

Chapter six The goodness of fit test

2) Pearson chi-square test 2:

2.1) The multi-nominal distribution is the sampling distribution, each category has a probability and the summation of probability is 1.

category	1	2	k
probability	p_1	p_2	p_k

$$p_1 + \dots + p_k = 1,$$

$$f(x_1, \dots, x_k) = \frac{n!}{x_1! \times \dots \times x_k!} \times p_1^{x_1} \times \dots \times p_k^{x_k}, \quad x_1 + \dots + x_k = n \quad \text{that is trial number.}$$

2.2)

$$X_i = np_i + \varepsilon_i, \quad E(\varepsilon_i) = 0, \quad E[(\varepsilon_i)^2] = np_i(1 - p_i), \quad i = 1, 2, \dots, k$$

$$X_i - np_i = \varepsilon_i, \quad E\left[\frac{(\varepsilon_i)^2}{np_i}\right] = E\left[\frac{(X_i - np_i)^2}{np_i}\right] = 1 - p_i,$$

$$\sum_{i=1}^k E\left[\frac{(X_i - np_i)^2}{np_i}\right] = E\left[\sum_{i=1}^k \left(\frac{(X_i - np_i)^2}{np_i}\right)\right] = k - \sum_{i=1}^k p_i = k - 1,$$

$$\sum_{i=1}^k \left(\frac{(X_i - np_i)^2}{np_i}\right) \rightarrow \chi_{k-1}^2$$

$$H_0 : p_1 = p_{01}, p_2 = p_{02}, \dots, p_k = p_{0k} \quad H_1 : \text{against } H_0$$

$p_{01}, p_{02}, \dots, p_{0k}$ are known value and $p_{01} + p_{02} + \dots + p_{0k} = 1,$

$$E_i = np_{i0}, \quad i = 1, 2, \dots, k, \quad O_i = x_i,$$

O_i : The observed sample number of cell i,

E_i : The expected sample number of cell i,

$$\chi_v^2 = \sum_{i=1}^k \frac{(O_i - E_i)^2}{E_i}, \quad v = k - 1 - (\text{number of point estimator}) \text{ is degree of freedom of}$$

pearsson chi square test.

$$\chi_v^2 > \chi_{\alpha, v}^2 \Rightarrow \text{reject } H_0.$$

2.3) The process of test

H_0 : Population distribution is a continuous probability distribution,

H_1 : against H_0

In here, the parameters are unknown and giving a interval of parameters.

The values of parameter interval will put the null hypothesis and computed the p-value. The minimum of p-value which parameter is the best parameter value of population distribution by the sample data.

The sample size is n.

The process: i) The class number of frequency distribution: $k = \log_2(n) + 1.$

ii) The class limit of frequency distribution: Let each class probability are equally and the class limit will be gotten.

iii) The class sample number of frequency distribution (O_i):

The frequency distribution is done and getting the O_i of each class.

iv) The class expected number of frequency distribution (E_i):

$$E_i = n \times \text{the probability of each class.}$$

2.4) Example (The simulated sample data and computing the result by the P_S_CCC)

The Pearson chi square goodness of fit test using the best fitting method and equally probability, the fitting method is used the parameter value changed to find the best population distribution

1.H0:Uniform distribution	13.H0:Gumbel distribution
2.H0:Normal distribution	14.H0:Triangular 1 distribution
3.H0:Shifted exponential distribution	15.H0:Trapezoid distribution
4.H0:Pareto 1 distribution	16.H0:U-quadratic distribution
5.H0:Pareto 2 distribution	17.H0:Semi-circle distribution
6.H0:Rayleigh distribution	18.H0:Logistic distribution
7.H0:Double exponential distribution	19.H0:Weibull distribution
8.H0:Log normal distribution	20.H0:Pareto 3 distribution
9.H0:Gamma distribution	
10.H0:Beta distribution	
11.H0:Cauchy distribution	
12.H0:Arcsin distribution	

The parameter values are the point estimator value.

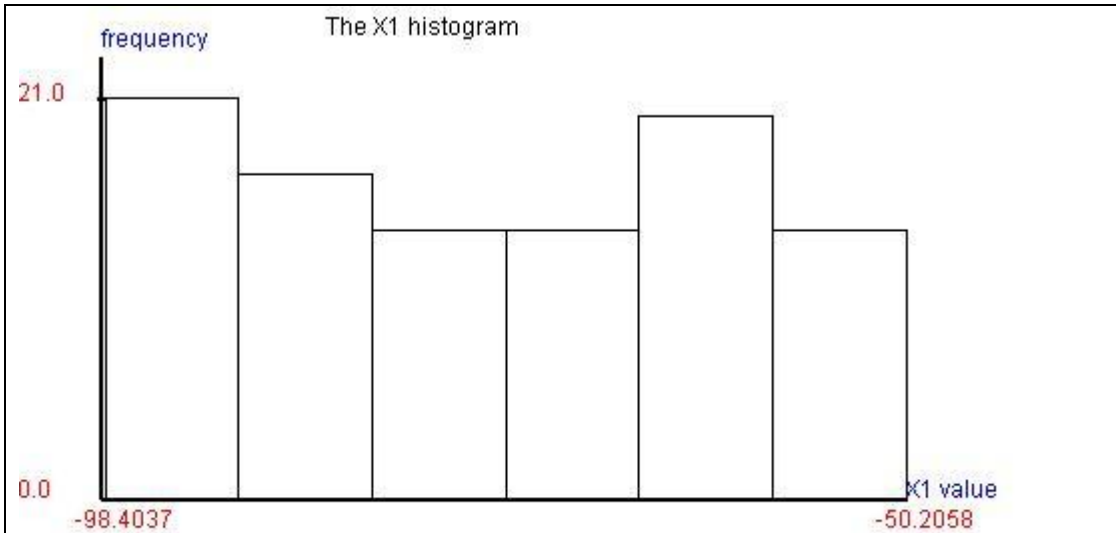
2.4.1)The population distribution is uniform distribution.

X1 is Uniform($\alpha=-100.000000$, $\beta=-50.000000$),

X1
-70.5407120066
-86.3492651112
-78.8757022799
-91.3780080771
-96.1721226616
-90.0414941823
-67.0029882665
-59.0158292915
-82.2447135923
-84.2093685656
-60.1149859026
-72.9781383234
-50.2058995488
-72.5183567492
-54.2509080457
-50.3076531004
-91.2930485891
-77.6100185257
-88.2770938360
-64.7113308192
-68.9146622455
-70.7329531707
-62.2187520531
-95.0866484299
-76.7959745982
-95.8443056358
-55.9044919065
-66.1687281010
-60.6735265592
-77.1564901416
-93.1998061143
-57.8088305591
-86.3995683109
-74.2960658693
-72.1606026195
-84.4649327909
-62.2423730121
-74.0463733568
-56.4333045929

-71.4866312664
-98.1838013420
-82.7798276495
-98.1792097776
-79.7277827827
-60.0566618435
-58.6476119323
-87.8511518142
-55.0599690467
-51.3216791283
-97.5807845966
-51.1642453359
-54.3644492604
-80.2716813095
-62.1369743666
-88.5246692072
-74.5360117876
-95.5192801078
-67.3061866468
-77.2506378320
-86.0763900967
-97.5449701189
-62.3083705813
-81.9151317514
-97.8640613104
-59.1481908994
-72.4367577759
-93.0941806564
-54.6017956306
-62.5767504302
-90.0345565144
-89.8426412020
-93.5760793119
-86.1847826361
-78.6529089059
-96.2300919434
-62.2328311172
-98.4037952637
-79.4487009516
-77.2084325188
-73.1684385938
-91.8571420869
-83.9021780090
-94.1712752683
-98.0786824761
-65.4268910160
-61.8931195428
-65.4751918821
-77.2310044464
-87.0698516281
-59.2246801210
-52.2645907672
-96.0530431248
-95.2176749359
-53.6229684153
-64.8977554552
-55.3311746503
-71.5608396032
-83.3213102456
-60.2898784470
-86.6608116878

X1 is Uniform(alpha=-100.000000,beta=-50.000000).



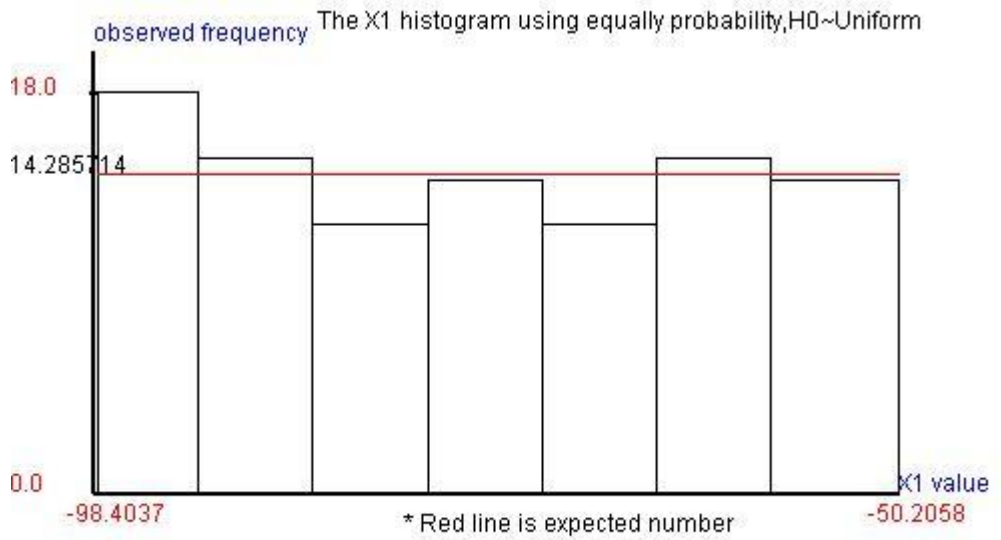
alpha point estimated value=-98.403795 (MLE)
 beta point estimated value=-50.205900 (MLE)
 alpha value from -99.377490 to -97.430100
 beta value from -51.179594 to -49.232205

pearson goodness of fit

class	[1]	[2]	[3]	[4]	
[5]	[6]	[7]			
lower limit	-99.22170	-92.31959	-85.41749	-78.51538	-71.61328
	-64.71117	-57.80907			
upper limit	-92.31959	-85.41749	-78.51538	-71.61328	-64.71117
	-57.80907	-50.90696			
observed no	18.00000	15.00000	12.00000	14.00000	
	12.00000	15.00000	14.00000		
probability	0.14286	0.14286	0.14286	0.14286	
	0.14286	0.14286			
expected no	14.28571	14.28571	14.28571	14.28571	
	14.28571	14.28571			
chi square	0.96571	0.03571	0.36571	0.00571	

0.36571 0.03571 0.00571

degree of freedom=4
H0: $X_1 \sim \text{Uniform}(\alpha=-99.221699, \beta=-50.906960)$,
pearson chi-square test statistic =1.780000
p-value=0.776100

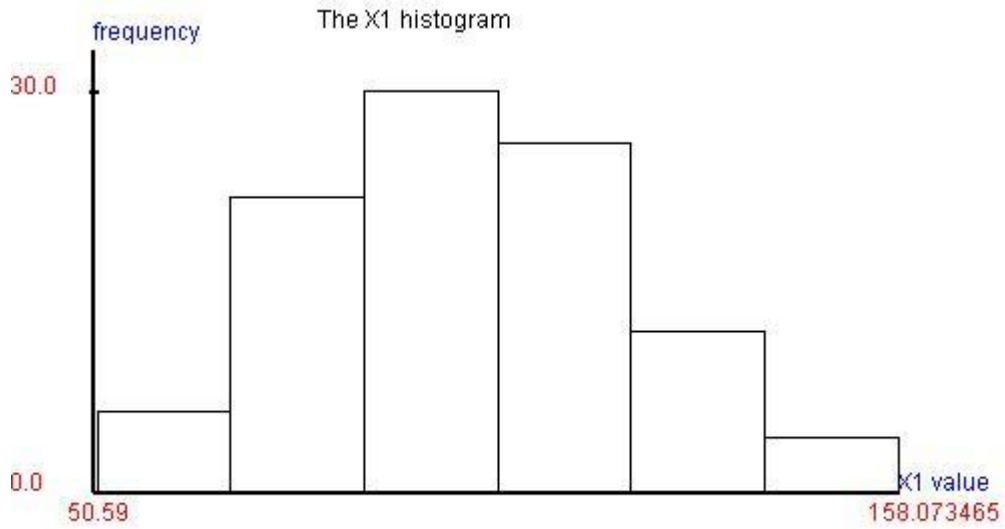


2.4.2)The population distribution is normal distribution.
X1 is Normal($\mu=100.000000$, $\sigma^2=400.000000$),

X1
123.1902471628
100.1575336434
97.5752279285
105.3734177112
125.0873749853
128.7861488598
112.0911740117
81.4266296330
88.9666186513
100.1084161096
116.7317458123
123.8277947275
96.3560242813
50.5948302895
118.2266027184
93.0752211037
118.6936140283
95.4433389824
100.4109595187
105.5079967529
74.4634767884
143.1182418831
119.5622400496
82.3571766607
106.6343478126
92.7039910492
117.6791787786
72.3102005705
109.5031449680
91.5239916690
121.7964506774
116.3953794363
96.3536930314
83.2115906040
72.4318390315
127.7343979449
106.9696852309
111.9134263981
92.2132478675
141.1151840418
79.9393464337
103.2356868091
129.1570566169
108.2303347319
119.0948612880
122.8972631087
63.8133605001
91.3396541274
89.4062760973
99.1579982586
67.8699431978
74.1998413023
88.0000680221
83.4937074287
85.5979126044
73.9185932528
120.2499434789
86.7892614321
116.9178732785
87.4186393870
93.2547660429
123.3515065164
98.6968475516
72.0705451304
90.6817658217
115.9649446358
67.2488584618
72.4403933282
110.1578848455
79.4805712647
90.4977063963

104.6787086078
 120.5140314809
 83.8118201917
 84.5221858939
 120.4760681021
 94.8578194075
 81.4042502346
 108.0039908606
 68.2835244714
 149.4584084166
 91.8664680645
 107.2406441197
 86.2839255086
 91.1291301211
 64.4445105994
 97.4187578795
 102.2311788283
 86.1568834983
 105.1149376883
 127.4678750884
 87.5958567991
 128.5544885776
 69.3662054879
 130.0984986160
 85.5764986125
 102.7646588557
 122.5957355356
 158.0734653811
 71.7155540105

X1 is Normal($\mu=100.000000$, $\sigma^2=400.000000$)

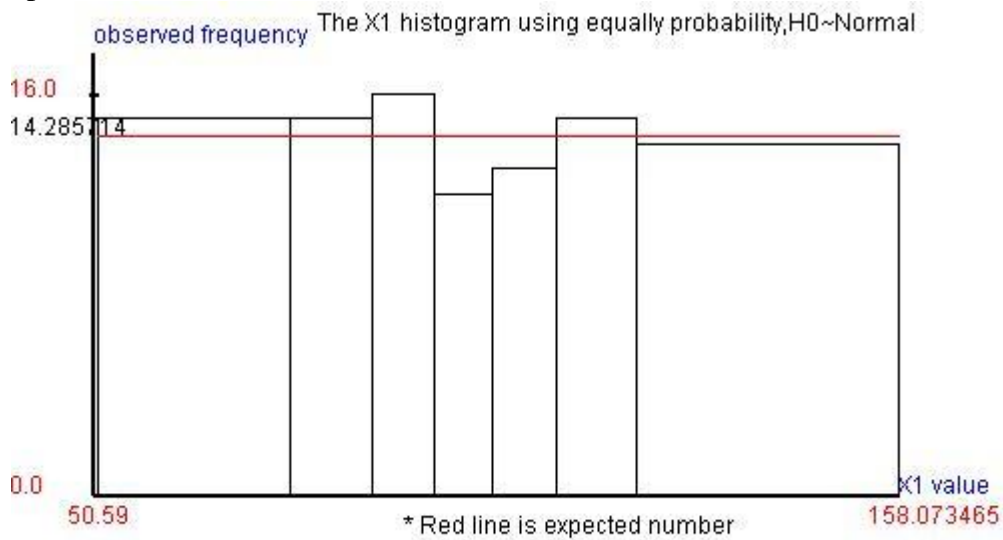


mu point estimated value=99.979013 (MLE)
 sigma point estimated value=20.910352 (MLE)
 mu value from 95.796943 to 104.161083
 sigma value from 17.425293 to 26.137940

pearson goodness of fit

class	[1]	[2]	[3]	[4]
lower limit		76.55307	87.48417	95.88801
103.73101	112.13296	123.05967		
upper limit	76.55307	87.48417	95.88801	103.73101
112.13296	123.05967			
observed no	15.00000	15.00000	16.00000	12.00000
13.00000	15.00000	14.00000		
probability	0.14286	0.14286	0.14286	0.14286
0.14286	0.14286	0.14286		
expected no	14.28571	14.28571	14.28571	14.28571
14.28571	14.28571	14.28571		
chi square	0.03571	0.03571	0.20571	0.36571
0.11571	0.03571	0.00571		

degree of freedom=4
 H0: X1~Normal(mu=99.811730,sigma*sigma=474.438810), sigma=21.781616
 pearson chi-square test statistic =0.800000
 p-value=0.938400



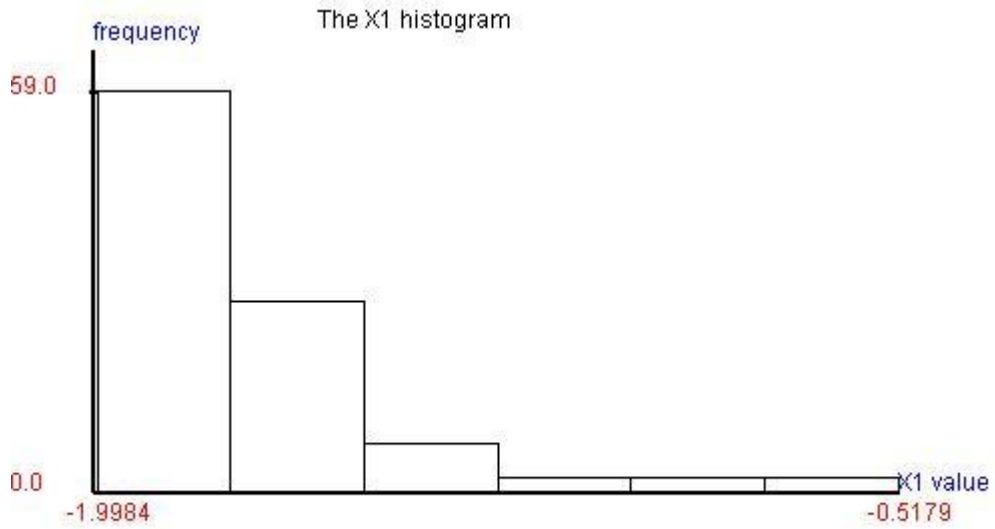
2.4.3)The population distribution is shifted exponential distribution.

X1 is Exponential($\lambda=4.000000, c=-2.000000$),

X1
-0.7599082012
-1.9229258679
-1.9702015586
-1.9019410597
-1.7236974853
-1.8047386565
-0.5179882442
-1.9603472159
-1.3024933657
-1.9072198655
-1.6868484510
-1.9943223119
-1.5952887841
-1.1434870938
-1.5850934387
-1.8735572324
-1.8914460056
-1.9984314116
-1.3546978159
-1.5861765478
-1.6248832046
-1.9969667765
-1.6074166308
-1.4459201857
-1.8895612664
-1.7270065954
-1.9311697828
-1.8847305132
-1.8425607263
-1.6666104112
-1.9466892358
-1.9833890305
-1.7854215846
-1.9750056139
-1.7292578990
-1.4493814347
-1.9294820655
-1.6980258600
-1.6453669253
-1.5215962227
-1.9645054865
-1.7652625368
-1.7115834830
-1.9859631843
-0.9394326987
-1.1491446555
-1.6654067165
-1.7064059078
-1.7383063811
-1.9958654430
-1.9079501988
-1.9870682586
-1.9846151108
-1.9754029355
-1.4709868240
-1.8188722318
-1.9284731627
-1.7527492298
-1.9923891081
-1.7933811557
-1.8956909658
-1.9615269706
-1.8589440286
-1.6418050454
-1.7975128319
-1.9266049612
-1.5281706102
-1.6258239529
-1.9432747064
-1.7196998060
-1.9620132792

-1.8867469476
 -1.7065228772
 -1.9423116636
 -1.9241863779
 -1.8816276147
 -1.8173886064
 -1.6548085887
 -1.9257828130
 -1.6434373821
 -1.9182312527
 -1.9131267021
 -1.9222583474
 -1.7997939985
 -1.7301979207
 -1.3466471307
 -1.8129000950
 -1.4627193189
 -1.8538275531
 -1.5607549109
 -1.8870107912
 -0.9769433768
 -1.8259703393
 -1.9216988824
 -1.9445003206
 -1.9532143753
 -1.5912932078
 -1.7592810883
 -1.8011078686
 -1.5094510811

X1 is Exponential(lamda=4.000000,c=-2.000000),|



lamda point estimated value=3.829464 (MLE)

c point estimated value=-1.998431 (MLE)

lamda value from 3.191220 to 4.786830

c value from -1.952784 to -2.044079

pearson goodness of fit

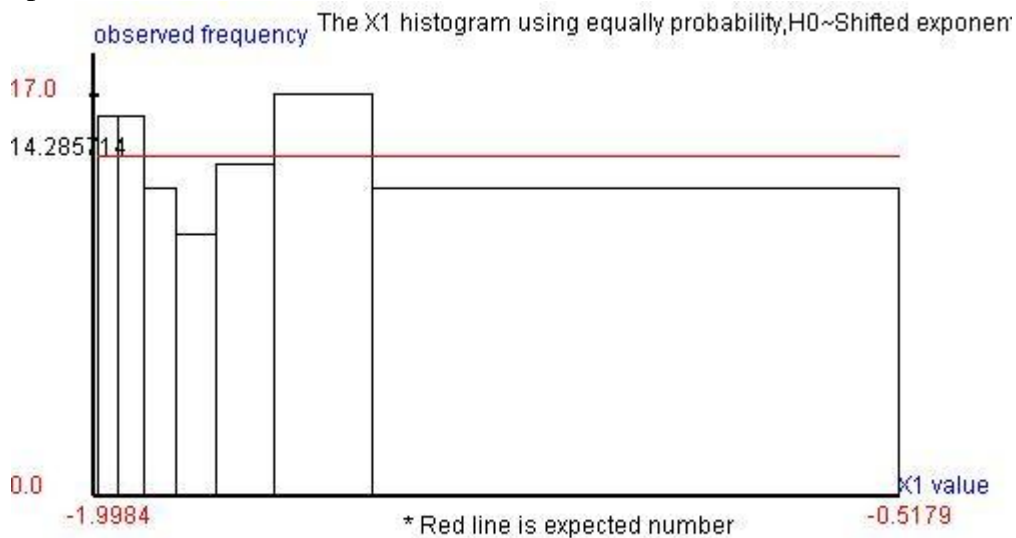
class	[1]	[2]	[3]	[4]	[5]	[6]	[7]
lower limit	-1.99843	-1.95818	-1.91057	-1.85230	-1.77717		
	-1.67129	-1.49029					
upper limit	-1.95818	-1.91057	-1.85230	-1.77717	-1.67129		
	-1.49029						
observed no	16.00000	16.00000	13.00000	11.00000			
	14.00000	17.00000	13.00000				
probability	0.14286	0.14286	0.14286	0.14286			
	0.14286	0.14286	0.14286				
expected no	14.28571	14.28571	14.28571	14.28571			
	14.28571	14.28571	14.28571				
chi square	0.20571	0.20571	0.11571	0.75571			
	0.00571	0.51571	0.11571				

degree of freedom=4

H0: $X_1 \sim \text{Shifted exponential}(\lambda=3.829464, c=-1.998431)$,

pearson chi-square test statistic =1.920000

p-value=0.750400



2.4.4)The population distribution is pareto1 distribution.

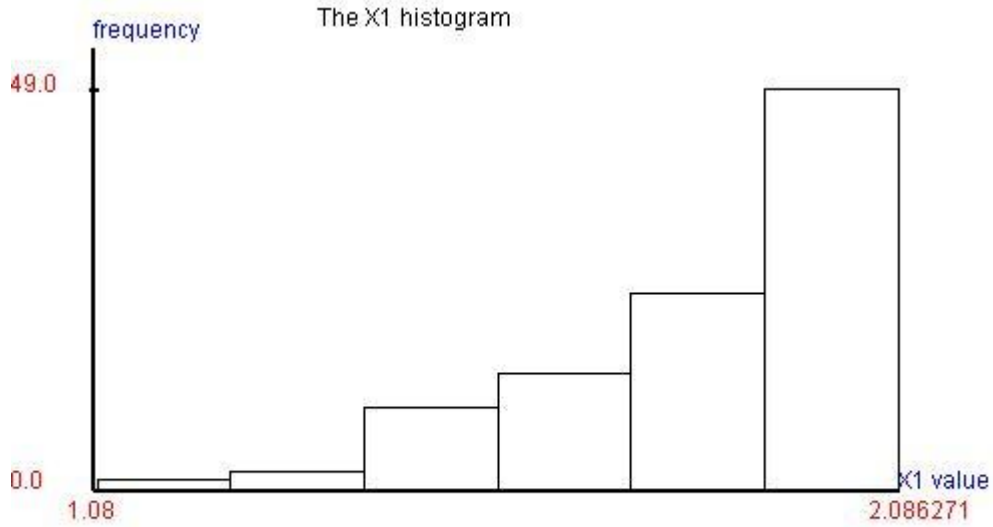
X1 is Pareto1(lamda=7.300000,c=2.100000),

X1

1.5646848298
1.9808577670
1.9533134681
2.0839551761
2.0462409071
2.0522565065
1.9671914374
1.6539312054
2.0412613268
1.9156989343
1.8722207441
1.9611773272
2.0085902289
2.0737251400
1.5381306906
1.9015447330
1.9560517758
1.4360406235
2.0160261699
1.9523557968
1.9355210205
2.0862714512
1.9078633242
1.7526339018
1.5457563997
1.8519875133
1.6546057028
1.5476461374
1.5656035871
1.9163463894
1.9144124480
1.6959210734
1.8134809694
2.0361726825
1.2539890627
1.5370502633
1.9753968653
2.0630994475
1.9331002595
1.7602581200
2.0289064850
2.0506415409
1.8089369259
1.8140012308
2.0634322073
1.6938231573
2.0492057597
2.0460449765
1.9042533091
1.9301607022
1.3978661884
1.9422797662
1.7490639903
1.9809021400
1.9368078609
2.0195284103
1.7680310080
1.8923539963
1.5578598098
1.9542874794
1.0854590304
1.9009659910
2.0843319877
1.9531611465
2.0448013619
2.0618404552
1.9590702328
1.8865390599
1.7347384500
1.7003336187
1.8876260594

1.9807770467
 2.0164582673
 2.0436965639
 2.0567071556
 1.9280662986
 1.6297488034
 2.0110454576
 2.0564529612
 1.6295299711
 2.0323969424
 1.7757237360
 1.9616182109
 1.9856390115
 1.4766654684
 1.8561291711
 1.8241708946
 1.8127940504
 2.0779125478
 1.8138927521
 1.4958806065
 1.7523949829
 1.9744356348
 1.7293999275
 2.0168153888
 1.9568482069
 1.9002720013
 1.6806507600
 1.6290002939
 1.8993817887

X1 is Pareto1(lamda=7.300000,c=2.100000),



lamda point estimated value=8.106167 (MLE)

c point estimated value=2.086271 (MLE)

lamda value from 6.755139 to 10.132709

c value from 2.046617 to 2.125926

pearson goodness of fit

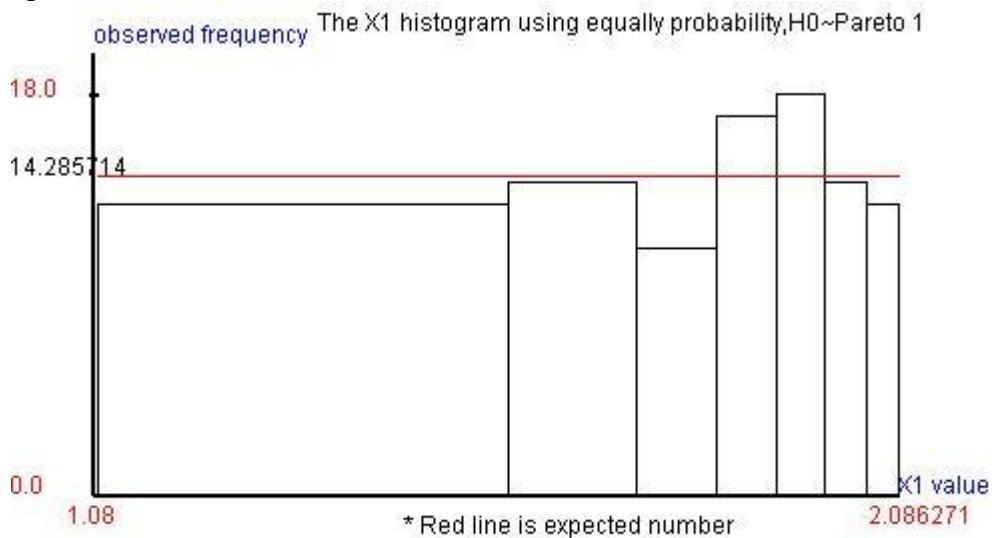
class	[1]	[2]	[3]	[4]
lower limit	0.00000	1.60054	1.76019	1.86087
1.93578	1.99595	2.04650		
upper limit	1.60054	1.76019	1.86087	1.93578
1.99595	2.04650	2.09024		
observed no	13.00000	14.00000	11.00000	17.00000
18.00000	14.00000	13.00000		
probability	0.14286	0.14286	0.14286	0.14286
0.14286	0.14286	0.14286		
expected no	14.28571	14.28571	14.28571	14.28571
14.28571	14.28571	14.28571		
chi square	0.11571	0.00571	0.75571	0.51571
0.96571	0.00571	0.11571		

degree of freedom=4

H0: $X_1 \sim \text{Pareto 1}(\lambda=7.289719, c=2.090237)$,

pearson chi-square test statistic =2.480000

p-value=0.648200



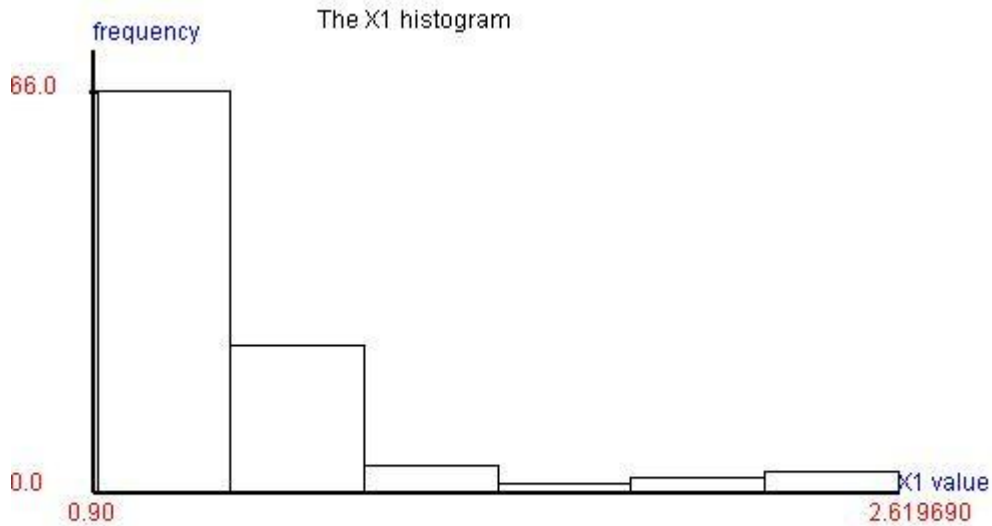
2.4.5)The population distribution is pareto2 distribution.

X1 is Pareto2(lamda=4.200000,c=0.900000),

X1
1.1776115420
1.1731498397
0.9151622623
1.0101937965
1.3907510591
0.9186196731
1.5965375059
1.2427684462
1.0255742675
1.3808845307
0.9895290292
0.9993334707
1.0252851857
1.4138133451
0.9721032082
1.1463348750
0.9167227572
2.1926579272
1.2832665351
1.0013301032
1.1234096417
1.1979003580
0.9953442610
1.0941856418
0.9971222579
0.9587871504
1.3525704642
1.3328594200
0.9621620599
2.5606968203
1.0736759813
1.4651152318
1.2036183126
1.0180292554
1.3818866547
1.2332964099
1.4565543985
0.9416528205
0.9270721691
0.9005685775
0.9525750421
1.3604256709
0.9060171291
1.1829500573
1.0929269275
1.1872465186
0.9497246476
0.9901320926
1.0303758591
0.9606924659
0.9840771520
1.1063661098
1.3633859527
1.3411935407
1.0623148227
0.9103660399
0.9785352636
1.1866948172
0.9461888829
1.1230773923
1.0533204058
1.0913820302
0.9668256992
0.9086624908
2.0495718811
1.0016681032
1.1305337801
0.9159516347
1.0737058680
1.0719973336
0.9271824527

1.2677282619
 1.2322949360
 1.0609372224
 1.5064062216
 0.9778750696
 2.6196902661
 1.3036211242
 2.5363456154
 0.9266569967
 1.4435922205
 0.9618106840
 1.0600205101
 1.0483148097
 1.4155897626
 1.3071556373
 1.6803254708
 0.9844868115
 1.0594963776
 1.0765512811
 0.9082374434
 1.0952452474
 1.5019507531
 0.9441641635
 0.9040176862
 1.2753882480
 1.0676215232
 1.8365529409
 0.9112028504
 0.9341311232

X1 is Pareto2(lamda=4.200000,c=0.900000),|



lamda point estimated value=4.092347 (MLE)

c point estimated value=0.900569 (MLE)

lamda value from 3.410289 to 5.115434

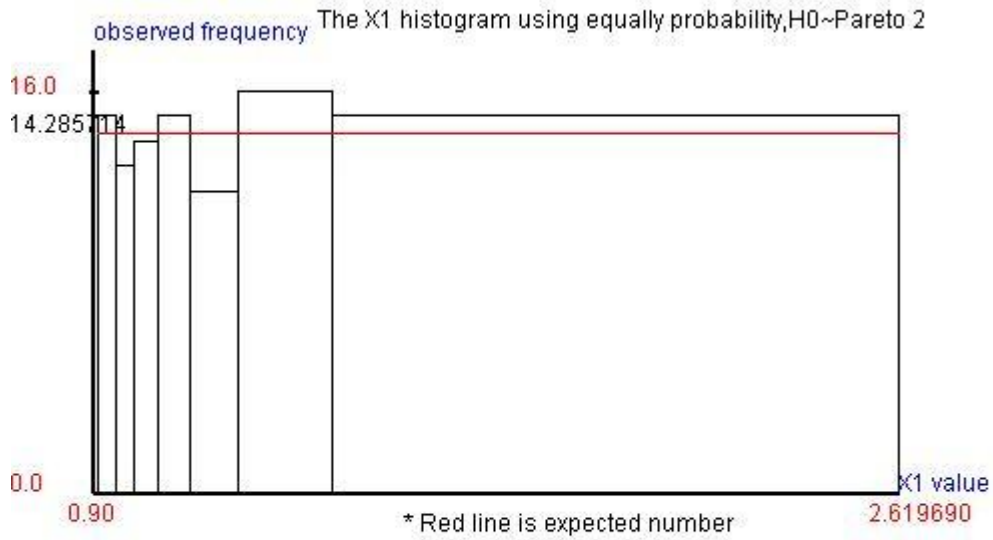
c value from 0.887312 to 0.913825

pearson goodness of fit

class	[1]	[2]	[3]	[4]
[5]	[6]	[7]		
lower limit	0.90932	0.94124	0.98045	1.03066
1.09921	1.20364	1.40565		
upper limit	0.94124	0.98045	1.03066	1.09921
1.20364	1.40565			

observed no	15.00000	13.00000	14.00000	15.00000
12.00000	16.00000	15.00000		
probability	0.14286	0.14286	0.14286	0.14286
0.14286	0.14286	0.14286		
expected no	14.28571	14.28571	14.28571	14.28571
14.28571	14.28571	14.28571		
chi square	0.03571	0.11571	0.00571	0.03571
0.36571	0.20571	0.03571		

degree of freedom=4
H0: $X_1 \sim \text{Pareto } 2(\lambda=4.467628, c=0.909318)$,
pearson chi-square test statistic =0.800000
p-value=0.938400



2.4.6)The population distribution is rayleigh distribution.

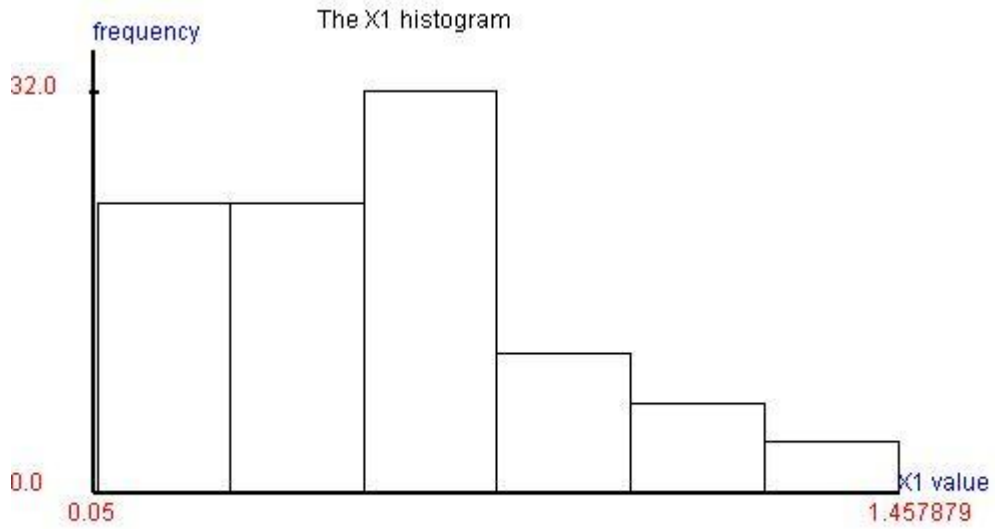
X1 is Rayleigh($\lambda=2.300000, c=0.000000$),

X1

0.7580384281
0.8282751107
0.5362654635
0.3203675641
0.1006071330
0.7119247017
0.6905046496
1.1017090980
0.7014629578
0.2558746317
0.8732048489
0.7069904026
0.9039046379
0.1604624957
0.2592989177
1.2077727873
0.6225399871
0.3657089470
0.7964389353
0.8845355714
0.5298011258
1.1342578767
0.9424031702
0.3602005439
0.1745334270
1.3160100969
0.5964436048
0.4552475329
0.3762009705
0.2409547225
0.6937356824
0.6812893028
0.6284635991
0.2144722786
0.4990913579
0.9477646589
0.3112880399
0.5801019531
0.1207527403
0.1101185601
1.0664907530
0.8441921729
1.4578794334
0.2028795915
0.2546567136
0.7240447356
0.5475261988
0.6704743067
0.5068725672
0.1144186524
0.5916951198
0.4676674068
0.4074450285
0.1594442363
0.0984849614
0.5961314502
0.5946558976
0.8732124576
0.5896304291
0.2359542136
0.6854610901
0.4442417374
0.4379637074
1.2696291987
0.1475583042
0.7769829219
1.0008622229
1.1014984129
0.2836281641
0.1741107866
0.4898397164

0.6581118368
 0.5309663003
 0.3344266402
 0.4515296187
 0.3462371297
 0.3890268539
 0.6919973498
 0.7147757925
 0.2787817242
 0.5765806824
 1.0747801970
 1.2895281364
 0.5566610079
 0.4045953729
 0.2188691813
 0.5848271615
 0.5436139921
 0.4924273191
 0.2723061317
 0.4126102795
 0.7172279881
 0.7309035087
 0.2429686523
 0.6908124898
 0.3047603257
 0.3548493267
 0.3140667326
 0.0538461916
 0.6174495566

X1 is Rayleigh(lamda=2.300000,c=0.000000),



lamda point estimated value=2.801656 (MLE)

c point estimated value=0.053846 (MLE)

lamda value from 2.334714 to 3.502071

c value from 0.059969 to 0.047723

pearson goodness of fit

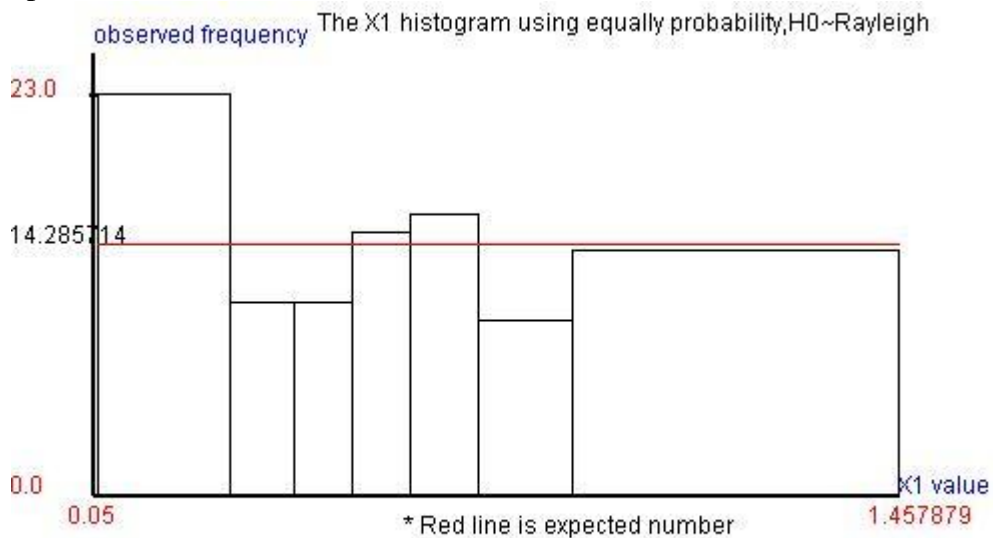
class	[1]	[2]	[3]	[4]
lower limit	0.05385	0.28841	0.40040	0.50077
0.60378	0.72254	0.88725		
upper limit	0.28841	0.40040	0.50077	0.60378
0.72254	0.88725			
observed no	23.00000	11.00000	11.00000	15.00000
16.00000	10.00000	14.00000		
probability	0.14286	0.14286	0.14286	0.14286
0.14286	0.14286	0.14286		
expected no	14.28571	14.28571	14.28571	14.28571
14.28571	14.28571	14.28571		
chi square	5.31571	0.75571	0.75571	0.03571
0.20571	1.28571	0.00571		

degree of freedom=4

H0: $X_1 \sim \text{Rayleigh}(\lambda=2.801656, c=0.053846)$,

pearson chi-square test statistic =8.360000

p-value=0.079200



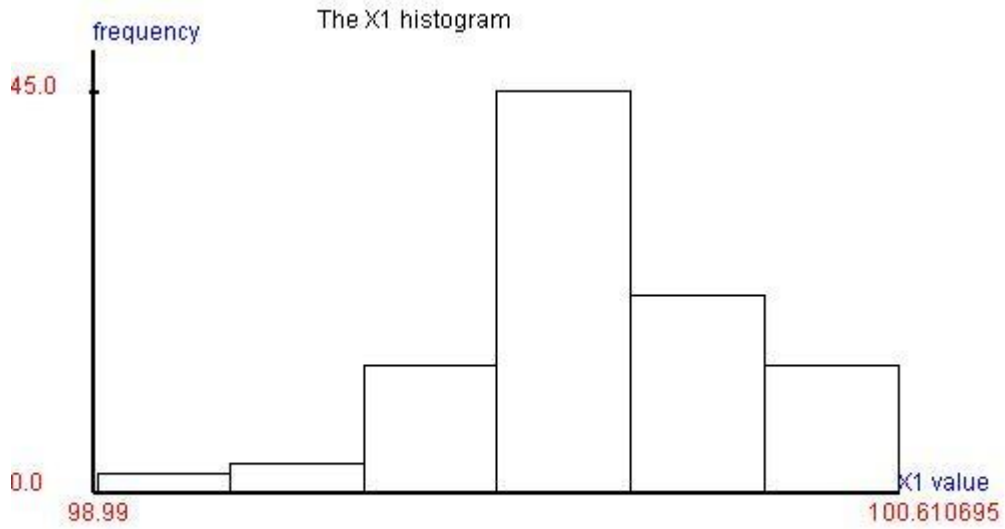
2.4.7)The population distribution is double exponential distribution.

X1 is Double Exponential($\lambda=4.000000,\mu=100.000000$),

X1
100.4596187809
99.7707646597
99.9998175604
100.4730967754
99.7164439211
99.8697312776
98.9964142495
99.9266560912
100.0985657647
100.5301026983
99.9687338594
100.2502383447
100.2463268456
99.9781818692
100.1536853763
100.1010942558
99.9704641939
99.2203435381
99.6952278752
100.3177566818
99.9285309338
100.0114537350
99.3845707068
99.8830818733
100.0382364119
100.3687196855
100.3998921831
100.1313001666
100.0430776004
99.6516132512
100.0730330729
100.0019183077
99.9002123936
99.9522867974
99.6647870113
100.1822950268
99.5821429378
99.9855428673
99.8528911042
100.2725099089
100.2570370541
100.1270389200
99.9005381628
99.5569398249
100.0808266665
100.1128518677
100.4351459689
100.3420201115
100.0037927363
100.1792004142
99.6942715656
100.0331242431
100.3745583078
100.3069837875
100.0004786570
100.0462518661
99.7972296313
100.1677971740
99.8874827109
100.0502881638
99.7456760120
100.0951319950
99.7283948966
99.2711857671
99.9072681106
100.0620165026
99.9905289582
99.9787615525
100.3783999787
99.9155843006
99.7896664255

99.9515518847
 99.8371602964
 100.3579838235
 99.8456334095
 99.9676370711
 100.0242505538
 100.1101122991
 99.9919302539
 99.8783612711
 100.4067257644
 100.0279071472
 99.9836413490
 99.5296818689
 99.8856010054
 99.9116824576
 100.0858616087
 100.6106951954
 99.9358244952
 100.0372075467
 99.8458475592
 99.8067032687
 100.2551396907
 99.9283708575
 99.5653877135
 99.5362963974
 100.5757579670
 99.9636616873
 100.3321915464
 100.3470197981

X1 is Double Exponential(lamda=4.000000,mu=100.000000),

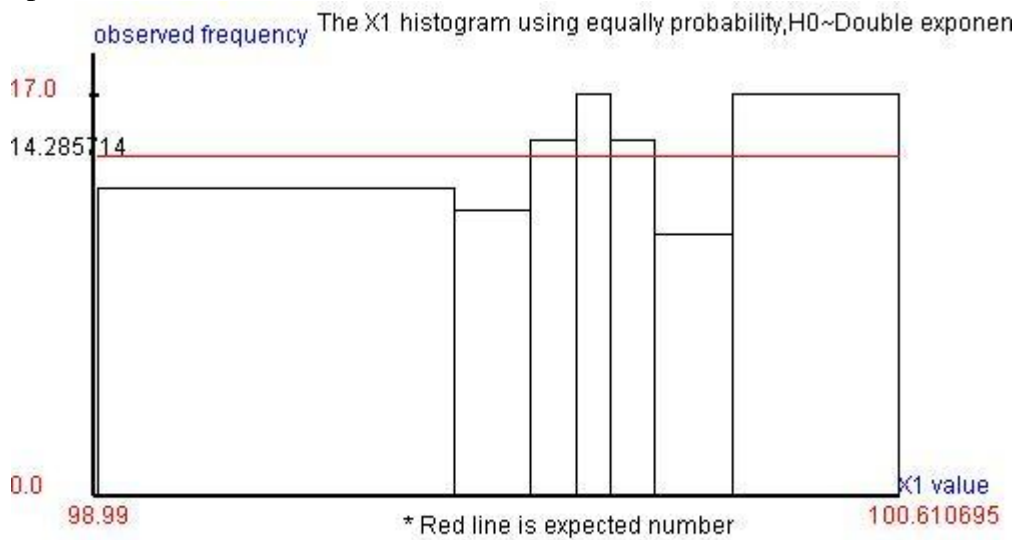


lamda point estimated value=4.773403 (MLE)
 mu point estimated value=99.995874 (MLE)
 lamda value from 3.977836 to 5.966754
 mu value from 97.979987 to 102.011760

pearson goodness of fit

class	[1]	[2]	[3]	[4]
lower limit		99.71558	99.87067	99.96138
100.03036	100.12108	100.27617		
upper limit	99.71558	99.87067	99.96138	100.03036
100.12108	100.27617			
observed no	13.00000	12.00000	15.00000	17.00000
15.00000	11.00000	17.00000		
probability	0.14286	0.14286	0.14286	0.14286
0.14286	0.14286	0.14286		
expected no	14.28571	14.28571	14.28571	14.28571
14.28571	14.28571	14.28571		
chi square	0.11571	0.36571	0.03571	0.51571
0.03571	0.75571	0.51571		

degree of freedom=4
 H0: X1~Double exponential(lamda=4.469479,mu=99.995874),
 pearson chi-square test statistic =2.340000
 p-value=0.673400



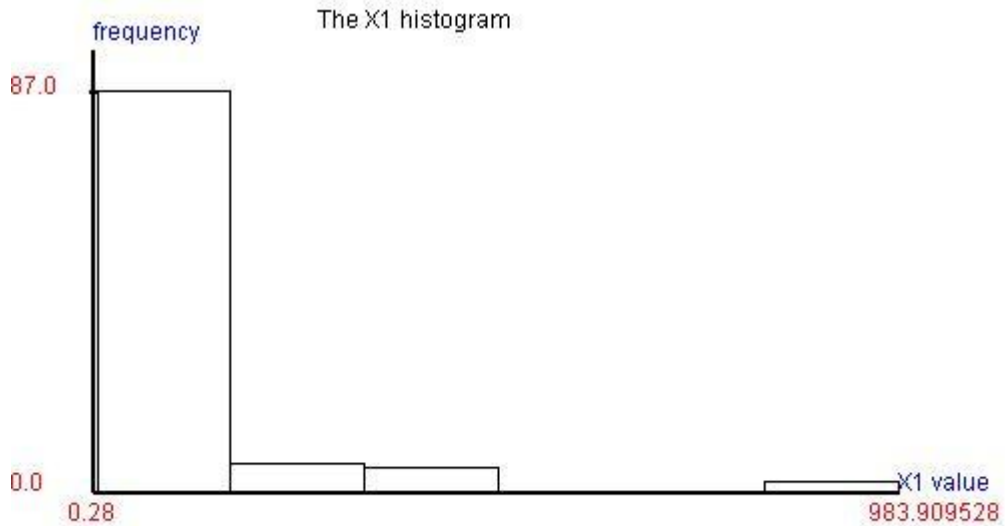
2.4.8)The population distribution is lognormal distribution.

X1 is Log normal($\mu=3.000000,\sigma=2.000000$),

X1
12.9731415093
1.5551114341
8.7995475548
65.5421351415
20.2343124193
8.7819362732
145.5677981742
3.5231444143
983.9095288162
18.8559561145
4.0687636340
341.9827673688
3.4936509475
107.9860490486
80.4688549064
1.5552378989
54.0474652809
490.0105298963
454.6008620797
872.4887124264
187.8314156083
2.2774671071
196.1647250851
5.0984636113
1.5296756433
11.4941711720
0.7387722300
26.7318120118
6.5133069283
72.2715474244
118.0279193265
18.9343347280
14.3052191817
208.3213074486
2.5952546222
9.9731726068
2.5939661896
14.2521529496
48.2459135788
32.3908731476
4.1986685564
94.9427562942
0.5215254141
74.0237759173
2.0617288624
9.9103327230
34.0427104475
309.6178280449
11.2601850691
0.2814238976
6.7503050275
23.0440234147
416.1566824195
211.3848810953
6.0343817796
45.7257244778
58.7678539450
0.9828132083
31.1963182496
9.4750328572
27.2761979666
2.3617413138
0.8621921883
83.3486528958
12.2348738083
134.9166644109
0.5188204116
12.3049959877
25.1556222520
26.3313304600
29.6621909095

2.9957893662
 256.0420838598
 109.5894331164
 1.6368342577
 11.3512212499
 12.9011183691
 145.6921527472
 72.4615976462
 11.0866271310
 56.4195608020
 8.0986147107
 62.2553243460
 2.9487530661
 2.4028813343
 1.9701854560
 4.1749164066
 73.4827053770
 3.1052282752
 70.8753852878
 2.8853710961
 0.2954235550
 51.7209143218
 8.4800917867
 5.5272456581
 65.7059835336
 1.2075802968
 3.7128367288
 110.2075022608
 410.4337260331

X1 is Log normal($\mu=3.000000$, $\sigma=2.000000$),

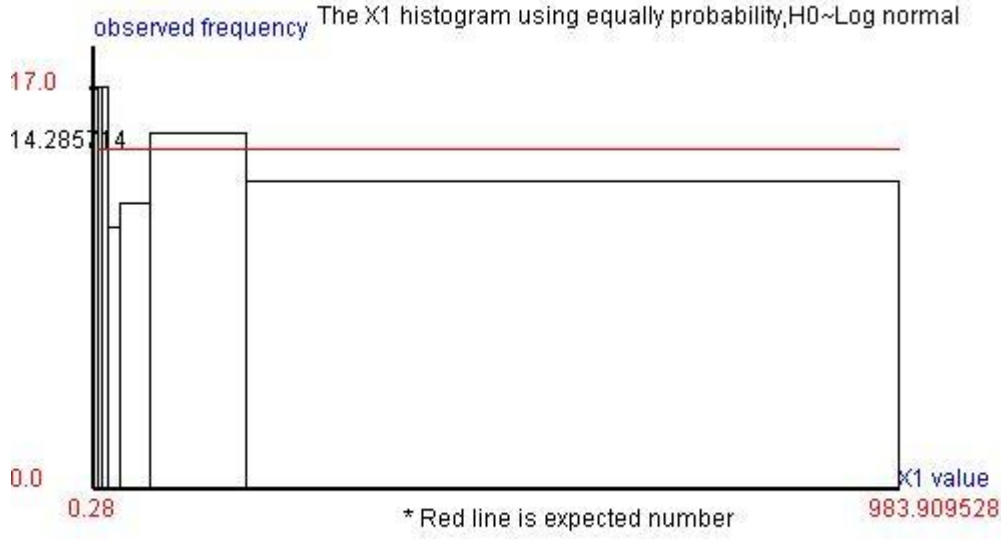


mu point estimated value=2.855866 (MLE)
 sigma point estimated value=1.912250 (MLE)
 mu value from 2.473416 to 3.238315
 sigma value from 1.593541 to 2.390312

pearson goodness of fit

class	[1]	[2]	[3]	[4]
lower limit		2.29120	6.42833	14.20846
29.78577	65.82341	184.60133		
upper limit	2.29120	6.42833	14.20846	29.78577
65.82341	184.60133			
observed no	15.00000	17.00000	17.00000	11.00000
12.00000	15.00000	13.00000		
probability	0.14286	0.14286	0.14286	0.14286
0.14286	0.14286	0.14286		
expected no	14.28571	14.28571	14.28571	14.28571
14.28571	14.28571	14.28571		
chi square	0.03571	0.51571	0.51571	0.75571
0.36571	0.03571	0.11571		

degree of freedom=4
 H0: X1~Log_Normal(mu=3.024144,sigma*sigma=4.225773),sigma=2.055668
 pearson chi-square test statistic =2.340000
 p-value=0.673400



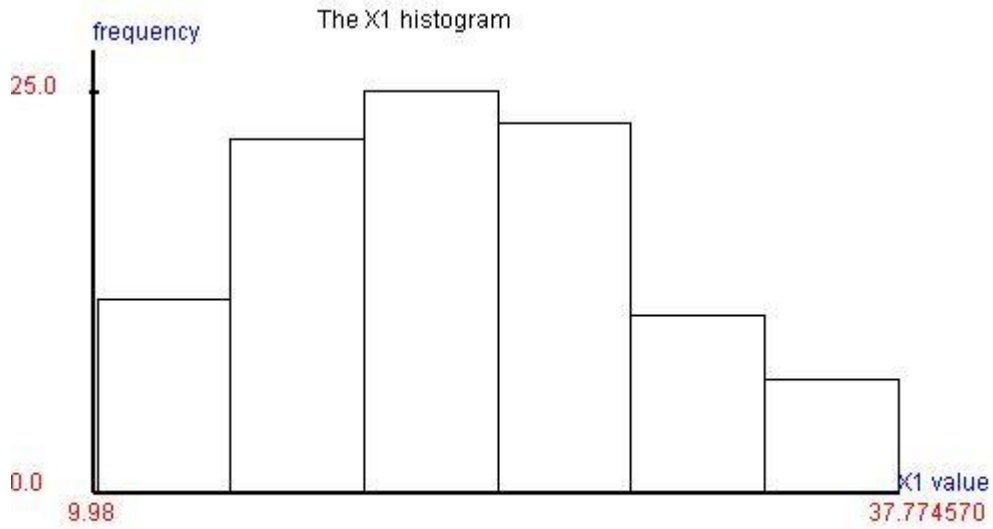
2.4.9)The population distribution is gamma distribution.

X1 is Gamma(alpha=11.000000,beta=2.000000),

X1
24.1230556839
32.3114072610
29.9127802076
21.3047881723
24.9112279539
17.8992360336
12.5278002462
23.4231706894
20.3919829389
17.8519267530
33.9463909080
19.7079158803
21.1937357199
12.1198920361
12.3235885356
29.5584277084
27.4432036356
12.0978834925
35.8966740643
24.9492895016
17.0069885590
18.0755499406
17.2313356741
16.3597201542
25.2948126372
24.2058897733
24.3051325016
26.3888389486
17.7288474708
9.9851912879
23.9520338113
14.9899294707
19.3614350635
29.3758437144
20.8552545210
21.9873102458
20.5668616479
31.0214489838
19.2793687679
16.3171673653
29.3675380597
28.2115938541
14.9002342095
34.1554191859
28.5672227693
15.2569178502
15.3119569063
37.7745709170
26.2945303297
23.7551017346
20.0954384363
21.7592955641
14.2154347826
26.3389504056
35.7081219934
26.5261883695
25.1795225036
18.7114705045
11.4781977573
28.3460492036
25.4482050083
21.7771300984
18.9747275597
26.3124982057
31.6868956855
17.9070563806
15.9275102450
28.5773291041
27.4732708465
22.2870444036
26.5430526222

21.9630581875
 30.8381419825
 33.4008879614
 24.3096252418
 12.0295385766
 22.8151388932
 18.6234813147
 14.7406679261
 13.9387871195
 18.7899081499
 21.0819758289
 13.1875558866
 14.1575050795
 27.2032329656
 21.4406546019
 32.0096645610
 19.4625225025
 20.5942938877
 22.0532199517
 26.5921120181
 15.8899138967
 23.0647171706
 15.5617435045
 35.0918150276
 23.0170972866
 12.7508597983
 26.5117938873
 17.4933694032
 23.6246911924

X1 is Gamma(alpha=11.000000,beta=2.000000),

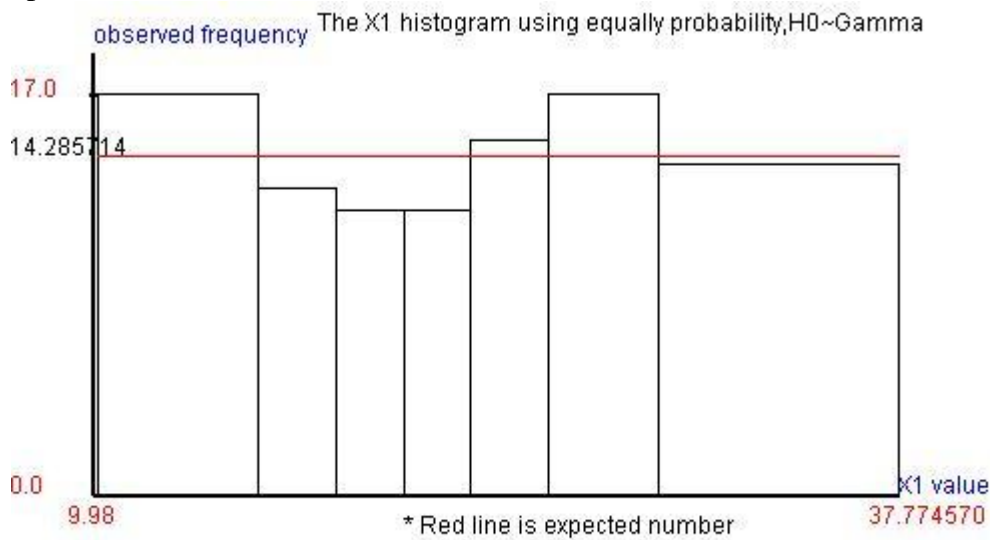


alpha point estimated value=12.000000 (MME)
 beta point estimated value=1.867088 (MME)
 alpha values are 11.500000, 12.000000 and 12.500000
 beta value from 1.493670 to 2.240506

pearson goodness of fit

class	[1]	[2]	[3]	[4]
lower limit	9.98519	15.54983	18.29715	20.62048
22.96068	25.65636	29.46286		
upper limit	15.54983	18.29715	20.62048	22.96068
25.65636	29.46286			
observed no	17.00000	13.00000	12.00000	12.00000
15.00000	17.00000	14.00000		
probability	0.14286	0.14286	0.14286	0.14286
0.14286	0.14286	0.14286		
expected no	14.28571	14.28571	14.28571	14.28571
14.28571	14.28571	14.28571		
chi square	0.51571	0.11571	0.36571	0.36571
0.03571	0.51571	0.00571		

degree of freedom=4
 H0: $X_1 \sim \text{Gamma}(\alpha=11.500000, \beta=1.949240)$,
 pearson chi-square test statistic =1.920000
 p-value=0.750400



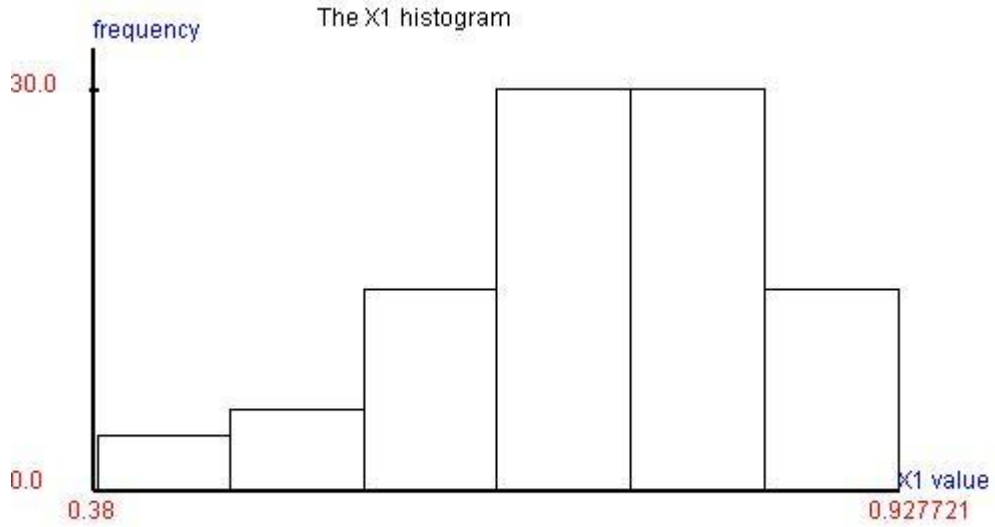
2.4.10)The population distribution is beta distribution.

X1 is Beta(alpha=10.000000,beta=4.000000),

X1
0.8908614494
0.7298336193
0.7552517110
0.6578450372
0.7704437990
0.6407354591
0.7572030663
0.4675236684
0.7745815165
0.6156419050
0.6239323929
0.5084337471
0.7155399325
0.7809078245
0.7550384967
0.6975314627
0.8447847777
0.6430022097
0.6843780380
0.8292504274
0.6874900003
0.8451829098
0.7287141615
0.6065945044
0.6808073222
0.6993859441
0.7062481729
0.7868830380
0.7609458471
0.8779229443
0.9134522025
0.8194607884
0.7154322816
0.7686762840
0.7283987920
0.6920120294
0.9277212226
0.7799022162
0.6731490773
0.6919744146
0.8373589816
0.6199580780
0.7851276773
0.7568659177
0.6797730014
0.7762155098
0.7435989562
0.6973778703
0.6826346702
0.7634774063
0.5131396760
0.6984635616
0.3870599030
0.5259342215
0.6167603885
0.5796656204
0.7516678448
0.8804645691
0.7236890204
0.6533406850
0.7371086816
0.7975208300
0.8447365957
0.7028691658
0.9074973949
0.8830153050
0.4224510422
0.6735425516
0.7319515756
0.8788568786
0.7225383842

0.7660632898
 0.5039364433
 0.7191484468
 0.6130779704
 0.8021047182
 0.7717882952
 0.6347833465
 0.7680077834
 0.6102771968
 0.7354804286
 0.8139402082
 0.7984347090
 0.5603100056
 0.8887460589
 0.8379358740
 0.7160784493
 0.7971471617
 0.6676839981
 0.7546514726
 0.8918257868
 0.8882831399
 0.6193769984
 0.6347167632
 0.5608993044
 0.6414480480
 0.4169818465
 0.7652191582
 0.7772942972
 0.7976579085

X1 is Beta(alpha=10.000000,beta=4.000000),

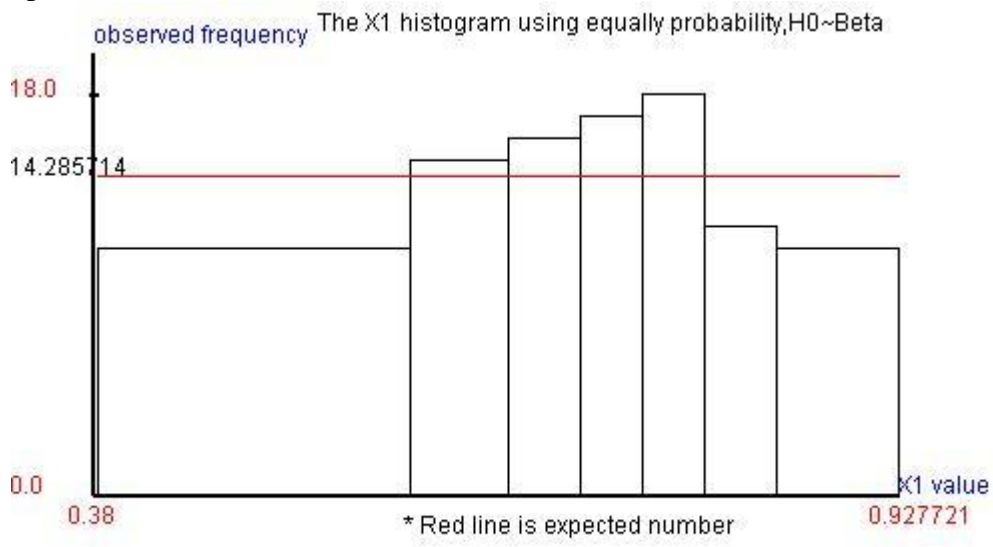


alpha point estimated value=10.500000 (MME)
 beta point estimated value=4.000000 (MME)
 alpha values are 10.000000 ,10.500000 and 11.000000,
 beya values are 3.500000 ,4.000000 and 4.500000,

pearson goodness of fit

class	[1]	[2]	[3]	[4]
lower limit	0.08855	0.59879	0.66488	0.71320
0.75548	0.79735	0.84561		
upper limit	0.59879	0.66488	0.71320	0.75548
0.79735	0.84561	0.99787		
observed no	11.00000	15.00000	16.00000	17.00000
18.00000	12.00000	11.00000		
probability	0.14286	0.14286	0.14286	0.14286
0.14286	0.14286	0.14286		
expected no	14.28571	14.28571	14.28571	14.28571
14.28571	14.28571	14.28571		
chi square	0.75571	0.03571	0.20571	0.51571
0.96571	0.36571	0.75571		

degree of freedom=4
 H0: $X_1 \sim \text{Beta}(\alpha=10.500000, \beta=4.000000)$,
 pearson chi-square test statistic =3.600000
 p-value=0.462800



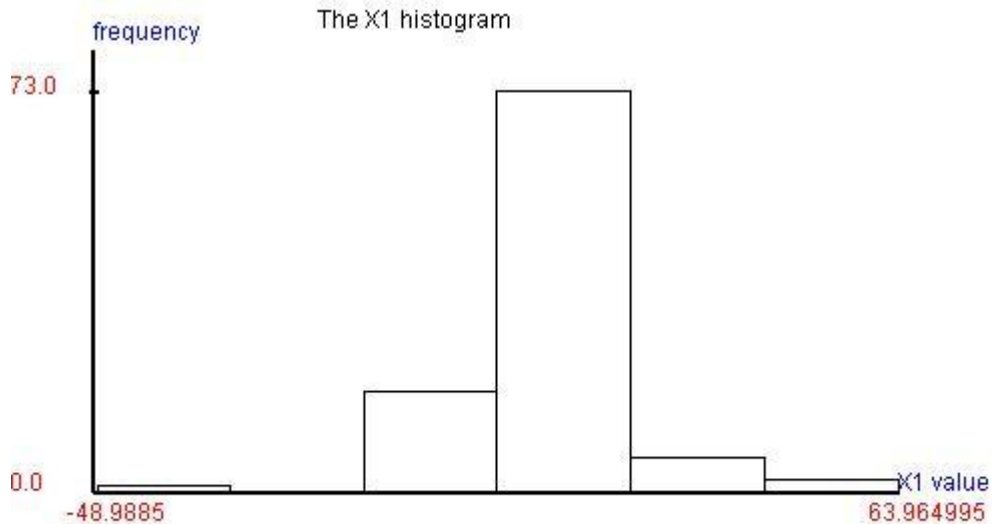
2.4.11)The population distribution is cauchy distribution.

X1 is Cauchy($\mu=10.000000,\sigma=2.000000$),

X1
11.1557788395
-1.4221559094
9.8710177410
11.7814492932
10.7082291597
11.7006280744
8.6873231746
6.4494478344
-2.9722305907
8.2492684931
10.9453213712
13.8824114091
7.1434744414
9.9282045691
7.3993746612
11.8599045817
10.9097315139
23.3337160813
13.2909469331
11.3207637235
8.1318735612
16.2651249148
4.2552119493
17.4724345111
10.0302674858
19.4910981658
11.3493907279
20.2148146535
-4.3854751301
8.7518794146
10.1959161864
13.7995630273
17.2388227928
13.0969304020
8.6818992415
1.3087418247
49.5939200067
8.3425617007
9.0820299458
-6.1407292316
10.4737605882
13.4233750656
9.4348731502
6.9683182998
8.7937841693
14.6130699351
7.1252420072
7.7788326247
11.0993706238
10.7378570488
10.0987073406
10.8999296430
63.9649953206
32.7376868434
25.9768219465
24.8840561187
8.1746922980
11.4282944259
33.7450492768
11.7791648720
9.0917635516
9.1488589874
8.2949218671
9.3005207032
8.9375486620
6.3428805188
-5.0264864545
30.2687193395
-0.0288083821
9.2159652704
39.7441516535

8.5347200720
 15.1181344764
 10.1838334460
 13.0775215364
 10.3269344865
 -48.9885816663
 36.7476565190
 5.1631096055
 9.9662641782
 19.1705164545
 9.2649789868
 -5.5602435666
 11.8766826052
 9.7619320648
 8.9946225830
 10.1445056465
 5.2144423769
 10.2541543360
 12.0116131409
 10.5361821904
 11.1112705219
 28.0385319761
 11.4080900630
 8.6240208798
 11.0278886860
 9.9086996343
 1.6615270322
 23.7303736172
 14.6695592946

X1 is Cauchy(mu=10.000000,sigma=2.000000),



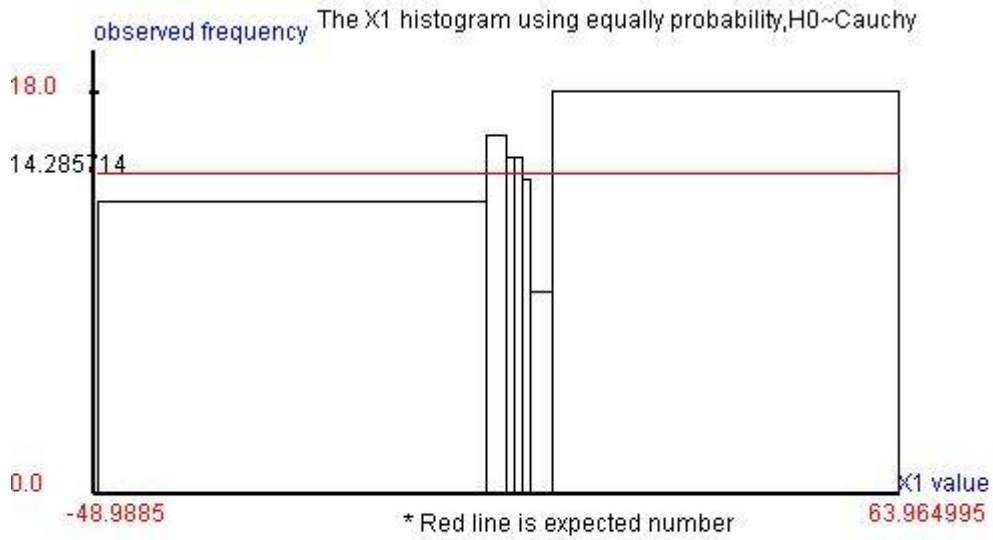
mu point estimated value=10.225035
 sigma point estimated value=2.426153
 mu value from 9.739805 to 10.710266
 sigma value from 2.021794 to 3.032691

pearson goodness of fit

class	[1]	[2]	[3]	[4]
[5]	[6]	[7]		
lower limit		5.81724	8.68767	9.96513
10.98958	12.26704	15.13747		
upper limit	5.81724	8.68767	9.96513	10.98958
12.26704	15.13747			

observed no	13.00000	16.00000	15.00000	15.00000
14.00000	9.00000	18.00000		
probability	0.14286	0.14286	0.14286	0.14286
0.14286	0.14286	0.14286		
expected no	14.28571	14.28571	14.28571	14.28571
14.28571	14.28571	14.28571		
chi square	0.11571	0.20571	0.03571	0.03571
0.00571	1.95571	0.96571		

degree of freedom=4
H0: $X_1 \sim \text{Cauchy}(\mu=10.477355, \sigma=2.244192)$,
pearson chi-square test statistic =3.320000
p-value=0.505700



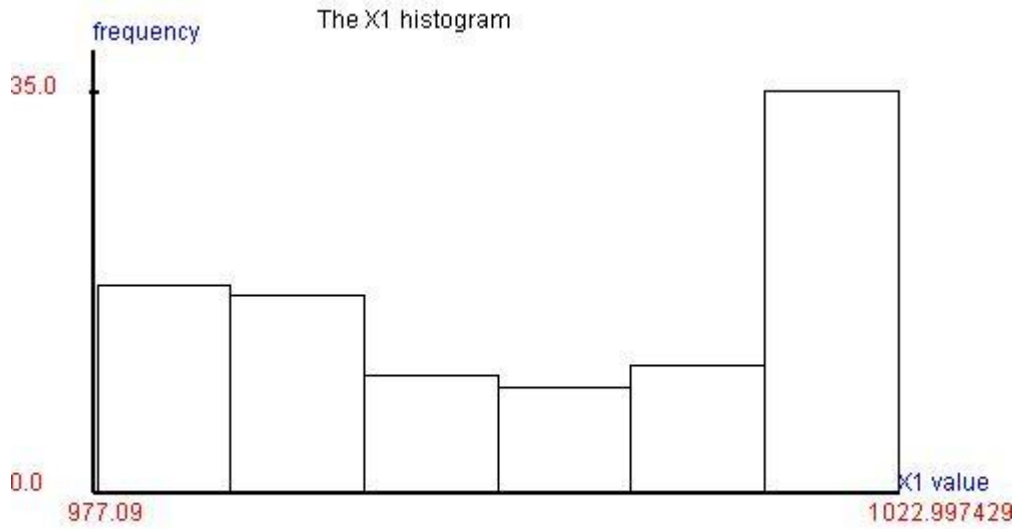
2.4.12)The population distribution is arcsin distribution.

X1 is $\text{Arcsin}(\mu=1000.000000,c=23.000000)$,

X1
1019.3485463152
1018.8761128712
1009.4497846537
997.2813778938
985.8373093406
977.6603763829
1022.9974293944
1018.3689359592
986.4091114154
977.0969871649
1022.7110530238
988.7011293205
1017.6008592756
985.8763563337
1004.8486023236
989.8162507500
1022.5270958791
1016.9285974068
981.4302884511
992.4414771151
984.4991176612
1009.6499905230
998.6951818927
985.4869355124
1002.7667645635
991.6395921468
987.9557984478
978.9156883498
1019.0688992431
1015.9727840898
1022.3447134616
988.1944687189
994.1766041797
1016.6229313224
1016.5697548285
1003.3819879973
1022.9425246582
1015.3274009881
1005.6354072368
1015.7120208587
991.9502163224
984.4116258171
995.0459865722
979.1824794327
1019.5795058070
982.4770625812
1009.2955764247
984.4442668639
1016.3683189475
1017.3577298985
1017.9340564778
1022.0326120682
1022.1224671760
989.1650759242
1017.2607289786
979.4755793241
988.0780790095
1010.9439265989
1016.4089735685
989.4112455482
986.5322785998
1014.3160356873
980.4904293195
979.3078508999
994.3034931065
1018.3024304800
1017.3012598304
997.0419856799
1003.7373092418
996.9242139157
1002.9142160701

999.7090844739
 1020.2886675422
 1003.6334530546
 1017.4786518966
 1015.6942249183
 1014.9696785507
 981.2775763253
 977.2192150075
 1022.7558746231
 1022.0408531132
 1018.7609358392
 981.3732397075
 980.8193953228
 991.6899186022
 977.3657165386
 991.1279924092
 1016.9909942050
 994.4530392480
 1022.3716260981
 1022.9592013971
 986.3719226275
 1005.9098973457
 1011.0211968219
 1014.4276851863
 1012.1477178036
 1013.4402776850
 1018.1201729052
 1003.9297115117
 977.6047176665

X1 is Arcsin(mu=1000.000000,c=23.000000),

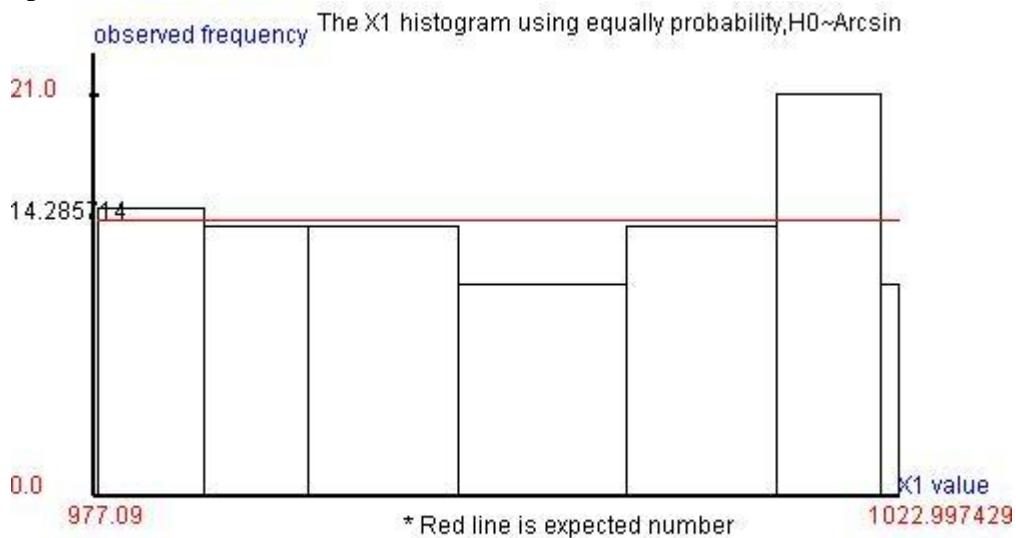


mu point estimated value=1002.518359
 c point estimated value=22.950221
 mu value from 997.928315 to 1007.108403
 c value from 19.125184 to 28.687776

pearson goodness of fit

class	[1]	[2]	[3]	[4]
lower limit	977.09699	983.22506	989.19526	997.82244
upper limit	1007.39788	1016.02506	1021.99525	1022.99743
observed no	15.00000	14.00000	14.00000	11.00000
expected no	14.28571	14.28571	14.28571	14.28571
chi square	0.03571	0.00571	0.00571	0.75571

degree of freedom=4
 H0: $X_1 \sim \text{Arcsin}(\mu=1002.610160, c=21.515832)$,
 pearson chi-square test statistic =4.720000
 p-value=0.317200



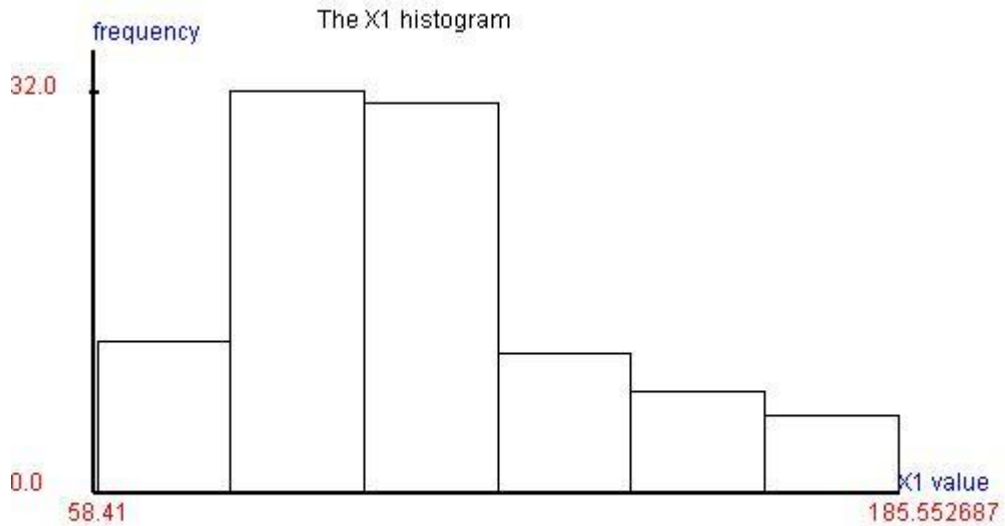
2.4.13)The population distribution is gumbel distribution.

X1 is Gumbel($\mu=100.000000,\sigma=23.000000$),

X1
87.0294658430
90.4683007880
132.8542331747
79.1041322557
84.1551396920
112.7192438625
117.9396060194
105.4150092826
97.8670373146
94.6121326857
145.9710108343
145.0585673575
133.9161401179
118.1569335531
115.0606075068
88.6436797754
91.2469911890
91.7567061293
102.4400331836
93.9633450556
100.7240677685
89.5155912032
115.7948688668
99.5866263083
185.5526872187
150.7569954866
170.0461392687
89.1741360868
99.2237142766
95.1559929376
80.7788222693
124.5849972228
121.2095935139
62.2904231308
115.8008248640
95.1332959243
128.3307730345
154.9180931654
115.8963825562
132.5340006794
111.3195152538
101.6761116317
81.0104185936
70.5696578637
93.0748130483
91.0521216593
143.9066738991
74.3938124795
75.6777397491
79.2498588809
175.1250068908
147.0412783561
102.2092032292
120.0782897063
166.1472917195
58.4157265189
74.6814427744
103.3941110718
115.7483954714
102.9154559724
117.2208573372
61.5232319534
101.6613507473
96.7962452493
142.3826327858
88.8889087639
106.4048514408
104.1079289187
103.1926275577
114.0051905678
98.0937254364

168.8219046191
 103.1815041116
 131.6445931622
 85.3686578013
 79.5048829160
 106.5822081751
 97.8019087405
 116.5527572015
 94.4963431866
 104.4796026353
 90.5247396927
 109.7022791668
 63.4007366722
 130.9269332831
 179.7004346810
 152.7486583579
 90.6428266078
 107.0143675646
 102.9181943847
 140.4291425093
 103.7429957400
 127.0864521837
 78.8878950599
 141.2950347748
 99.4851113023
 80.2493583940
 88.2141015644
 155.2333927041
 96.3857450009

X1 is Gumbel(mu=100.000000,sigma=23.000000),

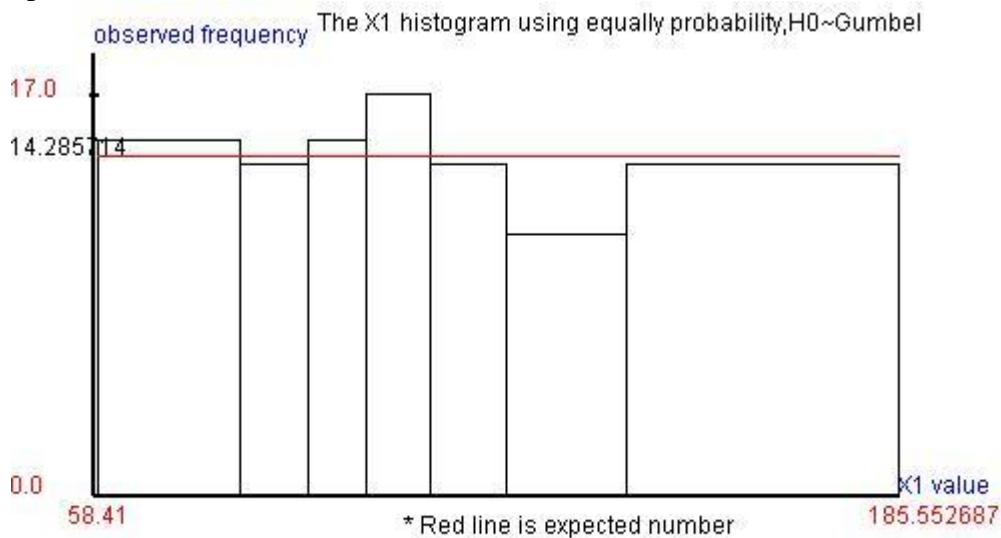


mu point estimated value=96.646091
 sigma point estimated value=21.475221
 mu value from 92.351046 to 100.941135
 sigma value from 17.896018 to 26.844026

pearson goodness of fit

class	[1]	[2]	[3]	[4]
lower limit		81.16361	91.80299	101.25072
111.27217	123.56302	142.42166		
upper limit	81.16361	91.80299	101.25072	111.27217
123.56302	142.42166			
observed no	15.00000	14.00000	15.00000	17.00000
14.00000	11.00000	14.00000		
probability	0.14286	0.14286	0.14286	0.14286
0.14286	0.14286	0.14286		
expected no	14.28571	14.28571	14.28571	14.28571
14.28571	14.28571	14.28571		
chi square	0.03571	0.00571	0.03571	0.51571
0.00571	0.75571	0.00571		

degree of freedom=4
 H0: X1~Gumbel(mu=97.247397,sigma=24.159624),
 pearson chi-square test statistic =1.360000
 p-value=0.851100



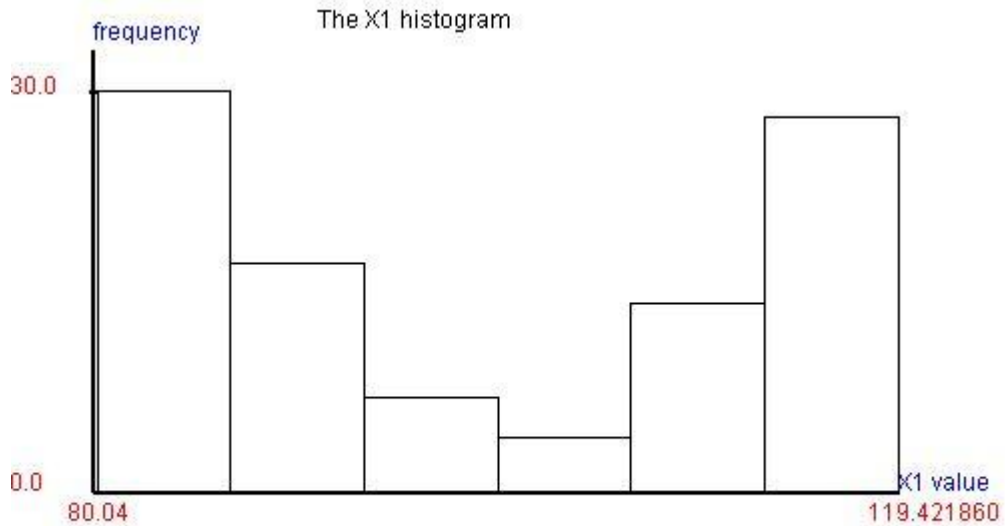
2.4.14)The population distribution is triangular 1 distribution.

X1 is Triangular1($\mu=100.000000,c=20.000000$),

X1
80.9594288971
119.4218605307
87.8006672850
83.2027612626
102.4281927741
89.5589853976
104.5840739178
86.6523472272
89.0904543576
89.1905506133
118.9687987803
86.4901647688
83.3454655948
81.2550356873
83.5137417647
117.5269363091
116.0848602895
117.1752148537
90.0160818201
117.7140545190
114.1741821170
113.5252553914
83.5419433157
94.6224525780
81.5636329746
108.1914768749
86.3039238272
110.7089910007
81.6195410993
118.3454749627
107.6214265934
119.2960527750
112.7886684067
93.3277151363
84.1868196992
84.3378779232
84.7306846850
117.4306042690
110.6490498418
84.0663377819
118.3456836420
117.8249306428
85.1954252535
117.1738914417
83.1035255149
83.5833633447
93.4818402852
90.2182308991
83.7430113731
103.5917582706
109.3283456360
86.6922048585
92.5007036378
86.3609664462
104.1131127964
118.3121584524
114.3753298695
110.0824198689
88.3587436388
118.7856290106
115.2397579598
85.9732245842
95.7775269219
117.4399182015
96.0043555717
118.6075606498
114.9452723062
118.0443148639
114.0240529861
89.9397972317
110.3877676714

112.3404736542
 85.5655320297
 82.9844563767
 89.4072532982
 90.4258007388
 85.6070514566
 112.3573968369
 110.6196846397
 114.0644156334
 111.9286312050
 107.7242943141
 82.5635435787
 89.7530762996
 86.4216423579
 112.9857487162
 82.0976646107
 80.9043851693
 115.4231982752
 116.1482119095
 90.0077531861
 86.0716639499
 98.9539763180
 80.0429804085
 84.3449338687
 115.3698095745
 108.3238427348
 89.1491260207
 88.4546959433
 96.5837626295

X1 is Triangular1(mu=100.000000,c=20.000000),

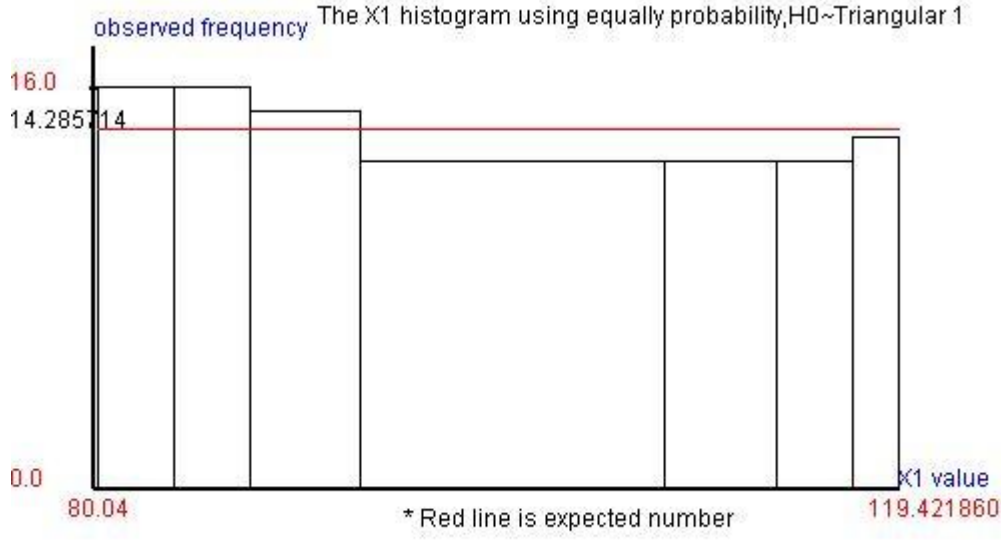


mu point estimated value=99.241916
 c point estimated value=19.689440
 mu value from 95.304028 to 103.179804
 c value from 16.407867 to 24.611800

pearson goodness of fit

class	[1]	[2]	[3]	[4]
lower limit	80.04298	83.79209	87.55857	93.02912
upper limit	107.97496	113.44551	117.21199	119.42186
observed no	16.00000	16.00000	15.00000	13.00000
expected no	14.28571	14.28571	14.28571	14.28571
chi square	0.20571	0.20571	0.03571	0.11571

degree of freedom=4
 H0: X1~Triangular 1(mu=100.502040,c=19.771479),
 pearson chi-square test statistic =0.800000
 p-value=0.938400



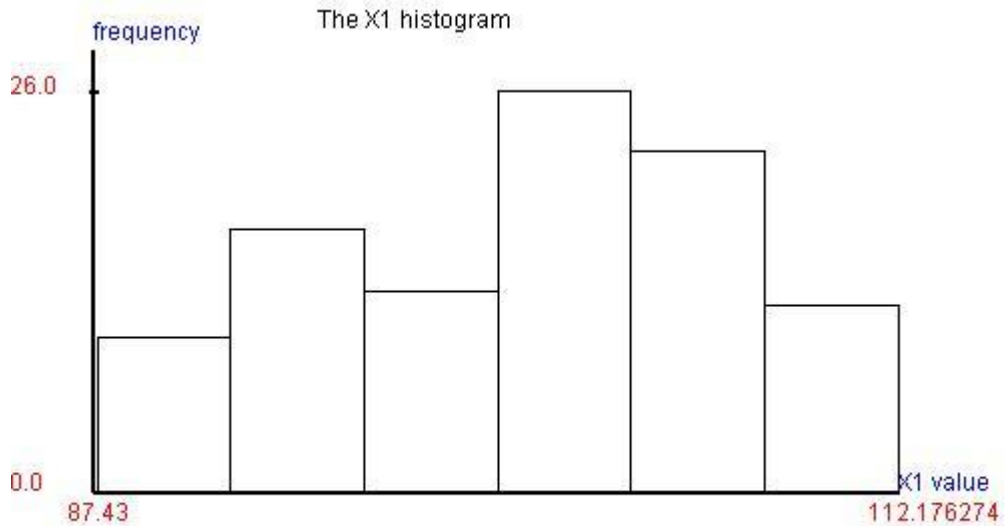
2.4.15)The population distribution is trapezoid distribution.

X1 is Trapezoid($\mu=100.000000,c=10.000000$),

X1
101.7526323802
89.1201052084
88.4587184617
99.0054608443
98.5587196571
100.5396813508
90.4244138918
100.1031430713
100.3625098496
99.3815749898
102.3706025141
87.7461372892
105.3040603598
101.3867674692
106.5473762334
90.1078179714
106.2876407407
97.8630502196
107.8616145568
95.4506529947
100.6940159503
108.8698084810
106.3479131507
93.2706990869
93.6451424489
102.6716030142
111.8051498988
93.1369182992
92.4008906984
98.9347596089
89.3711532746
92.2971847366
95.1139500390
111.6565399065
105.4624583063
102.9205307802
109.2926132473
102.5316452061
95.0902598446
106.4872020652
102.6841726128
95.5654094080
108.8819056067
105.6504952168
95.9756531142
100.6199830776
87.4347290444
105.5523692925
90.4631799516
102.6961697112
109.7608817273
108.1714233887
108.4940576505
87.8912963999
108.3392780106
97.2562628046
101.7787127923
99.2410535109
98.8779376034
102.6514828717
102.8683691365
107.4517093835
100.8220198574
107.6192440107
101.4785339364
102.8680690833
94.0893663440
94.0277575564
109.8053995430
106.6946051137
107.6046513853

102.4718960614
 109.9436051183
 102.9998779735
 95.4550881490
 97.8186214467
 104.3965140502
 107.7332362027
 91.3700330580
 93.1877968874
 94.3091922463
 103.3203565791
 91.6950469414
 104.6448930475
 103.2167412519
 93.7464635947
 112.1762744531
 96.5589115194
 104.3364313040
 104.2689502571
 104.6998426823
 102.6988038384
 94.6613099786
 98.2045648304
 106.6816076327
 97.3648470410
 102.4466280358
 105.5592081509
 106.4699450508
 100.0081422213

X1 is Trapezoid(mu=100.000000,c=10.000000),

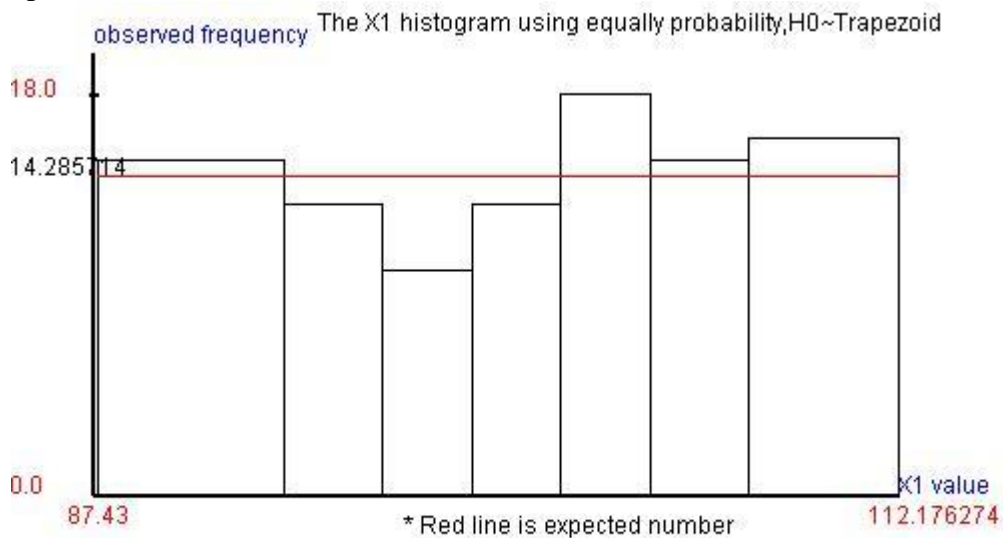


mu point estimated value=100.663941
 c point estimated value=8.247182
 mu value from 99.014505 to 102.313378
 c value from 6.343986 to 11.781688

pearson goodness of fit

class	[1]	[2]	[3]	[4]
lower limit	87.43473	93.21157	96.25961	99.01989
101.78017	104.54045	107.58849		
upper limit	93.21157	96.25961	99.01989	101.78017
104.54045	107.58849	112.17627		
observed no	15.00000	13.00000	10.00000	13.00000
18.00000	15.00000	16.00000		
probability	0.14286	0.14286	0.14286	0.14286
0.14286	0.14286	0.14286		
expected no	14.28571	14.28571	14.28571	14.28571
14.28571	14.28571	14.28571		
chi square	0.03571	0.11571	1.28571	0.11571
0.96571	0.03571	0.20571		

degree of freedom=4
 H0: $X_1 \sim \text{Trapezoid}(\mu=100.400031, c=9.660984)$,
 pearson chi-square test statistic =2.760000
 p-value=0.598700



2.4.16)The population distribution is U quadratic distribution.

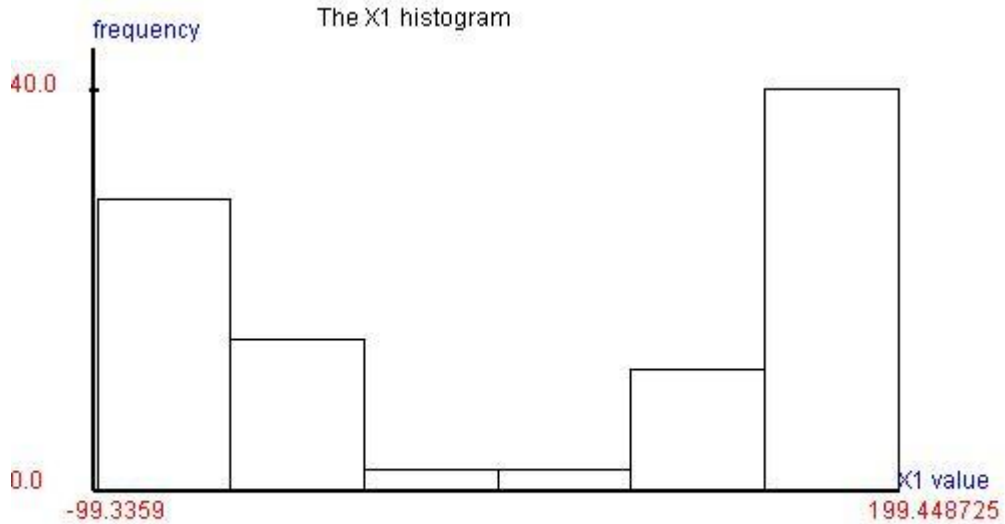
X1 is U-quadratic(a=-100.000000,b=200.000000),

X1

169.9391274828
111.3575520858
127.0222541845
-43.5297133046
-98.2633928338
158.2611586701
170.7295970856
180.0121935730
-94.4306722789
107.1192432805
-1.1388827723
174.7837223633
-37.0741733785
196.2642611562
153.1470550567
-74.3991934076
-21.9367155459
-6.5884071391
195.9789219352
-65.4137820426
161.2672204055
-88.4875063253
-39.7375332765
-46.4582797943
197.6866186721
-86.2305307875
157.2904679462
182.0708991339
-56.7116997302
154.8092639452
0.2627400345
129.8411812222
191.2145058590
-8.0117840834
183.1722875117
119.6297374207
146.8917392135
-88.2222258779
-86.7785967405
-94.0735211305
191.7281245933
132.1987210927
-19.4317692204
193.9685960973
178.2656453877
195.7681031196
181.5422080030
186.2145253755
-52.5708208512
-44.2284671064
-93.0060428830
-49.2526264607
-35.9944686034
129.8238936574
128.9659136397
-92.3836082868
-86.8238107357
161.3033582311
-94.4090759900
-78.6411916406
-99.3359688641
175.1567779023
189.6287525528
-69.0981798531
185.3613102763
196.0907930322
199.4487257869
194.2252613782
125.3068822974
160.5517617449
34.0378146213

144.9679791885
 -24.6376611041
 -67.5265973384
 -89.3427695127
 -78.2369449975
 -96.4910169702
 196.7079064756
 195.5143566116
 179.7551848280
 149.0415495616
 -51.0766839037
 182.0363449015
 -89.7006965590
 153.8866186239
 -98.6383709865
 161.8669661298
 79.9313989230
 155.7214725196
 -81.2699089033
 181.2234421554
 179.9638844726
 -31.5327174268
 174.6186500685
 -83.6203615442
 183.7371043448
 -58.2108434786
 -97.6143132109
 87.2931435634
 -28.0175568373

X1 is U-quadratic(a=-100.000000,b=200.000000),

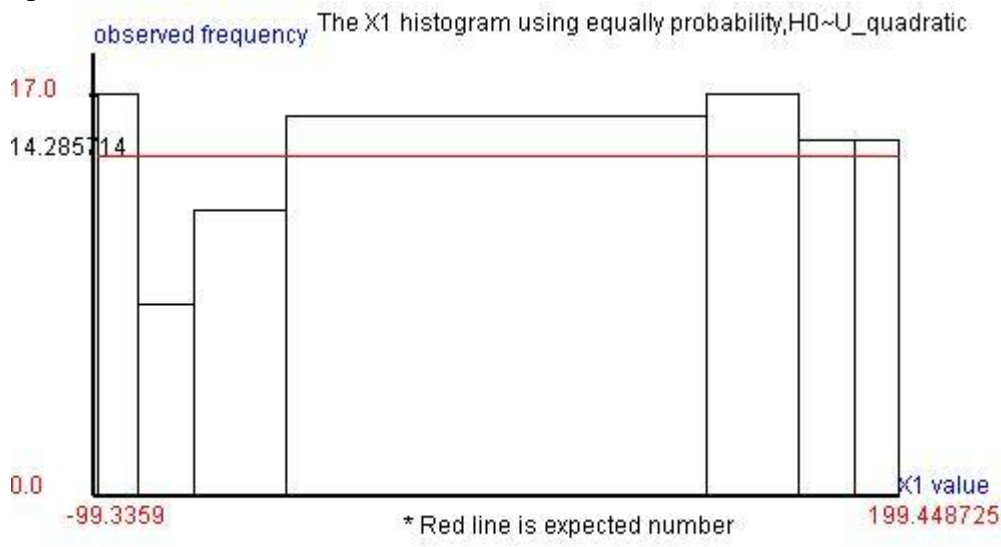


a point estimated value=-99.335969
 b point estimated value=199.448726
 a value from -99.933538 to -98.738399
 b value from 198.851156 to 200.046295

pearson goodness of fit

class	[1]	[2]	[3]	[4]
lower limit	-99.33597	-84.04938	-63.07593	-28.45995
128.15732	162.77043	183.74163		
upper limit	-84.04938	-63.07593	-28.45995	128.15732
162.77043	183.74163	199.44873		
observed no	17.00000	8.00000	12.00000	16.00000
17.00000	15.00000	15.00000		
probability	0.14286	0.14286	0.14286	0.14286
0.14286	0.14286	0.14286		
expected no	14.28571	14.28571	14.28571	14.28571
14.28571	14.28571	14.28571		
chi square	0.51571	2.76571	0.36571	0.20571
0.51571	0.03571	0.03571		

degree of freedom=4
 H0: $X_1 \sim U_quadratic(a=-99.933538, b=199.639948)$,
 pearson chi-square test statistic =4.440000
 p-value=0.349700



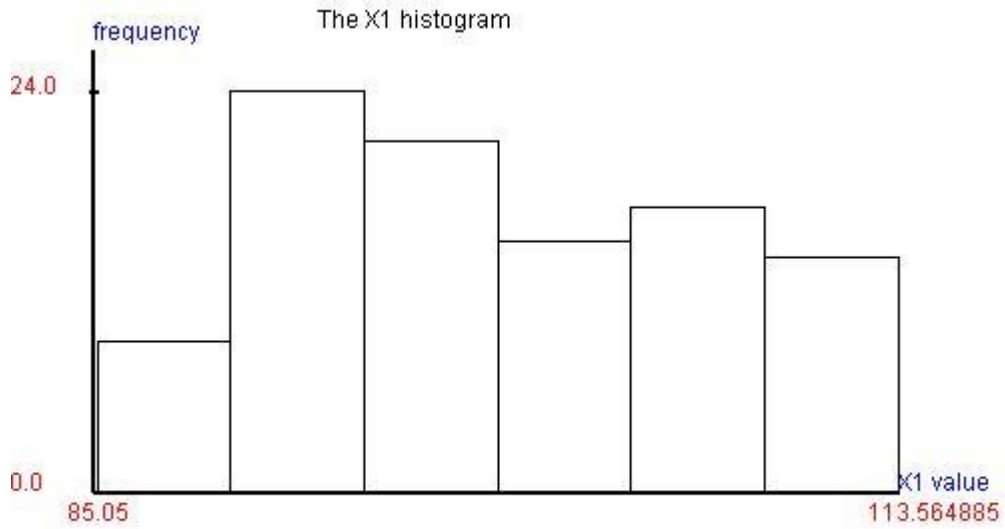
2.4.17)The population distribution is semi circle distribution.

X1 is Semi-circle($\mu=100.000000,R=15.000000$),

X1
109.2331523764
101.8873678331
103.7225836039
85.0534451914
112.7141088800
106.2857267929
97.0356376025
95.5224963146
94.9841172117
103.8795348759
106.5768261562
98.4533398600
94.5326367345
92.4320181723
91.2099512768
106.8247108896
92.3259729868
90.4818727444
85.7754112970
107.1286046466
113.2955870518
87.7366680921
104.6204506492
90.7930376687
106.7116793504
105.0835877797
106.5423592088
92.3020432112
92.4094251295
88.0013287691
107.5155147500
109.3831315637
94.7393511448
111.1308306847
94.4238332627
103.8687477774
110.5700497473
93.6646516564
111.0474460130
99.3805200883
94.9069303401
103.4863927947
90.5308626702
91.6495834677
90.1670262917
91.2416420033
111.3154838093
100.3404376106
113.0620709399
98.2090933799
103.1110965408
89.0695792406
97.7583870012
104.1760005998
95.0491992304
101.1316377303
109.2578321729
98.5156405657
95.5245628774
92.0964685070
97.8407077533
95.6451161931
105.5377253494
111.6094969721
104.4933398657
96.7425682240
90.8620042687
93.0638209563
93.5550345111
95.2700590414
96.0679298789

93.2841099618
 102.9604225944
 97.7565415781
 106.3221859840
 91.3272198232
 103.4562894824
 104.6370084642
 111.6506394576
 100.6022527025
 86.8807851188
 92.8564537621
 96.0215540317
 89.3327277176
 89.0996058086
 101.3787057318
 94.1921584750
 88.8414064836
 97.6263220614
 113.2963602729
 90.0462506047
 95.9218668731
 93.2836476581
 106.0319600777
 106.2646343623
 99.5187108060
 97.5587275419
 113.5648852280
 103.7184744035
 105.4468238828

X1 is Semi-circle(mu=100.000000,R=15.000000),

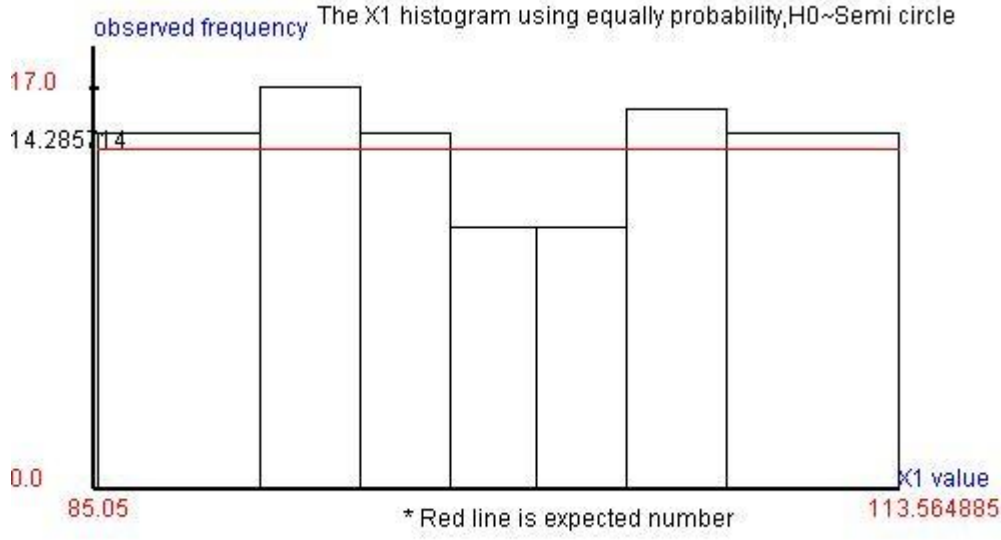


mu point estimated value=99.234462
 R point estimated value=14.255720
 mu value from 96.383318 to 102.085606
 R value from 11.879767 to 17.819650

pearson goodness of fit

class	[1]	[2]	[3]	[4]
lower limit	85.05345	90.88507	94.42384	97.62012
100.73339	103.92967	107.46757		
upper limit	90.88507	94.42384	97.62012	100.73339
103.92967	107.46757	113.56489		
observed no	15.00000	17.00000	15.00000	11.00000
11.00000	16.00000	15.00000		
probability	0.14286	0.14286	0.14286	0.14286
0.14286	0.14286	0.14286		
expected no	14.28571	14.28571	14.28571	14.28571
14.28571	14.28571	14.28571		
chi square	0.03571	0.51571	0.03571	0.75571
0.75571	0.20571	0.03571		

degree of freedom=4
 H0: X1~Semi-circle(mu=99.177439,R=13.839928),
 pearson chi-square test statistic =2.340000
 p-value=0.673400



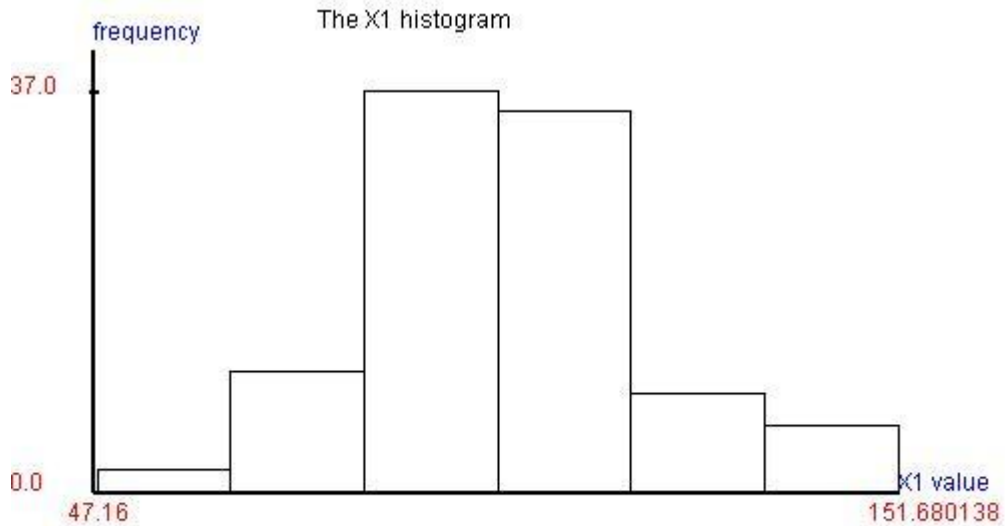
2.4.18)The population distribution is logistic distribution.

X1 is Logistic($\mu=100.000000$, $\sigma=10.000000$),

X1
110.5950918168
114.3367284228
90.9888647489
101.2805203517
96.2672068992
118.8555776642
112.1345941200
107.3865124208
99.1735567128
74.9506248092
98.7326701651
92.9285958079
111.8078224911
96.9793493473
120.0679887353
117.1449663046
80.3584785543
104.7350536533
77.3888598506
76.3314511620
85.1293117551
102.2538480450
91.9883819055
87.4393938989
108.5201047709
99.5335356678
53.3344800298
85.6228565968
100.7281280209
110.8287176379
110.3994910602
111.5327009906
84.3012498173
81.4756147004
100.7328672354
86.6330890446
79.4155010474
125.9790330663
119.6601827280
106.2833342533
102.1072305086
100.4539947568
100.5772747078
141.5727088498
126.7279895838
104.5723600308
103.3811134237
97.6006801743
67.9438334125
86.3677057110
104.2091933324
113.1206153124
108.2176066430
151.6801386794
149.3756048768
82.3384941259
81.4812278280
102.4431605651
87.9146895513
96.9886303339
134.7918390242
78.1759176406
105.7189369136
95.6982725175
112.5472317626
114.8319155289
113.8691281750
94.6900683096
93.1422366763
122.5817304277
103.3457730647

101.5117226826
 98.3369711582
 86.6951813234
 114.4090272685
 120.6378205926
 99.2717102264
 98.0143555496
 101.3755378199
 112.8797858697
 93.8455780956
 80.0863606481
 132.6302699357
 83.1783645684
 93.6825799380
 138.1474594897
 138.6448373366
 92.3520135990
 97.4030339413
 85.1783694414
 91.9556786846
 95.9552036658
 88.2830615431
 71.1230288296
 91.7408697862
 97.1008056650
 105.0899311772
 97.0109861750
 94.0215643278
 47.1688754789

X1 is Logistic(mu=100.000000,sigma=10.000000),

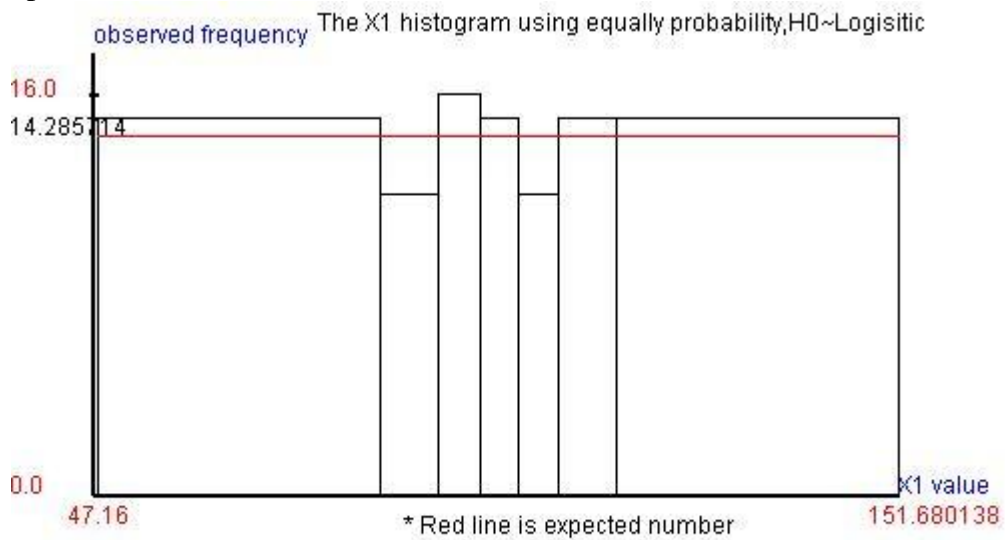


mu point estimated value=100.604346
 sigma point estimated value=9.960975
 mu value from 98.612151 to 102.596541
 sigma value from 7.662288 to 14.229964

pearson goodness of fit

class	[1]	[2]	[3]	[4]
lower limit		84.23179	91.74486	97.13943
102.07707	107.47164	114.98470		
upper limit	84.23179	91.74486	97.13943	102.07707
107.47164	114.98470			
observed no	15.00000	12.00000	16.00000	15.00000
12.00000	15.00000	15.00000		
probability	0.14286	0.14286	0.14286	0.14286
0.14286	0.14286	0.14286		
expected no	14.28571	14.28571	14.28571	14.28571
14.28571	14.28571	14.28571		
chi square	0.03571	0.36571	0.20571	0.03571
0.36571	0.03571	0.03571		

degree of freedom=4
 H0: X1~Logisitic(mu=99.608249,sigma=8.581763),
 pearson chi-square test statistic =1.080000
 p-value=0.897400



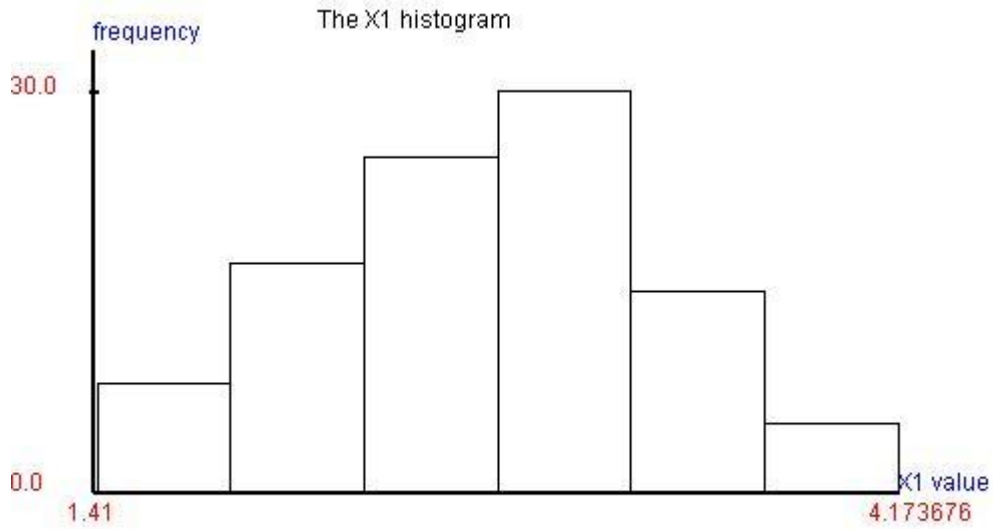
2.4.19)The population distribution is weibull distribution.

X1 is Weibull(alpha=1.000000,beta=2.000000,gamma=4.000000),

X1
2.9569944457
3.2344130861
3.0249582440
3.0353410663
2.9777669886
2.5042919264
3.2297130323
3.5069262215
2.9993508617
2.8347034175
3.3870898854
1.7353437212
3.2002578450
3.6069309444
2.5265222331
2.4369502281
2.1817197376
2.8095736053
2.4419662025
3.2288091374
2.1690088815
3.7688498741
1.8537288509
3.2055162009
3.2723514336
2.8561137789
2.5713969386
3.6592268979
1.7991294850
3.2068933766
2.2192107120
2.7413836395
2.5819462765
3.8253529037
3.3603102549
2.3592838712
3.5499183678
1.6421070717
1.9867362684
2.8166619539
2.7898777499
2.3350897463
2.2986610688
2.3811128369
3.0110434949
4.1448309767
1.9379940719
3.5870027988
3.0546889489
3.1943007038
2.7987601806
2.1927285375
3.1825939114
1.9901611991
2.7584660719
3.4084498248
2.5415627836
2.0844523949
2.6379834730
3.3734950651
1.8661872350
3.4763876673
2.1837801442
3.4557168899
2.0419713638
2.8978119632
1.9458661500
2.8615596492
3.1739363594
1.6768354784
3.1972600469

2.7463998818
 2.7317425645
 1.9834006586
 3.3265953931
 3.1766719641
 2.1324200783
 4.1736768544
 2.6567413978
 2.3477864471
 3.4729973285
 2.5245124835
 2.7865686144
 2.1386613371
 2.8723777594
 3.9120474718
 2.8710477267
 2.7151616310
 2.8081202533
 2.2598626573
 2.8098386019
 2.7159667452
 2.3503933347
 1.4141822121
 3.3227812374
 2.6932335565
 1.7467688996
 2.1724697337
 2.9895453403
 2.4362383372

X1 is Weibull(alpha=1.000000,beta=2.000000,gamma=4.000000),



alpha point estimated value=1.414182 (MLE)

beta point estimated value=1.471173

gamma point estimated value=1.471173

beta value from 0.543784 to 0.815676

gamma value from 0.200000 to 10.000000

pearson goodness of fit

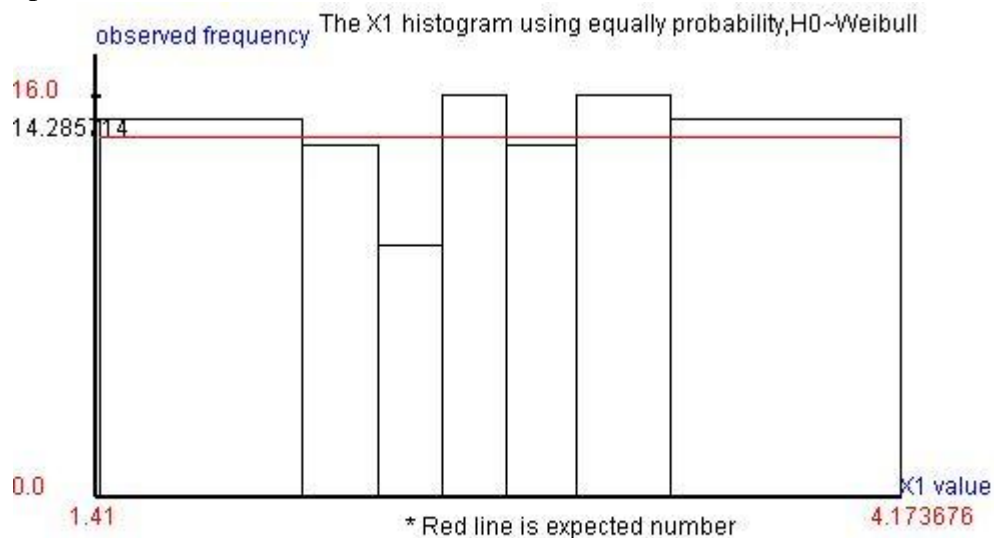
class	[1]	[2]	[3]	[4]
[5]	[6]	[7]		
lower limit	1.41418	2.11517	2.37768	2.59964
2.81794	3.06045	3.38404		
upper limit	2.11517	2.37768	2.59964	2.81794
3.06045	3.38404			
observed no	15.00000	14.00000	10.00000	16.00000
14.00000	16.00000	15.00000		
probability	0.14286	0.14286	0.14286	0.14286
0.14286	0.14286	0.14286		
expected no	14.28571	14.28571	14.28571	14.28571
14.28571	14.28571	14.28571		
chi square	0.03571	0.00571	1.28571	0.20571
0.00571	0.20571	0.03571		

degree of freedom=4

H0: $X_1 \sim \text{Weibull}(\alpha=1.414182, \beta=1.501823, \gamma=2.454000)$,

pearson chi-square test statistic = 1.780000

p-value=0.776100



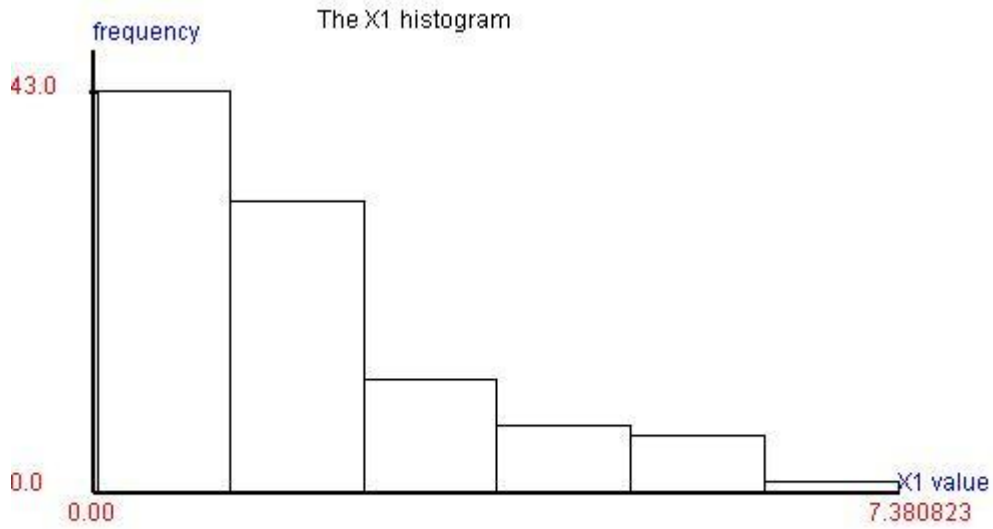
2.4.20)The population distribution is pareto 3 distribution.

X1 is Pareto3(lamda=4.000000,c=10.000000),

X1
0.2333575151
1.0124918110
0.6459560230
0.0032724592
3.6921911204
2.5155326702
1.1237113269
1.4837955547
0.8197324539
1.6398295438
1.0614746590
0.2724809350
0.6579498206
0.1300921869
0.4259353282
0.1643049488
1.7214895595
2.2162853512
1.2212771647
0.4293726925
1.5771669723
2.7827908474
3.1032715240
0.9250985538
0.7913732631
4.0087610399
0.6589652460
5.1893542523
0.0962839750
1.3790964442
0.5903578471
0.3672487403
0.0217555267
1.0831718081
1.9122536237
3.4240976149
1.4633403602
2.0799384967
1.6374517036
2.4307524818
1.8271252274
2.1592564685
0.9731136744
0.1641379067
5.4447780796
2.3016790279
0.2250291766
2.6903964914
1.6102016080
3.9429914315
0.1316969998
0.9495587936
2.0714351366
1.3405433626
0.4041939781
0.1942857455
1.9122262916
1.3369043660
0.5638450083
5.0250503558
3.5419473888
2.2096877083
0.1756831797
5.3081649769
5.7012668520
0.4196837036
2.2138809467
0.8525352974
2.7229220168
0.1505223570
2.7740966741

0.8636378889
 3.0610756221
 2.6409906730
 4.0460219276
 0.7579031840
 1.3683611987
 0.2395630524
 4.7604690137
 0.6317185449
 0.8838788935
 7.3808235874
 1.9078088107
 1.7404426024
 2.0327619921
 0.0641253679
 2.8274624991
 1.0001968840
 1.7782210356
 1.7733538092
 3.5641530093
 6.1400091291
 1.9579540949
 1.3719610283
 4.2251901730
 0.3910410902
 1.2755999471
 3.7541363906
 2.0133996047
 0.1572692280

X1 is Pareto3(lamda=4.000000,c=10.000000),



lamda point estimated value=3.118206 (MLE)

c point estimated value=7.380824 (MLE)

lamda value from 2.598505 to 3.897757

c value from 7.238195 to 7.523452

pearson goodness of fit

class	[1]	[2]	[3]	[4]
lower limit	0.00000	0.37547	0.79544	1.27600
1.84522	2.56050	3.58079		
upper limit	0.37547	0.79544	1.27600	1.84522
2.56050	3.58079	7.38082		
observed no	17.00000	13.00000	14.00000	16.00000
15.00000	11.00000	14.00000		
probability	0.14286	0.14286	0.14286	0.14286
0.14286	0.14286	0.14286		
expected no	14.28571	14.28571	14.28571	14.28571
14.28571	14.28571	14.28571		
chi square	0.51571	0.11571	0.00571	0.20571
0.03571	0.75571	0.00571		

degree of freedom=4

H0: X1~Pareto 3(lamda=3.009851,c=7.520600),

pearson chi-square test statistic =1.640000

p-value=0.801500

