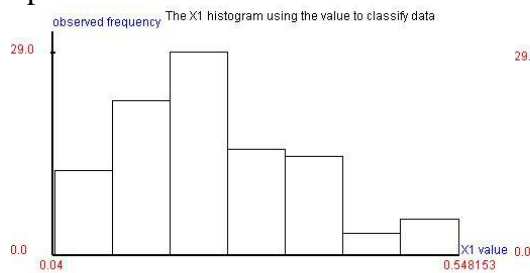
pearson chi square test statistic=6.497277
 degree of freedom=4
 p-value=0.483000

correction:

expected number ≥ 5 in each cell, the frequency table is adjusted

class	[1]	[2]	[3]	[4]
lower limit	0.04607	0.11779	0.18952	0.26125
0.33297	0.40470			
upper limit	0.11779	0.18952	0.26125	0.33297
0.40470	0.54815			
observed no	12.00000	22.00000	29.00000	15.00000
14.00000	8.00000			
probability	0.16018	0.24415	0.23893	0.17490
0.10324	0.07861			
expected no	16.01763	24.41467	23.89313	17.48975
10.32422	7.86060			
chi square	1.00772	0.23882	1.09153	0.35443
1.30871	0.00247			

degree of freedom=3
 pearson chi-square test statistic =4.003680
 p-value=0.261000



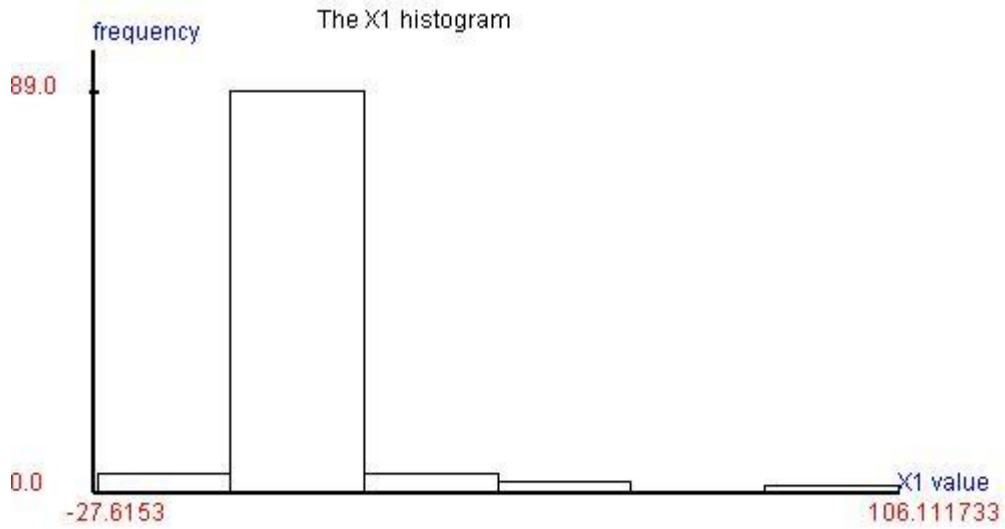
3.4.11)The population distribution is cauchy distribution.

X1 is Cauchy($\mu=5.000000$, $\sigma=2.000000$),

X1
12.9005475768
5.1955505968
4.7588325642
8.9215550940
7.1914660557
9.5190330122
49.7777431498
4.8763127854
3.4646475569
5.5047718654
3.0132171864
-10.5216031810
4.8170552795
3.8403242181
1.8405058371
6.2571709887
6.8355763878
5.2385040803
5.9906002739
0.0586655104
8.6708616271
5.0277337134
4.3843387781
-5.1583750961
4.7529293702
6.0575472265
5.8875734998
5.5914212326
6.7282255092
5.3206595993
5.3576733421
4.9262669825
2.5205292480
7.6643358479
5.6173808336
6.3872609423
3.5029353310
8.2118019805
23.0750235845
-5.6732769980
3.6366304757
5.9422953977
4.6283745672
14.5816195925
1.6940092572
5.0297653537
7.2705836200
1.1079170026
4.0245888948
-27.6153252935
5.3182537622
3.2770921596
4.8573817681
4.3992187716
6.1200483886
-13.6040618273
9.2583116497
5.1838196936
2.0896962905
8.0774936353
5.4591451194
3.1036827727
5.4671567097
6.1306010578
7.6188120279
21.6830737370
4.8571093849
7.5709950217
-0.7606828345
4.1584094227
7.0229445794

31.1371758340
 6.5686342935
 9.8939738291
 8.7016885055
 30.0878629560
 41.5672048272
 6.2993155175
 7.5170972415
 2.5977645895
 4.9554502046
 8.9142189730
 5.1904565717
 5.2677549675
 3.6848494066
 5.7804412685
 2.2001067043
 2.1829023002
 2.3844689519
 5.2905389538
 6.3747061785
 8.6418291030
 2.0493370024
 4.9994698411
 5.5741102330
 3.5307029149
 106.1117338753
 4.4159366576
 1.2522612492
 7.5396914154

X1 is Cauchy(mu=5.000000,sigma=2.000000),



H0: $X_1 \sim \text{Cauchy}(\mu, \sigma)$, μ, σ are unknown

μ point estimated value=5.304396

σ point estimated value=1.804922

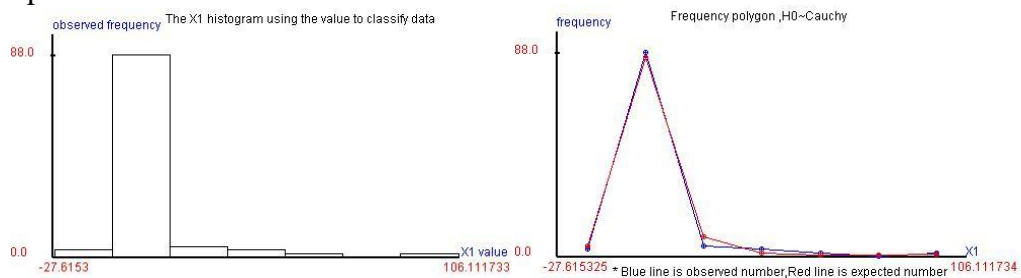
pearson goodness of fit

class	[1]	[2]	[3]	[4]
lower limit	-27.61533	-8.51146	10.59241	29.69627
48.80014	67.90400	87.00787		
upper limit	-8.51146	10.59241	29.69627	48.80014
67.90400	87.00787	106.11173		
observed no	3.00000	88.00000	4.00000	3.00000
1.00000	0.00000	1.00000		
probability	0.04135	0.85395	0.08119	0.01031
0.00403	0.00214	0.00703		
expected no	4.13502	85.39499	8.11888	1.03099
0.40260	0.21445	0.70307		
chi square	0.31155	0.07947	2.08960	3.76046
0.88648	0.21445	0.12541		

degree of freedom=4

pearson chi-square test statistic =7.467417

p-value=0.113100



3.4.12)The population distribution is arcsin distribution.

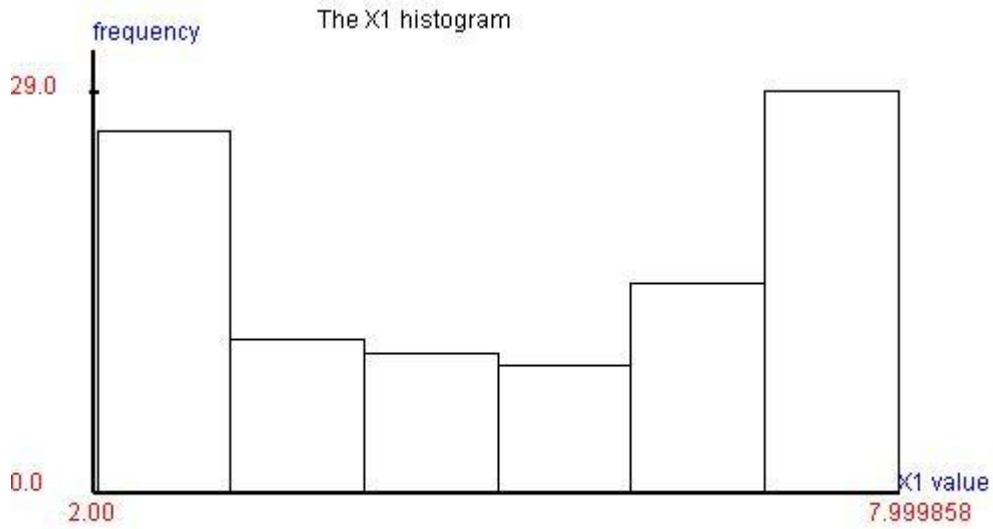
X1 is Arcsin($\mu=5.000000,c=3.000000$),

X1

7.9933281602
7.0766898203
2.1120717304
7.5065144737
3.3120906346
5.6892324574
7.4173412797
5.4941612449
7.5780070206
5.9948924858
4.2639110505
6.9746033104
7.7784984250
2.3925680661
2.3481827483
2.0049045345
4.7847298673
7.9851304292
2.0939739869
3.0556992355
5.1247945599
6.9650035073
4.0323517259
6.5938105165
2.0124043250
7.6425039328
7.3328014973
7.9998583185
6.3956587624
2.8209682480
7.8444872523
7.9983071518
2.3491408082
3.4670278699
7.9890324651
7.9066501250
2.1819693713
2.0901611213
5.6540306692
2.6094472272
6.8807945785
2.7478838200
4.3791646940
2.4930323529
6.7601568638
6.1745905765
4.8370930725
2.0012110608
3.9399043063
6.5846574609
7.9133307023
6.4984503400
7.7126281404
7.9121005175
4.8777378981
2.2279917604
3.0671396076
7.1820058844
7.3954436504
5.9377895848
3.1193263331
2.9598495243
7.6415351943
7.9711134720
5.6584773256
2.3014873842
6.1065024236
2.6430029833
5.4126451348
7.6202652711
7.6434127662

2.5856123894
 2.2568454690
 6.7954967202
 7.5333092641
 3.5075803413
 6.8403917345
 3.7312171499
 4.4330539337
 4.8946203657
 2.3106353674
 2.2978674295
 3.1139493390
 2.0210081400
 7.9952948610
 6.6317632618
 7.7390961904
 7.9980938348
 7.0025328124
 6.0920553876
 7.9998067937
 2.9101151531
 3.3179809469
 4.2936235669
 2.9890887127
 2.0352361925
 5.7301146567
 3.7999989720
 6.4828349024
 4.1052784404

X1 is Arcsin(mu=5.000000,c=3.000000).



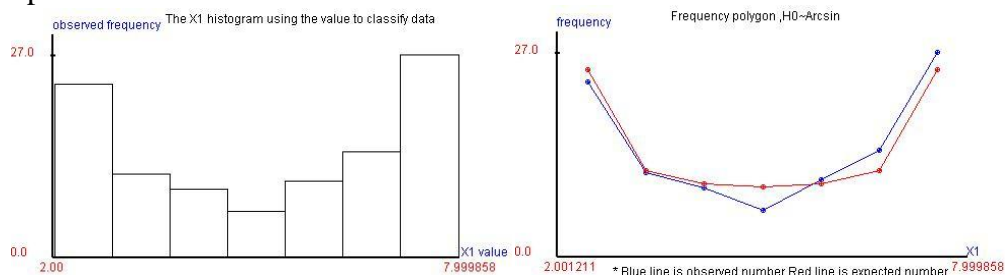
H0: $X_1 \sim \text{Arcsin}(\mu, c)$, μ, c are unknown
 μ point estimated value=5.000535 (MLE)
 c point estimated value=2.999324 (MLE)

pearson goodness of fit

class	[1]	[2]	[3]	[4]
lower limit	2.00121	2.85816	3.71511	4.57206
5.42901	6.28596	7.14291		
upper limit	2.85816	3.71511	4.57206	5.42901
6.28596	7.14291	7.99986		
observed no	23.00000	11.00000	9.00000	6.00000
10.00000	14.00000	27.00000		
probability	0.24675	0.11227	0.09535	0.09126
0.09535	0.11227	0.24675		
expected no	24.67517	11.22653	9.53540	9.12579
9.53540	11.22653	24.67517		
chi square	0.11373	0.00457	0.03006	1.07065
0.02264	0.68517	0.21904		

degree of freedom=4

pearson chi-square test statistic =2.145862
p-value=0.708900



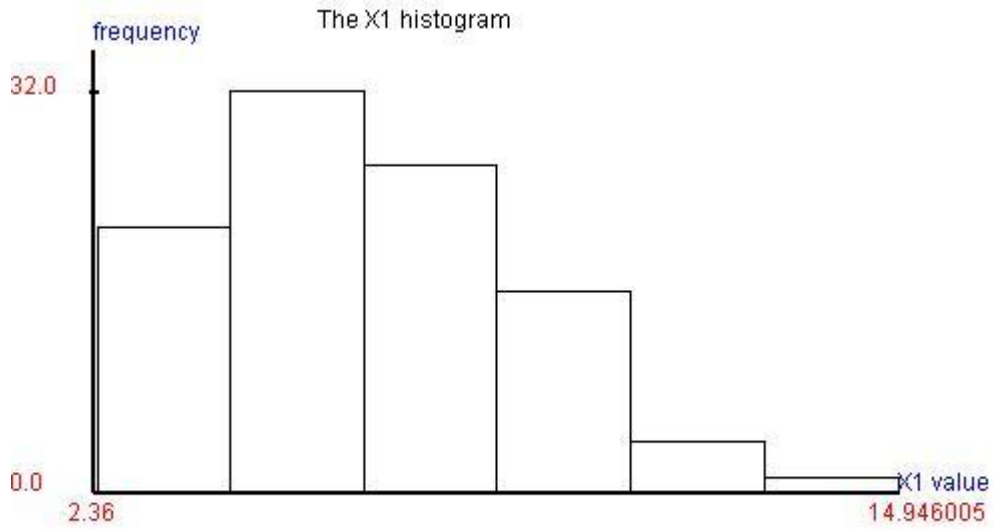
3.4.13) The population distribution is gumbel distribution.

X_1 is Gumbel($\mu=6.000000, \sigma=2.000000$),

X_1
4.8056001580
4.8426508095
8.9465078759
8.0826886204
9.4469575085
4.5545181967
11.0155923094
5.8886079734
10.2872363691
4.8890975637
6.4642632069
7.5071058914
4.2571723005
4.9961252334
9.2730444477
3.4082133754
2.3615301401
6.1862347861
9.1627344191
6.1571829441
7.4252406688
4.8935509526
8.5658928717
8.9942298374
4.4679168896
12.0409940300
9.9581304946
6.7426792884
9.0518812704
5.5907416324
8.1464764683
4.2358798304
4.1869766900
3.8834308406
8.5548711797
8.1030643423
7.4943649718
8.0377352550
4.4973817037
6.2402930333
7.4667412434
8.0235962472
6.0266600255
7.9134975286
5.0167679771
4.3706078264
6.7690587010
2.6665091804
10.9548477452
5.2228208916
3.0607980217
7.5903645230
6.4426347868
6.2998197157
4.2741208374
6.4592066972
5.6158846858
3.2163895630
14.9460056101
11.4758330792
8.3536412388
6.0283464019
6.3743920874
5.6600490386
9.0051634154
3.8116734370
5.3691067703
8.3268669108
6.4662096941
4.4671409354
3.7306226602

8.1980653841
 9.4461369193
 7.1294783073
 3.4961938277
 7.1924040396
 7.8820621039
 10.0693434746
 2.8482138971
 3.8677230135
 9.0066453918
 7.9243145894
 6.3503104608
 3.9504323311
 8.7769359169
 7.1611917209
 7.4841766027
 6.9457719942
 8.0546322903
 3.2037479047
 4.2492889541
 6.3696415711
 3.8833277782
 6.3246907305
 2.9128168197
 9.5986215551
 8.7296500476
 5.7242057276
 8.7941523541
 5.2266605687

X1 is Gumbel(mu=6.000000,sigma=2.000000),



H0: $X1 \sim \text{Gumbel}(\mu, \sigma)$, μ, σ are unknown

μ point estimated value=5.587248 (MME)

σ point estimated value=1.855642 (MME)

pearson goodness of fit

class	[1]	[2]	[3]	[4]
lower limit	2.36153	4.15931	5.95709	7.75488
9.55266	11.35044	13.14822		
upper limit	4.15931	5.95709	7.75488	9.55266
11.35044	13.14822	14.94601		
observed no	15.00000	24.00000	26.00000	26.00000
6.00000	2.00000	1.00000		
probability	0.11547	0.32527	0.29201	0.15593
0.06751	0.02695	0.01686		
expected no	11.54741	32.52675	29.20102	15.59314
6.75149	2.69463	1.68555		
chi square	1.03230	2.23525	0.35090	6.94553
0.08365	0.17906	0.27883		

pearson chi square test statistic=11.105522

degree of freedom=4, p-value=0.134000

correction:

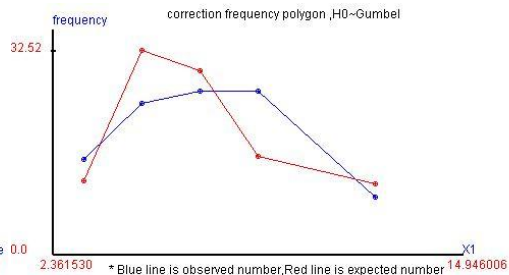
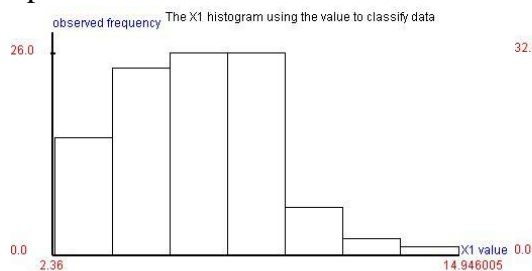
expected number ≥ 5 in each cell, the frequency table is adjusted

class	[1]	[2]	[3]	[4]
lower limit	2.36153	4.15931	5.95709	7.75488
9.55266				
upper limit	4.15931	5.95709	7.75488	9.55266
14.94601				
observed no	15.00000	24.00000	26.00000	26.00000
9.00000				
probability	0.11547	0.32527	0.29201	0.15593
0.11132				
expected no	11.54741	32.52675	29.20102	15.59314
11.13168				
chi square	1.03230	2.23525	0.35090	6.94553
0.40821				

degree of freedom=2

pearson chi-square test statistic =10.972189

p-value=0.004100



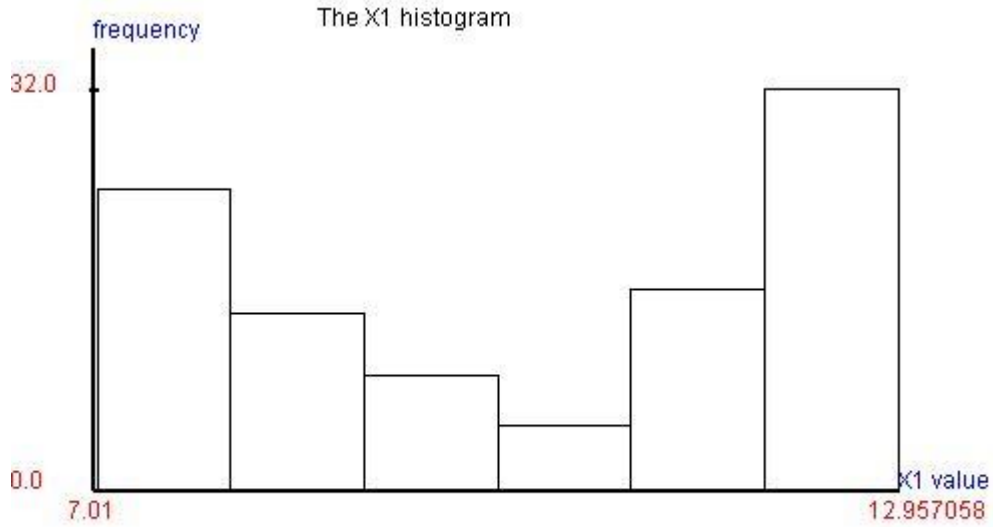
3.4.14)The population distribution is triangular 1 distribution.

X1 is Triangular1($\mu=10.000000,c=3.000000$),

X1
11.8009233399
12.8356782041
12.3835558235
8.7883269243
12.4932729014
8.1768858942
11.9710910998
10.7105929858
12.2869942584
9.2046003826
12.9506558982
10.3564005306
7.3405503056
8.5314140293
9.1642399486
12.0016010023
7.6809065199
11.5418531272
10.4335136482
9.3042101195
7.3528648768
12.0093488204
7.4639054652
7.4847773132
12.8340723348
7.2069103383
12.2132708110
7.0465289800
12.8698829761
11.1410306772
11.3056911870
11.5111295716
7.7767441898
12.6182449882
11.2884098673
12.5389008212
12.9315933819
7.6976409955
7.5303984884
11.0917844287
10.7332268666
12.3476341626
8.4503249134
12.6003981062
12.5237181568
7.2727251382
12.4034802862
7.2774014038
12.1907465251
8.1911477960
7.6308407479
7.7474479097
12.1150463772
12.9570581568
8.1629917960
7.4984246912
8.5531261833
12.1709323508
12.9159903556
8.9241373665
8.8534441371
7.3287127079
7.9181981640
11.8050145820
9.7024337574
9.3396574221
11.2128743253
11.3694847756
8.5494711194
7.6689047950
8.0755047590

11.8223615850
 7.0177105605
 12.7436710548
 12.8494801490
 9.0481704922
 12.8912591604
 11.9389355311
 7.6199452399
 8.5674554565
 12.0662047046
 12.2895282646
 7.7253888484
 9.3112308630
 9.7583319550
 12.6219001989
 12.8779060291
 8.3381342777
 10.9584397558
 7.5485800115
 12.4021108221
 7.2436696032
 8.7248247857
 12.6665574626
 7.0943970343
 11.2738976445
 9.3368321114
 11.3973492484
 11.7832432257
 11.8490484866

X1 is Triangular1(mu=10.000000,c=3.000000),



H0: $X_1 \sim \text{Triangular 1}(\mu, c)$, μ, c are unknown

μ point estimated value=9.987384 (MLE)

c point estimated value=2.969674 (MLE)

pearson goodness of fit

class	[1]	[2]	[3]	[4]
lower limit	7.01771	7.86619	8.71467	9.56315
10.41162	11.26010	12.10858		
upper limit	7.86619	8.71467	9.56315	10.41162
11.26010	12.10858	12.95706		
observed no	23.00000	11.00000	11.00000	3.00000
7.00000	17.00000	28.00000		
probability	0.24490	0.16327	0.08163	0.02041
0.08163	0.16327	0.24490		
expected no	24.48980	16.32653	8.16327	2.04082
8.16327	16.32653	24.48980		
chi square	0.09063	1.73778	0.98577	0.45082
0.16577	0.02778	0.50313		

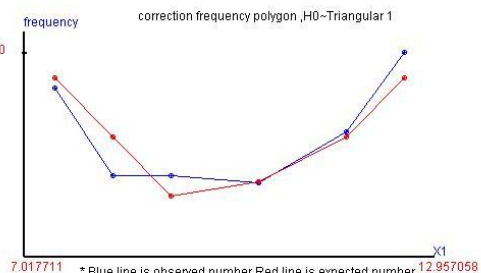
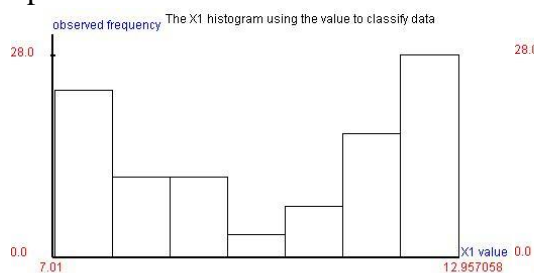
pearson chi square test statistic=3.961667
degree of freedom=4,p-value=0.784200

correction:

expected number ≥ 5 in each cell, the frequency table is adjusted

class	[1]	[2]	[3]	[4]
lower limit	7.01771	7.86619	8.71467	9.56315
11.26010	12.10858			
upper limit	7.86619	8.71467	9.56315	11.26010
12.10858	12.95706			
observed no	23.00000	11.00000	11.00000	10.00000
17.00000	28.00000			
probability	0.24490	0.16327	0.08163	0.10204
0.16327	0.24490			
expected no	24.48980	16.32653	8.16327	10.20408
16.32653	24.48980			
chi square	0.09063	1.73778	0.98577	0.00408
0.02778	0.50313			

degree of freedom=3
pearson chi-square test statistic =3.349167
p-value=0.340800



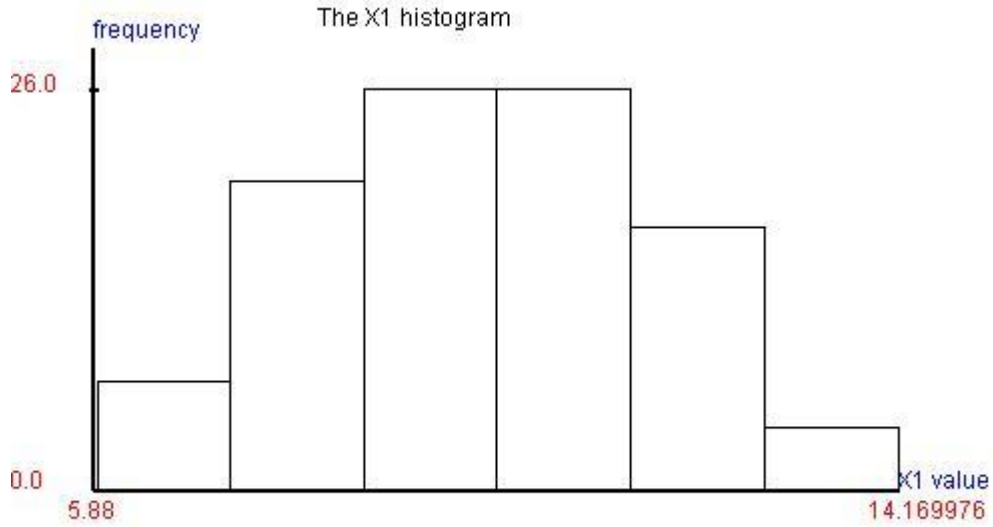
3.4.15)The population distribution is trapezoid distribution.

X1 is Trapezoid($\mu=10.000000,c=3.000000$),

X1
6.2175136609
11.6801283385
9.6051735916
9.5338032868
9.6731089728
10.2028026708
10.6398577709
6.6135445426
8.5871399112
10.5022268243
8.0463957448
9.7356218202
10.9775354935
5.8869600231
14.1699761122
11.4603701349
9.6350615497
10.5041796923
10.4251551642
8.4820283546
8.7050079738
9.3519913030
8.0787840304
9.9368950328
11.1465111446
13.4306272108
10.6795007196
11.0189789457
9.9434785199
7.7488055343
9.8242369037
11.7666979299
6.1293656218
9.7939138200
8.0693694982
9.5255823788
12.2845961989
8.7773651514
10.5242210569
6.9317241946
9.1158303795
9.8295552106
7.8338141664
7.7783442018
10.0621335291
10.8067194441
11.9817143837
11.1838458221
11.5451218971
7.9909917359
10.0382125985
10.3339254826
8.1018749563
11.7106565470
7.8297892946
8.5754730037
9.8793362428
11.0215516429
9.8919725264
10.4126372572
9.5767565069
9.1098838888
9.0738519951
10.3699440763
7.5102842853
7.4771988011
11.0843552986
9.5141759017
11.0067387704
6.8211521251
7.5555251234

11.7386257582
 11.2588016992
 11.7957476061
 8.1786568857
 11.7470569912
 9.1688702320
 11.9104715572
 8.3878934741
 10.8733303311
 12.7569935504
 9.0115535307
 11.5215252980
 12.9719440124
 7.5743714558
 13.0304618039
 10.8151180258
 9.8315398199
 7.2553359522
 11.4416260943
 11.8404799317
 11.8744843210
 11.9985990909
 8.1740132335
 10.5026625255
 9.1914564977
 9.4825589308
 10.4158393788
 11.0695855117
 8.0752524208

X1 is Trapezoid(mu=10.000000,c=3.000000),



H0: $X_1 \sim \text{Trapezoid}(\mu, c)$, μ, c are unknown

μ point estimated value=10.028468 (MLE)

c point estimated value=2.761005 (MLE)

pearson goodness of fit

class	[1]	[2]	[3]	[4]
lower limit	5.88696	7.07025	8.25354	9.43682
10.62011	11.80340	12.98669		
upper limit	7.07025	8.25354	9.43682	10.62011
11.80340	12.98669	14.16998		
observed no	6.00000	17.00000	13.00000	29.00000
24.00000	8.00000	3.00000		
probability	0.04592	0.13776	0.20918	0.21429
0.20918	0.13776	0.04592		
expected no	4.59184	13.77551	20.91837	21.42857
20.91837	13.77551	4.59184		
chi square	0.43184	0.75477	2.99739	2.67524
0.45398	2.42144	0.55184		

pearson chi square test statistic=10.286486

degree of freedom=4, p-value=0.172900

correction:

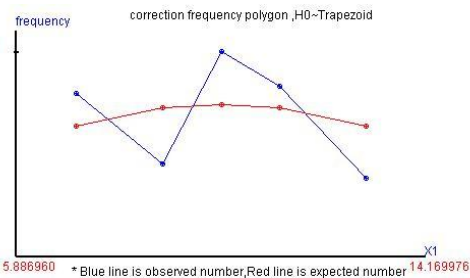
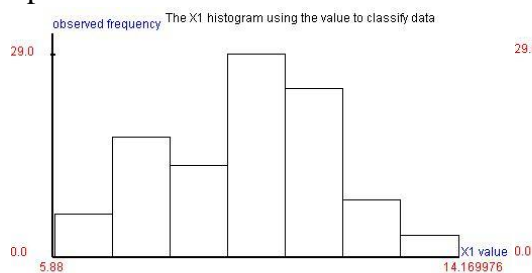
expected number ≥ 5 in each cell, the frequency table is adjusted

class	[1]	[2]	[3]	[4]
lower limit	5.88696	8.25354	9.43682	10.62011
11.80340				
upper limit	8.25354	9.43682	10.62011	11.80340
14.16998				
observed no	23.00000	13.00000	29.00000	24.00000
11.00000				
probability	0.18367	0.20918	0.21429	0.20918
0.18367				
expected no	18.36735	20.91837	21.42857	20.91837
18.36735				
chi square	1.16846	2.99739	2.67524	0.45398
2.95512				

degree of freedom=2

pearson chi-square test statistic =10.250190

p-value=0.005900



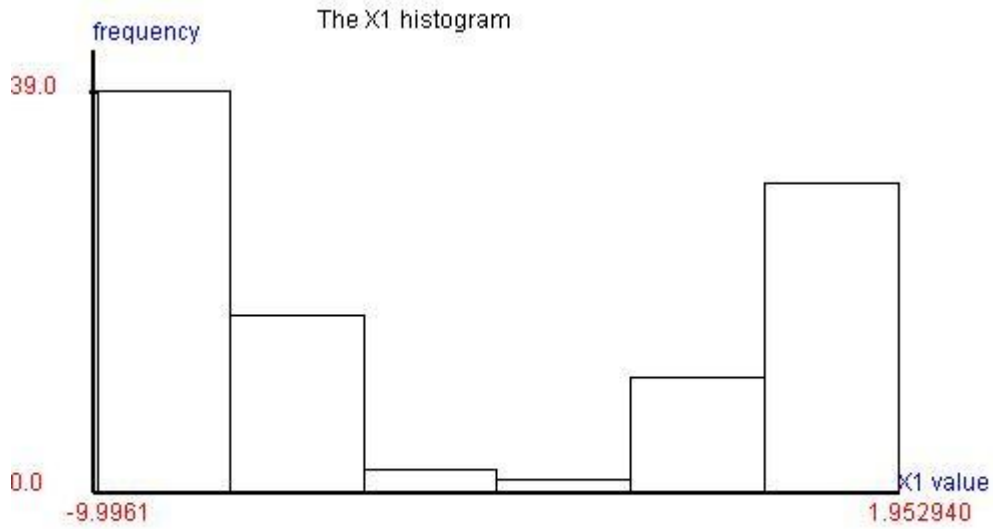
3.4.16)The population distribution is U quadratic distribution.

X1 is U-quadratic(a=-10.000000,b=2.000000),

X1
-9.3576464978
-6.5998969713
-9.0511036474
0.9789394897
0.5503686995
-9.6062076956
-7.9635181116
0.3694563503
-7.7653155498
-8.7467080093
-9.3729977222
-8.4432393510
1.2225043760
-7.9047764849
-8.3716949540
-6.8981770079
-1.8783232759
1.9529408344
-8.4366168413
-8.7692269884
-5.7069851828
-9.3485015009
-8.7620515244
0.7097079655
-0.5426045241
-8.8105600099
-6.8809025831
-7.9256501408
1.2501769893
-8.2364021593
-6.9317841299
-9.1196764858
-9.8259017668
-9.9961192991
1.8628227059
-8.6397597983
1.1109848681
-9.1725147759
-9.1476992793
-8.5126216370
1.8207744018
-1.7890987807
-9.6261114777
-9.9503539579
-9.1533937388
1.8037927716
-9.9824199339
0.6532062645
-8.7569136183
0.0540744997
-8.0363659486
1.7223275624
0.1085887463
1.6561263737
0.1320547197
-0.2323977309
-0.9726094725
1.8763400816
-0.8739552357
-8.4324993156
1.4741503838
-9.4565934647
0.9453085574
1.7414965891
-9.8111847520
-9.9738104203
-0.6964503830
-8.4491438359
-6.6812390004
-9.2357207291
-7.4172002521

-7.0306132846
 -0.6415396076
 0.9259403753
 -0.7661428965
 -7.5448239041
 -9.7246467618
 0.8310460941
 -0.5891071206
 -0.0634155908
 1.7729052960
 -9.4180597462
 0.7119347803
 -7.7167215606
 -8.1082760483
 0.3939344908
 -5.4988980884
 0.2519778856
 -9.8280062584
 -6.6835508138
 1.0476445919
 -6.9033529356
 -2.5571838409
 -9.5443823565
 -9.9195699077
 1.2227755010
 -7.9722223270
 -7.4037783231
 1.6561786189
 -8.5450446502

X1 is U-quadratic(a=-10.000000,b=2.000000),



H0: $X_1 \sim U_quadratic(a,b)$, a,b are unknown

a point estimated value=-9.996119 (MLE)

b point estimated value=1.952941 (MLE)

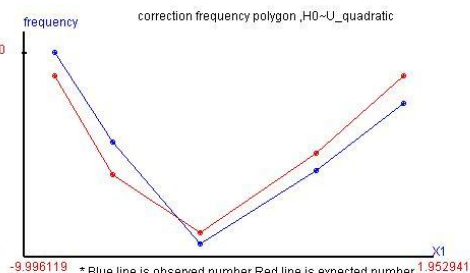
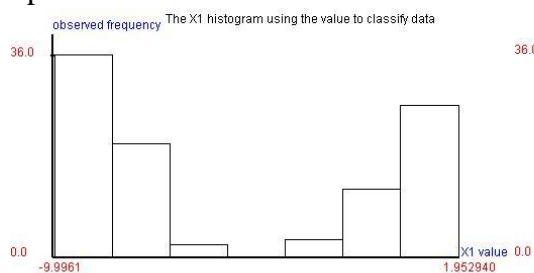
pearson goodness of fit

class	[1]	[2]	[3]	[4]	[5]
lower limit	-9.99612	-8.28911	-6.58210	-4.87509	-3.16808
upper limit	-1.46108	0.24593			
observed no	36.00000	20.00000	2.00000	0.00000	
expected no	31.78000	14.29000	3.79000	0.29000	
chi square	0.56037	2.28160	0.84541	0.29000	
pearson chi square test statistic	=5.225199				
degree of freedom	=4				
p-value	=0.632500				

correction:

expected number ≥ 5 in each cell, the frequency table is adjusted

class	[1]	[2]	[3]	[4]	[5]
lower limit	-9.99612	-8.28911	-6.58210	-3.16808	0.24593
upper limit	-8.28911	-6.58210	-3.16808	0.24593	1.95294
observed no	36.00000	20.00000	2.00000	15.00000	
expected no	31.78000	14.29000	4.08000	18.08000	
chi square	0.56037	2.28160	1.06039	0.52469	
pearson chi-square test statistic	=5.143226				
degree of freedom	=2				
p-value	=0.076400				



3.4.17)The population distribution is semi circle distribution.

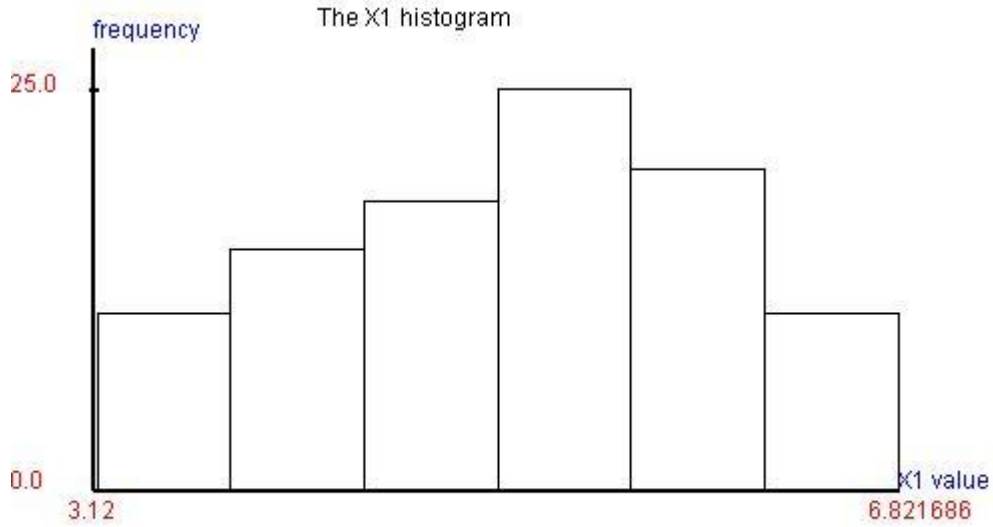
X1 is Semi-circle($\mu=5.000000,R=2.000000$),

X1

6.3233418562
5.5517253429
4.8932845440
4.3725520732
4.9907676178
4.2858998823
6.0003014213
5.1859625895
5.0848379036
3.8258530232
6.4600513410
3.1211038109
4.4831121939
6.2158001998
6.8216869526
3.7406123278
4.6334740101
5.1831362670
5.6034817189
3.5195781425
3.7267454521
6.1512062754
6.1868630097
4.5589362694
5.0007068293
3.8466332428
5.3863084241
4.7038683012
4.5963483655
3.9968962147
6.5845304056
5.2066584822
4.8803908331
5.9718892659
5.6779792448
5.0694457658
5.4938498826
4.1302162365
5.8493604378
4.0130968975
4.1125719185
4.6422733073
5.0539551647
5.4382750585
5.2000839459
6.7350700846
5.3860961791
5.2506200688
5.2150643487
6.1302336764
5.9157866693
3.9592494536
5.9275138845
5.5566733982
5.6193726369
6.5873089055
4.8698892222
4.4038226475
3.2559148098
3.7041104294
4.9530436232
6.1690795074
4.3758167464
4.9516274167
4.1499482134
4.6790000515
4.0078510114
6.3105624028
6.1373438607
3.4947609271
4.3239991790

3.2593501355
 5.7505699747
 3.6725266653
 6.3573839566
 3.1482263820
 5.3133231387
 5.3570082262
 4.3451511798
 5.6102774210
 4.6552888040
 4.7029310774
 3.6586919508
 3.7649756107
 6.4170752773
 3.9955365169
 4.6416432307
 5.1542681518
 6.1420830559
 5.3517317721
 5.5109265515
 5.2470369069
 6.0720631252
 6.0759473791
 5.5376138322
 5.8160948175
 5.7802945143
 3.6572791500
 5.3326491914
 6.8018336593

X1 is Semi-circle(mu=5.000000,R=2.000000),



H0: $X_1 \sim \text{Semi-circle}(\mu, R)$, μ, R are unknown

μ point estimated value=5.049752 (MLE)

R point estimated value=1.850292 (MLE)

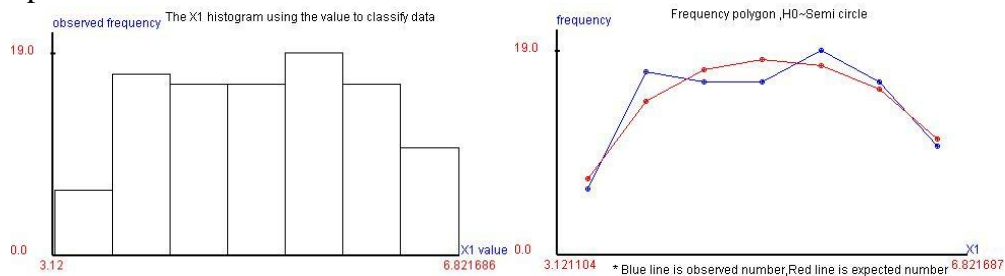
pearson goodness of fit

class	[1]	[2]	[3]	[4]
lower limit	3.12110	3.64976	4.17841	4.70707
5.23572	5.76438	6.29303		
upper limit	3.64976	4.17841	4.70707	5.23572
5.76438	6.29303	6.82169		
observed no	6.00000	17.00000	16.00000	16.00000
19.00000	16.00000	10.00000		
probability	0.06940	0.14230	0.17110	0.18110
0.17570	0.15340	0.10700		
expected no	6.94000	14.23000	17.11000	18.11000
17.57000	15.34000	10.70000		
chi square	0.12732	0.53921	0.07201	0.24584
0.11639	0.02840	0.04579		

degree of freedom=4

pearson chi-square test statistic =1.174949

p-value=0.882200



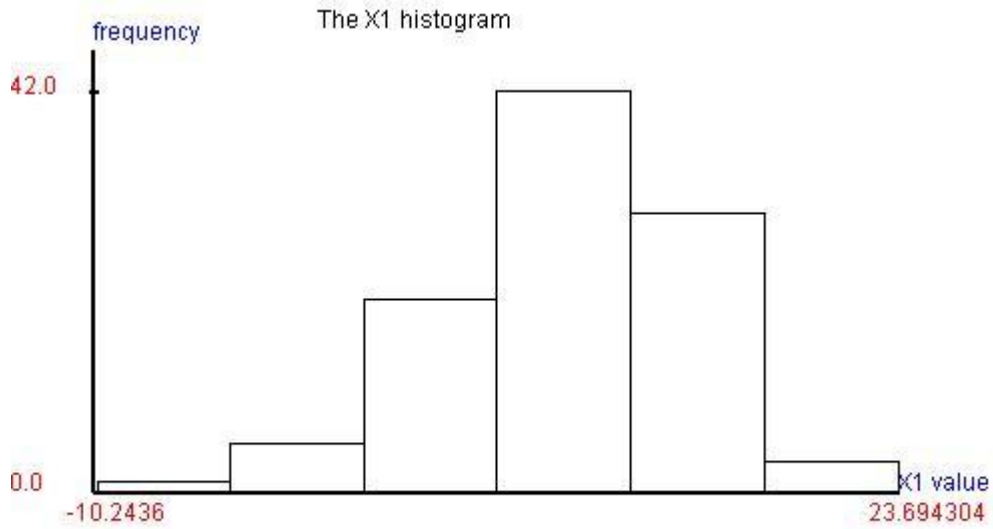
3.4.18) The population distribution is logistic distribution.

X_1 is Logistic($\mu=10.000000, \sigma=3.000000$),

X_1
8.4693686932
9.4072412322
5.2497110164
4.6103864493
11.9133326809
15.2044313422
6.9625364778
5.3658112135
6.4334568801
9.3111836705
0.2179632782
5.1316519047
16.3713649848
11.2719629327
-1.3643223773
9.3964994302
23.6943047626
4.6080294923
16.6865592719
16.0976256741
13.1388901415
15.5009270146
11.1859889644
12.7162370892
4.3648482057
13.4663295121
22.6893588246
2.3757556518
9.5419441782
14.9699489865
6.7747026043
4.3136696748
9.8674827508
10.6081682355
15.7669800257
14.6810361868
6.3121304011
15.3814506570
5.5765522155
6.8407133194
8.1203041191
17.8034110166
11.7805390083
12.3522685055
6.3091635018
9.5624888744
11.7654679390
5.0000943436
11.0684630642
11.3608271699
-10.2436428018
13.5916359682
14.2475324173
10.7296141961
1.5445610888
10.3461293238
14.1353972138
12.8035239611
8.6305177875
16.0512228181
10.9766126042
10.3461197136
5.1264404258
14.6686653219
8.8892998880
17.2142046299
3.9534111383
12.0478146735
10.8836073768
8.1780289249
4.4954609286

8.6825425980
 14.3193290272
 10.5036858227
 9.9773156301
 12.4643025977
 7.8275254537
 13.2119591625
 12.5742361580
 10.9562350655
 9.7308670448
 12.7725520736
 19.2485101197
 11.5569654508
 0.1053412855
 7.0337943929
 6.9622832238
 15.8004308926
 3.6657549710
 9.7480115676
 5.0857365738
 7.6309037583
 12.1364816831
 9.7206448875
 0.8546110945
 -1.3602948153
 12.9745712154
 17.0189880032
 4.9773362086
 17.4775744288

X1 is Logistic(mu=10.000000,sigma=3.000000),



H0: $X_1 \sim \text{Logistic}(\mu, \sigma)$, μ, σ are unknown

μ point estimated value=9.885096 (MME)

σ point estimated value=2.939818 (MME)

pearson goodness of fit

class	[1]	[2]	[3]	[4]
lower limit	-10.24364	-5.39536	-0.54709	4.30119
9.14947	13.99775	18.84603		
upper limit	-5.39536	-0.54709	4.30119	9.14947
13.99775	18.84603	23.69430		
observed no	1.00000	2.00000	7.00000	29.00000
39.00000	19.00000	3.00000		
probability	0.00550	0.02246	0.10221	0.30759
0.36425	0.15268	0.04530		
expected no	0.54986	2.24626	10.22148	30.75914
36.42496	15.26846	4.52985		
chi square	0.36852	0.02700	1.01530	0.10061
0.18204	0.91197	0.51667		

pearson chi square test statistic=3.122110

degree of freedom=4, p-value=0.873400

correction:

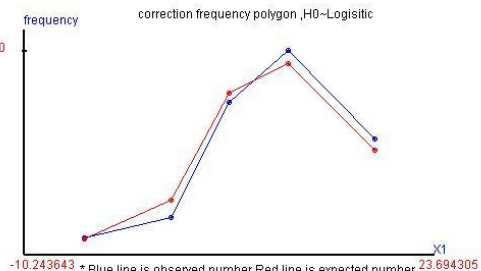
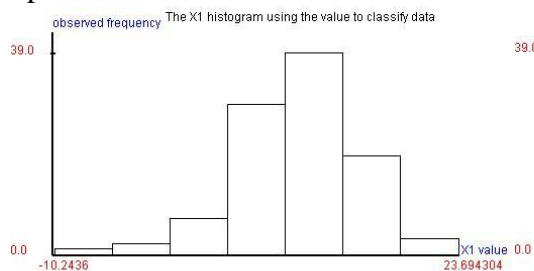
expected number ≥ 5 in each cell, the frequency table is adjusted

class	[1]	[2]	[3]	[4]
lower limit	-10.24364	-0.54709	4.30119	9.14947
13.99775				
upper limit	-0.54709	4.30119	9.14947	13.99775
23.69430				
observed no	3.00000	7.00000	29.00000	39.00000
22.00000				
probability	0.02796	0.10221	0.30759	0.36425
0.19798				
expected no	2.79611	10.22148	30.75914	36.42496
19.79831				
chi square	0.01487	1.01530	0.10061	0.18204
0.24484				

degree of freedom=2

pearson chi-square test statistic =1.557660

p-value=0.458900



3.4.19)The population distribution is weibull distribution.

X1 is Weibull(alpha=1.000000,beta=4.000000,gamma=2.000000),

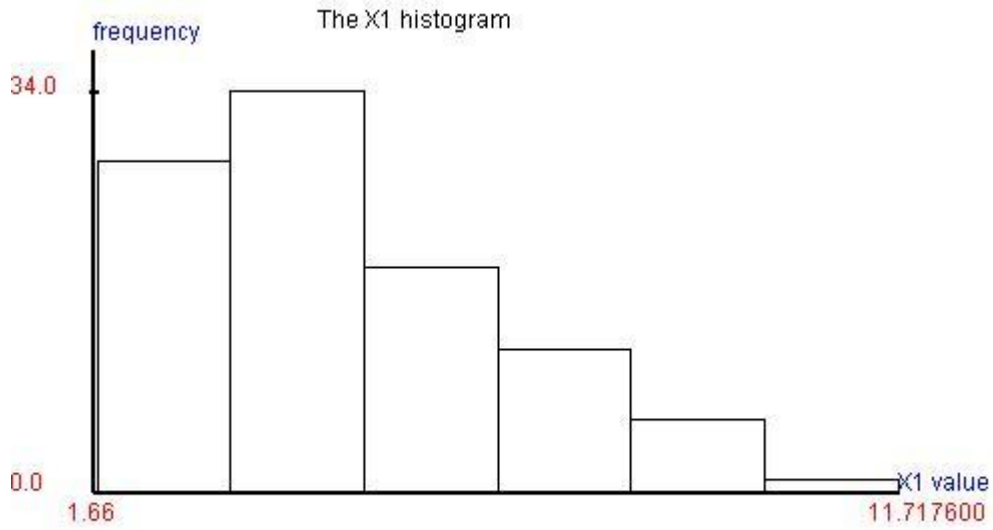
The parameter of gamma value is supposed to 2.

X1

4.4254130815
3.2446854642
2.8453890395
5.9574713971
7.9286849967
2.4477426093
7.0516886242
4.5904912683
4.3621802517
5.7270307159
7.0389240491
2.5407133407
4.7990620434
3.5347372439
4.0571512325
2.9944958698
7.5455627506
4.8006485779
4.9821182435
6.1611018698
7.9485050339
4.1788051889
9.1719742539
4.6178407000
4.7237400425
1.9841546752
4.0199589273
4.0199087287
3.8021466055
1.6650404157
5.0419362372
5.9888489547
2.4220598402
3.5557290881
5.4724251453
5.9711943282
4.7771861234
3.6954589992
3.4362441037
2.9660309478
8.5399875616
3.9369011358
2.2248548473
5.1894921690
4.0295967060
4.7192230576
4.6186535688
2.2900193762
2.8423634686
8.6447663174
3.3192158160
5.8394520908
6.7637584499
3.7712309014
4.2711490409
4.5165058645
3.5675745000
3.0475538610
11.7176009049
4.6296882754
6.0342030533
8.6899455673
4.8787712150
3.5739401619
3.6319761327
6.3895320156
5.9076046123
2.6528520147
7.1532186454

7.5011983426
 5.5424440516
 6.9532287000
 3.9921220341
 5.6958014705
 6.9913038926
 2.8433837410
 4.8864362521
 1.9078869225
 6.0385841916
 1.8340312518
 7.0200162984
 2.2584782929
 2.9849265901
 5.3623892966
 6.8218974422
 3.2105576374
 6.2491542726
 8.7713393336
 2.2089749804
 1.9993062922
 2.2805623840
 8.5919746842
 2.4725294559
 4.2415487564
 5.4749188774
 2.2000354939
 2.9719586801
 5.7697144523
 3.0298704704
 4.1971675112

X1 is Weibull(alpha=1.000000,beta=4.000000,gamma=2.000000),



H0: $X_1 \sim \text{Weibull}(\alpha, \beta, \gamma = 2.000000)$, α, β are unknown

alpha point estimated value=1.665040 (MLE)

beta point estimated value=3.684710 (MLE)

gamma value=2.000000 (hypothesis value)

pearson goodness of fit

class	[1]	[2]	[3]	[4]
lower limit	1.66504	3.10112	4.53720	5.97328
7.40936	8.84544	10.28152		
upper limit	3.10112	4.53720	5.97328	7.40936
8.84544	10.28152	11.71760		
observed no	25.00000	25.00000	25.00000	14.00000
9.00000	1.00000	1.00000		
probability	0.14092	0.31441	0.28981	0.16685
0.06558	0.01821	0.00422		
expected no	14.09237	31.44144	28.98113	16.68455
6.55770	1.82098	0.42184		
chi square	8.44262	1.31966	0.54689	0.43195
0.90959	0.37014	0.79243		

pearson chi square test statistic=12.813271

degree of freedom=5, p-value=0.076800

correction:

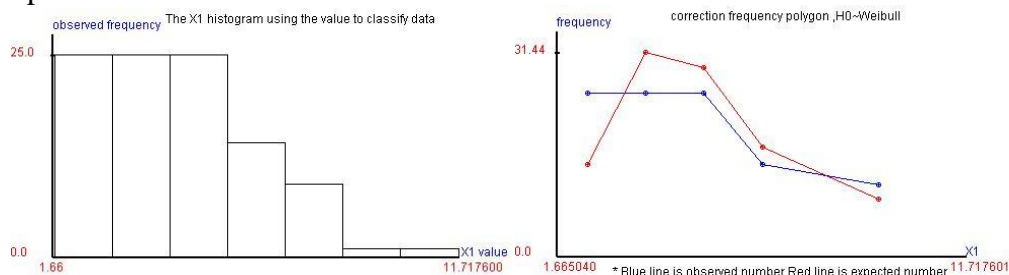
expected number ≥ 5 in each cell, the frequency table is adjusted

class	[1]	[2]	[3]	[4]
lower limit	1.66504	3.10112	4.53720	5.97328
7.40936				
upper limit	3.10112	4.53720	5.97328	7.40936
11.71760				
observed no	25.00000	25.00000	25.00000	14.00000
11.00000				
probability	0.14092	0.31441	0.28981	0.16685
0.08801				
expected no	14.09237	31.44144	28.98113	16.68455
8.80051				
chi square	8.44262	1.31966	0.54689	0.43195
0.54971				

degree of freedom=3

pearson chi-square test statistic =11.290824

p-value=0.010200



3.4.20)The population distribution is pareto 3 distribution.

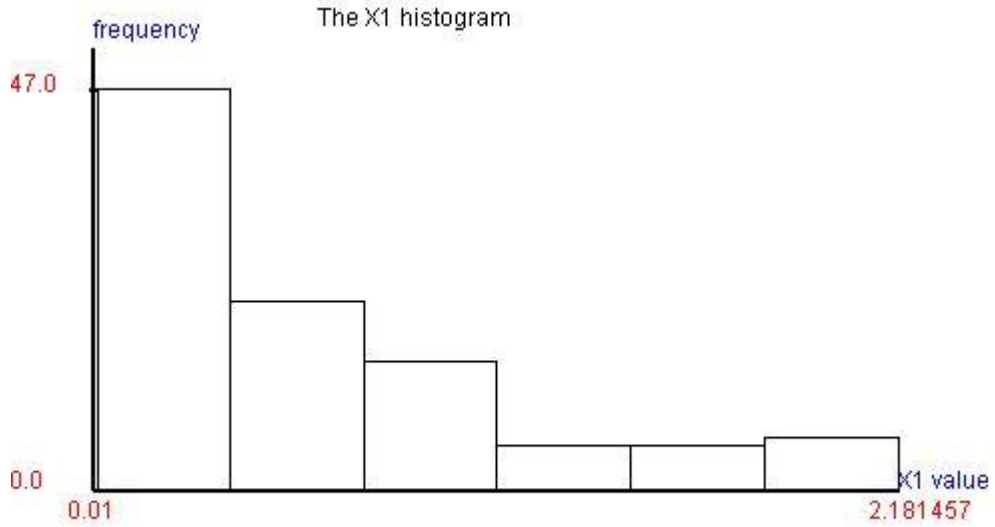
X1 is Pareto3(lamda=5.000000,c=4.000000),

X1

0.0636758882
0.3519267553
0.3293245960
0.3593538929
0.8435758627
0.5497461408
0.6098971147
0.6399836835
0.2636897365
0.0513461335
0.0142140919
0.3007623706
0.4645254226
0.1009599269
0.6229758449
0.4259985097
0.9291049108
1.5100947209
0.6031293292
1.0680847572
1.3888244004
0.7263093013
0.0448557115
0.8427871682
0.1083620981
0.2768496446
0.0104253659
0.0697317342
0.1945536925
0.3583225630
0.2476955989
0.7484420919
0.3619606885
0.2942014284
0.4544831895
1.0684963257
0.0967653772
0.0130170227
0.8640290339
0.5318718368
1.7645129699
0.0578374537
0.3339154698
0.0991590122
1.8059523927
1.9107727020
0.4050136228
1.2269855045
0.2051989561
0.0789194957
2.0047129744
0.2779906792
0.2077560412
0.8995332385
0.4355046455
1.4144666595
1.7701573954
0.9024210551
0.8757365857
1.9066855033
2.0533268568
0.2613614418
0.1315046883
2.1814575350
0.4973419931
0.4511465305
0.5348990866
0.4839695212
0.0369317107
0.5336854955
0.7666045484

1.4762144532
 0.2806364597
 0.3037207214
 0.1942204986
 0.3496916091
 0.6361282326
 0.4863944363
 0.1064606962
 0.2251343042
 0.2726391192
 0.2906265721
 1.8563417902
 0.7975645117
 0.3025171449
 1.3588343416
 0.1940355474
 1.2353171962
 0.1417736700
 0.8035698408
 0.5103296612
 0.2866664746
 0.0250342982
 0.3607275533
 0.4262256219
 0.0455597052
 1.0894972105
 0.3142427322
 0.7437075370
 0.4804329533

X1 is Pareto3(lamda=5.000000,c=4.000000),



H0: $X_1 \sim \text{Pareto } 3(\lambda, c)$, λ, c are unknown

λ point estimated value=2.384217 (MLE)

c point estimated value=2.181458 (MLE)

pearson goodness of fit

class	[1]	[2]	[3]	[4]
lower limit	0.01043	0.32057	0.63072	0.94087
1.25102	1.56116	1.87131		
upper limit	0.32057	0.63072	0.94087	1.25102
1.56116	1.87131	2.18146		
observed no	39.00000	27.00000	15.00000	5.00000
5.00000	4.00000	5.00000		
probability	0.31542	0.24134	0.18287	0.12924
0.08126	0.04032	0.00955		
expected no	31.54204	24.13399	18.28738	12.92356
8.12576	4.03197	0.95531		
chi square	1.76340	0.34035	0.59095	4.85801
1.20239	0.00025	17.12494		

pearson chi square test statistic=25.880294

degree of freedom=4, p-value=0.000500

correction:

expected number ≥ 5 in each cell, the frequency table is adjusted

class	[1]	[2]	[3]	[4]
lower limit	0.01043	0.32057	0.63072	0.94087
1.25102				
upper limit	0.32057	0.63072	0.94087	1.25102
2.18146				
observed no	39.00000	27.00000	15.00000	5.00000
14.00000				
probability	0.31542	0.24134	0.18287	0.12924
0.13113				
expected no	31.54204	24.13399	18.28738	12.92356
13.11303				
chi square	1.76340	0.34035	0.59095	4.85801
0.06000				

degree of freedom=2

pearson chi-square test statistic =7.612702

p-value=0.022200

