

Chapter six The goodness of fit test

1) Pearson chi-square test 1:

1.1) The multi-nominal distribution is the sampling distribution, each category has a probability and the summation of probability is 1.

category	1	2	k
probability	p_1	p_2	p_k

$$p_1 + \dots + p_k = 1,$$

$$f(x_1, \dots, x_k) = \frac{n!}{x_1! \times \dots \times x_k!} \times p_1^{x_1} \times \dots \times p_k^{x_k}, x_1 + \dots + x_k = n \quad \text{that is trial number.}$$

1.2)

$$X_i = np_i + \varepsilon_i, E(\varepsilon_i) = 0, E[(\varepsilon_i)^2] = np_i(1 - p_i), i = 1, 2, \dots, k$$

$$X_i - np_i = \varepsilon_i, E\left[\frac{(\varepsilon_i)^2}{np_i}\right] = E\left[\frac{(X_i - np_i)^2}{np_i}\right] = 1 - p_i,$$

$$\sum_{i=1}^k E\left[\frac{(X_i - np_i)^2}{np_i}\right] = E\left[\sum_{i=1}^k \left(\frac{(X_i - np_i)^2}{np_i}\right)\right] = k - \sum_{i=1}^k p_i = k - 1,$$

$$\sum_{i=1}^k \left(\frac{(X_i - np_i)^2}{np_i}\right) \rightarrow \chi_{k-1}^2$$

$$H_0 : p_1 = p_{01}, p_2 = p_{02}, \dots, p_k = p_{0k} \quad H_1 : \text{against } H_0$$

$p_{01}, p_{02}, \dots, p_{0k}$ are known value and $p_{01} + p_{02} + \dots + p_{0k} = 1,$

$$E_i = np_{i0}, i = 1, 2, \dots, k, O_i = x_i,$$

O_i : The observed sample number of cell i,

E_i : The expected sample number of cell i,

$$\chi_v^2 = \sum_{i=1}^k \frac{(O_i - E_i)^2}{E_i}, \quad v = k - 1 - (\text{number of point estimator}) \text{ is degree of freedom of}$$

pearsson chi square test.

$$\chi_v^2 > \chi_{\alpha, v}^2 \Rightarrow \text{reject } H_0.$$

1.3) The process of test

H_0 : Population distribution is a continuous probability distribution,

H_1 : against H_0

The sample size is n.

The process: i) The class number of frequency distribution: $k = \log_2(n) + 1.$

ii) The class limit of frequency distribution: Let each class probability are equally and the class limit will be gotten.

iii) The class sample number of frequency distribution (O_i):

The frequency distribution is done and getting the O_i of each class.

iv) The class expected number of frequency distribution (E_i):

$$E_i = n \times \text{the probability of each class.}$$

Note: There are 20 kinds of continuous probability distribution that is can be assigned to null hypothesis.

1.4) Example (The simulated sample data and computing the result by the P_S_CCC)

? The random variable X1
 ----- test the population distribution -----

1. Pearson chi square test ,
the frequency table is according the equally probability,
2. Pearson chi square test ,
the frequency table is according the equally probability,
the best fitting method getting the parametes and population distribution.
3. Pearson chi square test ,
the frequency table is used to tradition method.
4. Pearson chi square test ,
the frequency table is used to tradition method.
tthe best fitting method getting the parametes and population distribution.
5. Kolmogorov Simirnov test
6. P-P plot
7. Q-Q plot
8. Likelihood ratio chi square test ,
the frequency table is according the equally probability,
9. Likelihood ratio chi square test ,
the frequency table is used to tradition method.
10. The sample data estimated cumulative relative frequency estimated line.
11. return

確定

取消

The Pearson chi square test (goodness of fit) ,the equally probability frequency distribution, please select the population distribution

- | | |
|---------------------------------------|------------------------------------|
| 1.H0:Uniform distribution | 13.H0:Gumbel distribution |
| 2.H0:Normal distribution | 14.H0:Triangular 1 distribution |
| 3.H0:Shifted exponential distribution | 15.H0:Trapezoid distribution |
| 4.H0:Pareto 1 distribution | 16.H0:U-quadractic distribution |
| 5.H0:Pareto 2 distribution | 17.H0:Semi-circle distribution |
| 6.H0:Rayleigh distribution | 18.H0:Logistic distribution |
| 7.H0:Double expoenoidal distribution | 19.H0:Weibull distribution |
| 8.H0:Log normal distribution | 20.H0:Pareto 3 distribution |
| 9.H0:Gamma distribution | ** Above H0 population all do once |
| 10.H0:Beta distribution | |
| 11.H0:Cauchy distribution | |
| 12.H0:Arcsin distribution | |

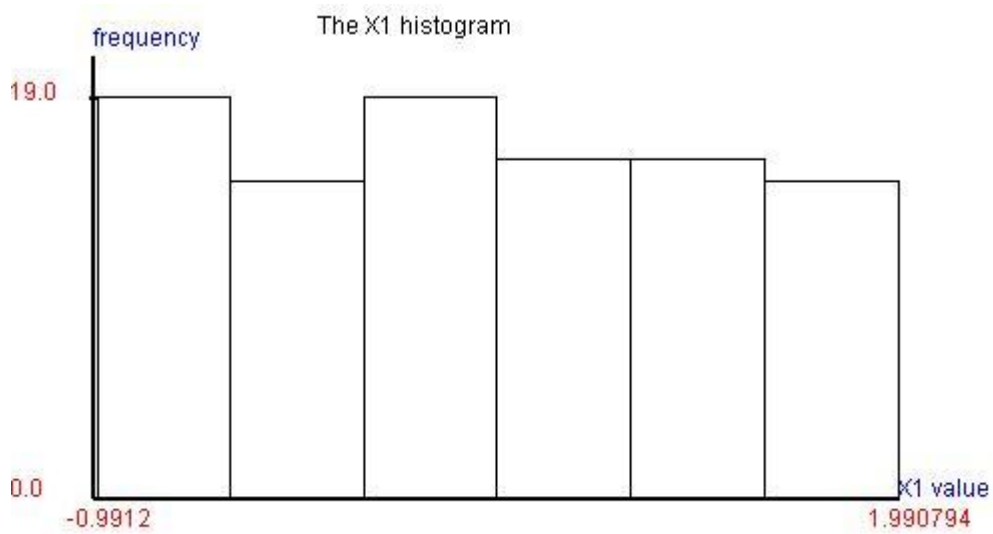
1.4.1)The population distribution is uniform distribution.

X1 is Uniform(alpha=-1.000000,beta=2.000000),

X1
1.9490281735
1.1275468963
-0.4792117472
-0.6693622657
0.8634497198
0.4294725915
1.5282097475
1.9907944225
-0.8447370682
0.6976870387
-0.0295300403
1.3996982609
1.8834823628
-0.6794357158
1.3443346097
0.9767624581
0.8284528120
1.0007356866
0.3691298118
0.3578717686
-0.1976189581
0.7410411746
0.2143931617
1.6241474373
1.3645974990
0.9163213279
0.4599934805
1.7537818654
-0.5283636410
0.0842390139
1.0697690515
-0.2936883676
0.0951521378
0.6928052956
0.1602833539
0.0436619314
0.3366625182
0.3409111912
0.8462057976
-0.5066974324
-0.9275896880
1.7477089686
1.9414445045
1.3627010379
-0.3412924856
0.0869613706
-0.1988906686
-0.7915098378
-0.2709598462
0.8436673962
0.0818757166
1.6684371636
0.6135715824
-0.1527480995
0.7042711105
0.8608218334
1.4137877736
-0.3606399475
1.7968349859
1.2602262349
1.2628307431
-0.1671671146
0.9043008764
-0.5483114588
1.8015169389
-0.3849475516
-0.7587836485
1.6746038076
0.4031503559
1.0557119327
1.2704942273

-0.4427743163
-0.8219572869
0.3732064460
0.3901420462
-0.0460241391
-0.7123540027
0.8157627955
-0.9912382062
-0.7148666312
-0.3990506618
1.3148001052
1.7909611082
1.2918431610
-0.7760390017
0.5649915344
0.3531763939
1.6068399498
1.9339203618
0.6897091077
-0.8067535782
-0.9482790511
1.0474126956
-0.8549597186
-0.8675942594
-0.9359981575
-0.2424892844
1.0225230275
0.0530364465
0.1619512743

X1 is Uniform(alpha=-1.000000,beta=2.000000),



X1 is mean= 0.4496395373, s.d.= 0.8797059964, variance= 0.7738826402,
 skewed coefficient= 0.0425943022, kurtosis coefficient= 1.7963526822, MAD=
 0.7547568458,
 Q1= -0.3174904266, median= 0.3966462011, Q3= 1.2602262349,
 MIN= -0.9912382062, MAX= 1.9907944225, Range= 2.9820326287,
 Mid-Range= 0.4997781081, C.V.= 1.9564694013, sample size=100

pearson goodness of fit

class	[1]	[2]	[3]	[4]
[5]	[6]	[7]		
lower limit	-0.99124	-0.56523	-0.13923	0.28678
0.71278	1.13879	1.56479		
upper limit	-0.56523	-0.13923	0.28678	0.71278
1.13879	1.56479	1.99079		
observed no	16.00000	16.00000	11.00000	16.00000
16.00000	11.00000	14.00000		
probability	0.14286	0.14286	0.14286	0.14286
0.14286	0.14286	0.14286		
expected no	14.28571	14.28571	14.28571	14.28571
14.28571	14.28571	14.28571		
chi square	0.20571	0.20571	0.75571	0.20571
0.20571	0.75571	0.00571		

degree of freedom=4

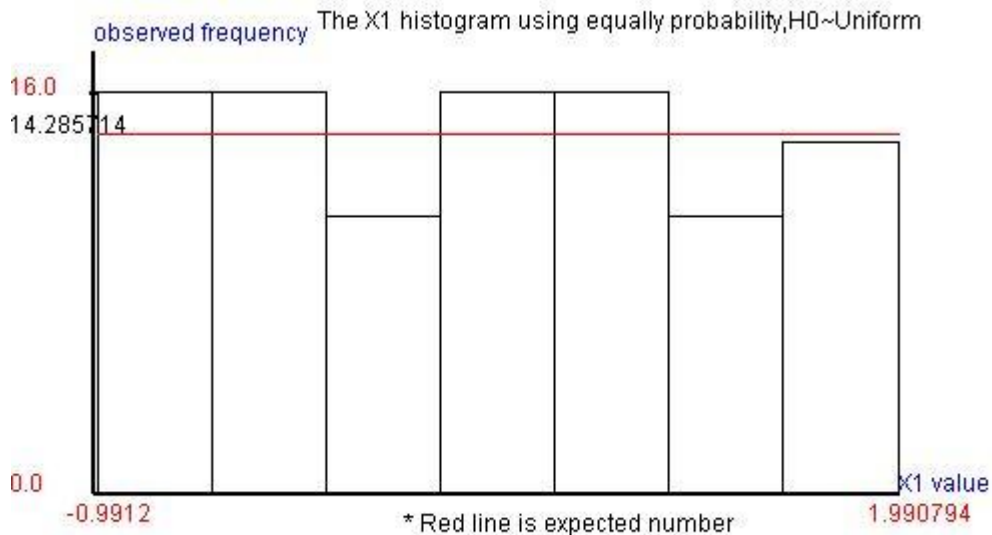
H0: X1~Uniform(alpha,beta), alpha,beta are unknown

alpha point estimated value=-0.991238 (MLE)

beta point estimated value=1.990794 (MLE)

pearson chi-square test statistic =2.340000

p-value=0.673400,



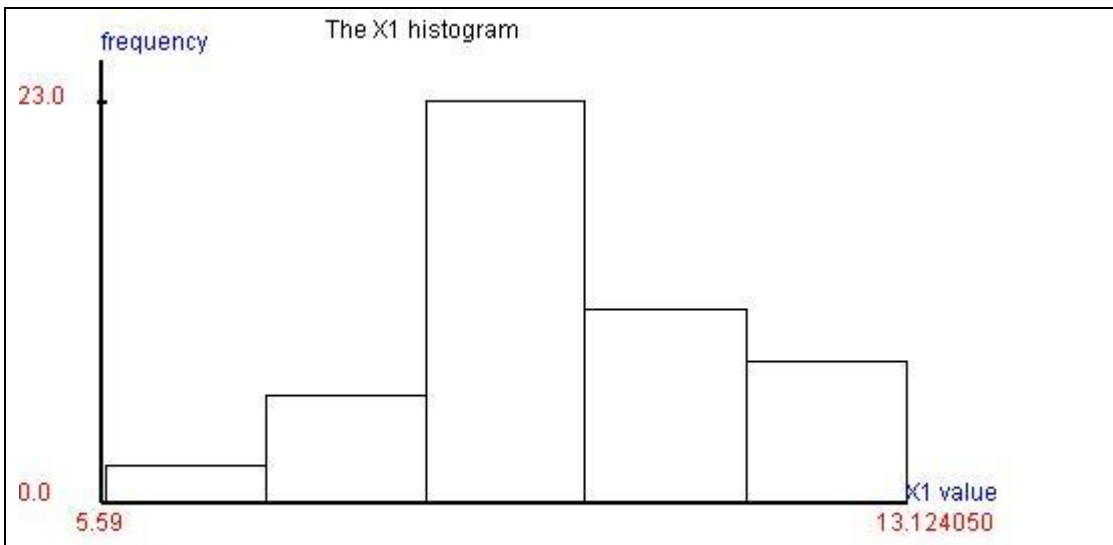
1.4.2)The population distribution is normal distribution.

X1 is Normal($\mu=10.000000$, $\sigma^2=4.000000$),

X1

9.1887746603
13.1240500625
9.1975237873
10.0434537721
9.0938145576
10.0053925710
5.5981195605
10.6965403839
10.8507182789
8.9891849467
11.6285741926
11.7252665052
9.6974731813
9.2415487942
9.2090708440
10.6179252966
8.8025631170
7.6905528163
8.6065550992
10.6089400308
10.7204434117
9.9491806926
7.5387494513
12.6967660633
12.6654014307
10.4701212248
8.7418229740
8.0013208465
9.1397596903
12.1678302290
7.6368378044
9.6830445060
10.8659674522
9.1814856981
9.7822281458
9.9655670867
12.8080575645
10.9107247006
9.3532360675
8.8303522251
10.1662346825
10.5945412176
11.4715357814
10.0715546200
11.7103740283
9.3621913602
10.0530770233
8.1202406818
6.3792092452
9.2068517906

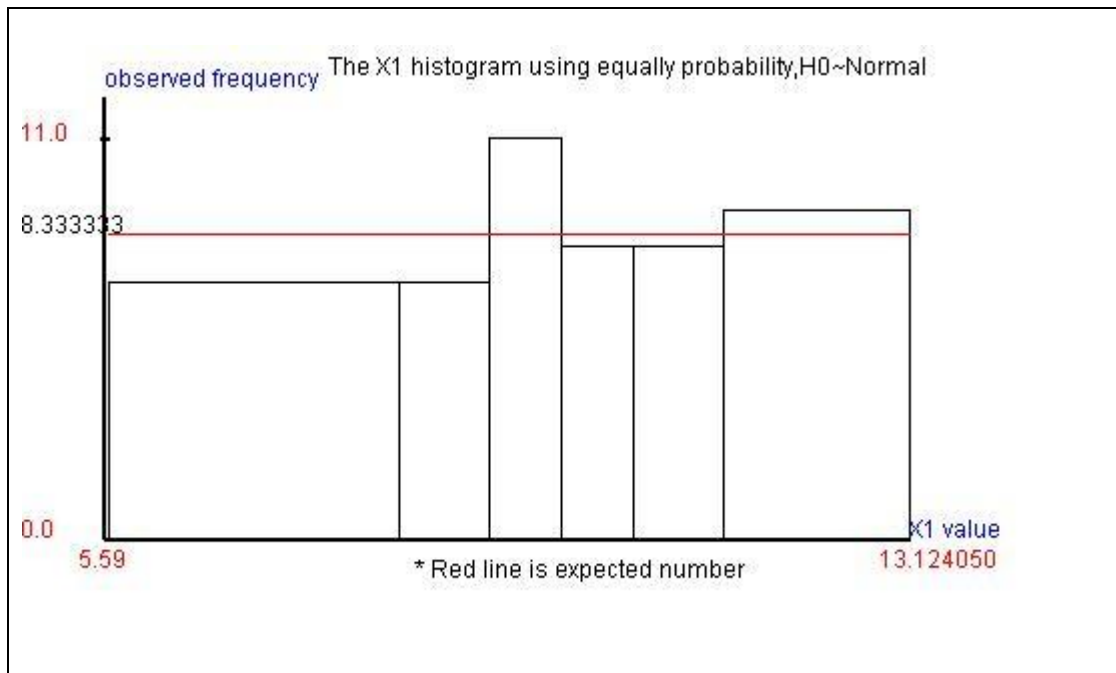
X1 is Normal($\mu=10.000000$, $\sigma^2=4.000000$),



pearson goodness of fit

class	[1]	[2]	[3]	[4]
lower limit		8.33118	9.17784	9.85725
10.53610	11.38258			
upper limit	8.33118	9.17784	9.85725	10.53610
11.38258				
observed no	7.00000	7.00000	11.00000	8.00000
8.00000	9.00000			
probability	0.16667	0.16667	0.16667	0.16667
0.16667	0.16667			
expected no	8.33333	8.33333	8.33333	8.33333
8.33333	8.33333			
chi square	0.21333	0.21333	0.85333	0.01333
0.01333	0.05333			

degree of freedom=3
H0: $X_1 \sim \text{Normal}(\mu, \sigma^2)$, μ, σ are unknown
population mean(μ) point estimated value=9.857215 (MLE,UMVUE)
population variance(σ^2) which point estimated value=2.487017 (UMVUE)
pearson chi-square test statistic =1.360000
p-value=0.714900



1.4.3)The population distribution is shifted exponential distribution.

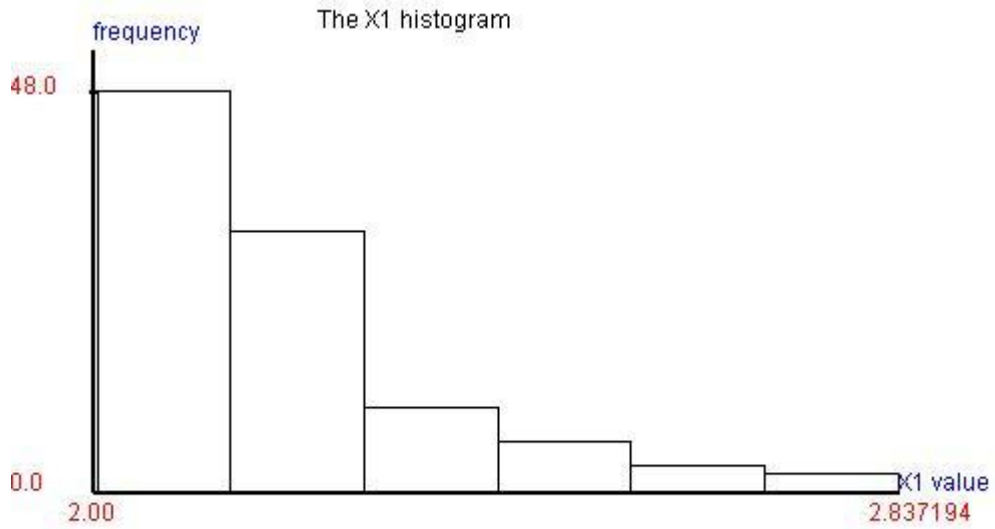
X1 is Exponential($\lambda=4.000000, c=2.000000$),

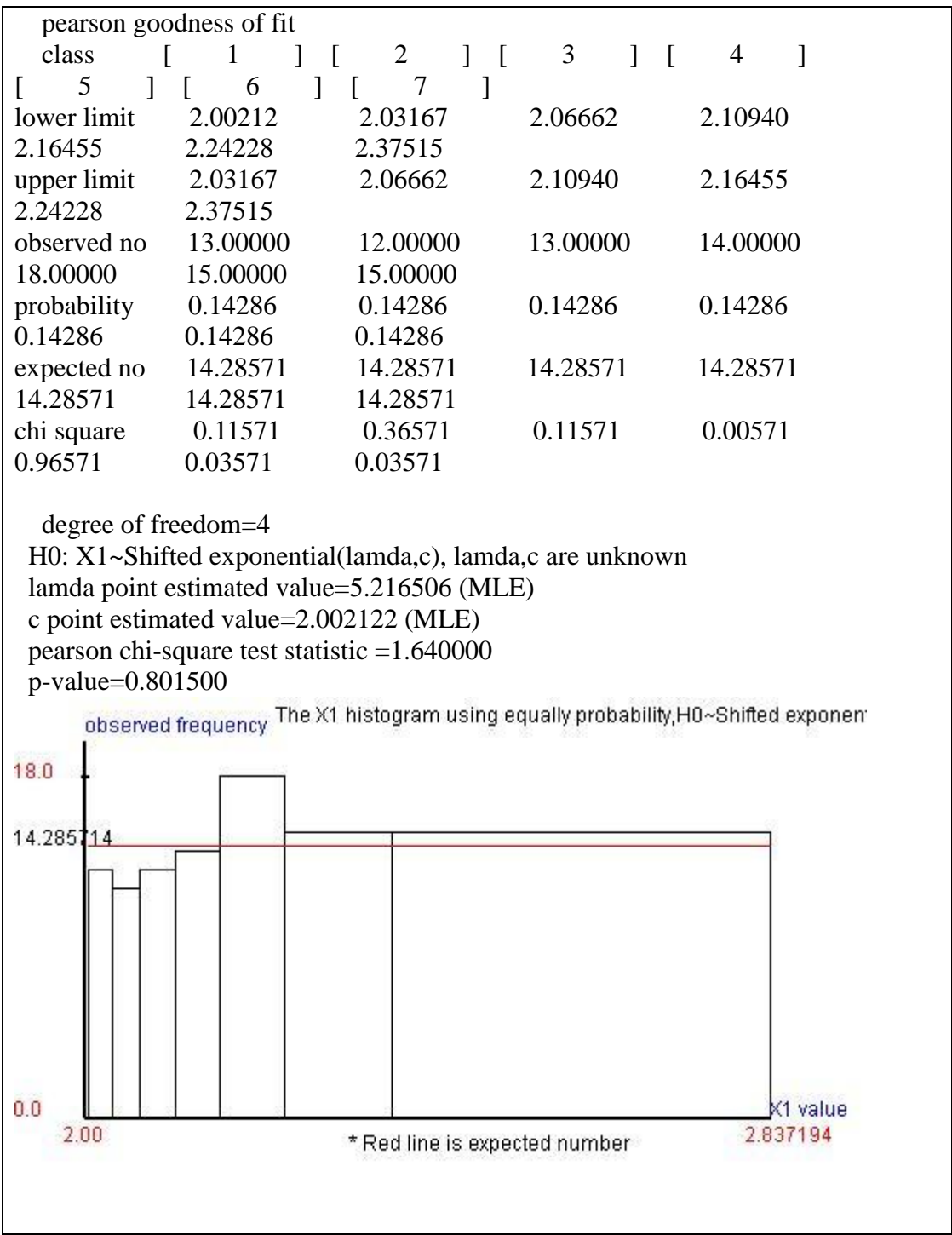
X1

2.2070218145
2.1657632563
2.0934818678
2.2094710812
2.0748253961
2.8371947589
2.0637025257
2.2240012037
2.0291130915
2.1960353195
2.3989730920
2.0024185439
2.0349514047
2.4070347307
2.1386018888
2.3236070188
2.1681252340
2.1631868677
2.2375093526
2.1104628198
2.1773704944
2.1361947563
2.2654166297
2.1208269176
2.0075679083
2.6670509762
2.2935064173
2.2812409815
2.0458333240
2.1665092537
2.0403293835
2.1267474549
2.1378442313
2.4320175105
2.1968508173
2.0840164573
2.0519555408
2.1273812623
2.2773211859
2.0879480538
2.2321838117
2.0692785625
2.0404349239
2.2551246295
2.4357998942
2.0422678305
2.1171757752
2.0482595179
2.2430063111
2.0942571122
2.4176850050
2.0544522510
2.1231515511
2.0641610900
2.2919321304
2.2131503911
2.1797629834
2.0195375517
2.0126612947
2.0865942966
2.0309617124
2.1416048583
2.0523244488
2.1070561674
2.0503206328
2.1072824266
2.1552947738
2.0064058472
2.0121212924
2.4275843757
2.1586470440

2.1997715186
 2.0802465075
 2.2427024324
 2.2363384854
 2.0316568608
 2.5732884100
 2.1895668009
 2.0061629419
 2.0886650529
 2.0125553923
 2.6159735221
 2.5509302864
 2.4088827003
 2.7110480016
 2.2921031716
 2.0308780526
 2.2666325161
 2.0736865737
 2.2734334671
 2.1276049595
 2.2978112861
 2.5171159431
 2.0756701411
 2.2666482143
 2.1746162600
 2.0021217425
 2.2252059374
 2.4411451380
 2.2697403995

X1 is Exponential(lamda=4.000000,c=2.000000),





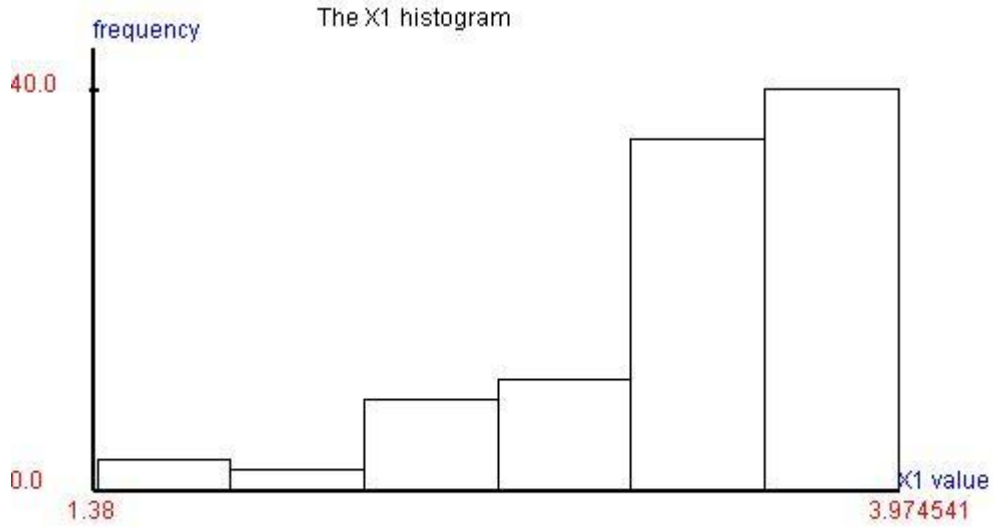
1.4.4)The population distribution is pareto1 distribution.

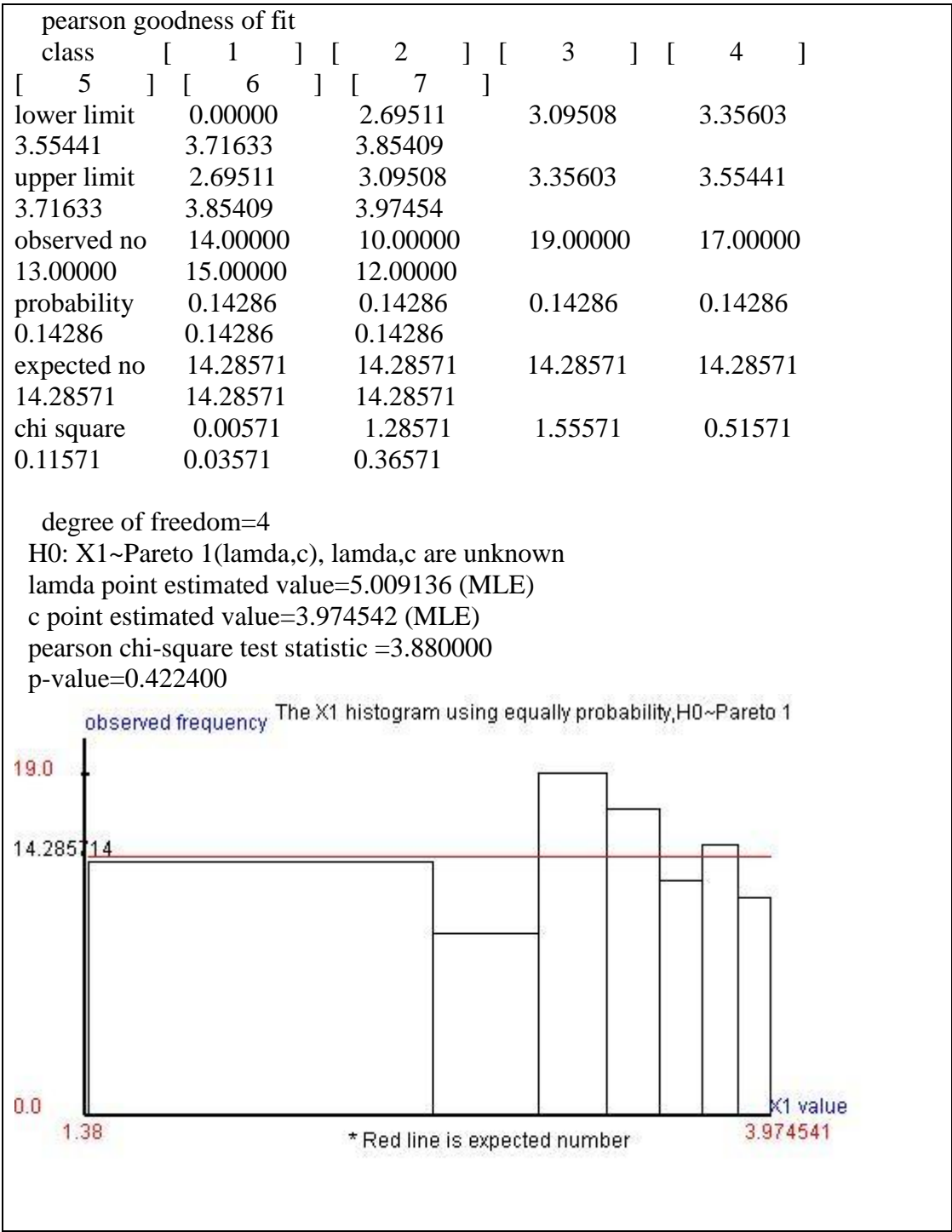
X1 is Pareto1(lamda=5.000000,c=4.000000),

X1
3.8114048214
3.0086651822
3.7132945330
3.1741152788
2.2613433323
3.0977650944
3.4185194289
3.1694099438
3.1831315988
3.7131429572
3.1785034825
3.1569103831
3.2738467568
2.3206044599
3.2127003171
3.3207716545
3.9474151102
3.6988303241
3.9156908924
3.9020952093
3.9048150205
2.8598078037
2.9928223545
3.8479775549
1.9029091727
1.5868119456
3.5176130501
2.8062373160
3.5363987597
3.0911605699
3.6961574356
3.5685688014
2.3912532835
3.3798138525
3.6950224047
3.7785360947
3.3607193705
3.4248596169
3.6304346666
3.6211910096
3.8371860943
3.7617503466
3.1921983503
3.8296764904
3.9198487251
3.4646374011
3.6085097343
2.4858281989
3.1574876055
3.9193421335
2.5165140694
2.9386930132
3.8187188261
3.8711264622
3.8740855780
3.3860917620
2.5776786280
2.9506804882
3.2899875165
3.7310496434
3.3711167582
3.9207708767
3.3880866142
3.4095035420
3.3004334833
3.3274886369
3.3647999982
3.9745418329
3.8299971439
1.5779963511
3.2460937645

3.4535808619
 1.3800934453
 3.7419027373
 3.4913252808
 3.8035327857
 3.7536111145
 3.6550364563
 3.6983515303
 3.5042113598
 3.7421584341
 2.6497764670
 2.1125085541
 2.6234589356
 3.9213053840
 3.4889781671
 3.7029153662
 3.1959891427
 2.9920056948
 3.3197787976
 3.7170880840
 3.1990718727
 2.4756861730
 3.7247140959
 3.9670338046
 3.5824515921
 3.2920663609
 2.9226042260
 3.5031274071
 2.8525885795

X1 is Pareto1(lamda=5.000000,c=4.000000),





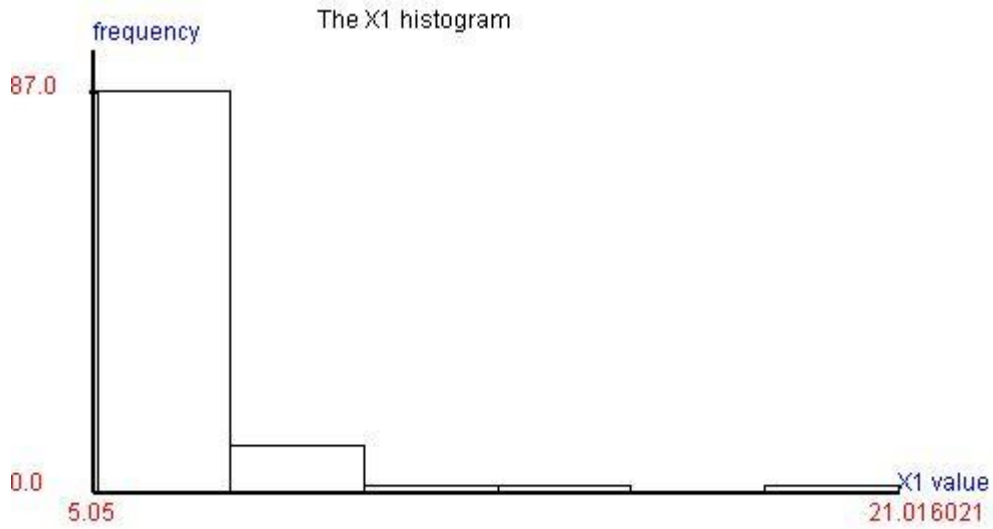
1.4.5)The population distribution is pareto2 distribution.

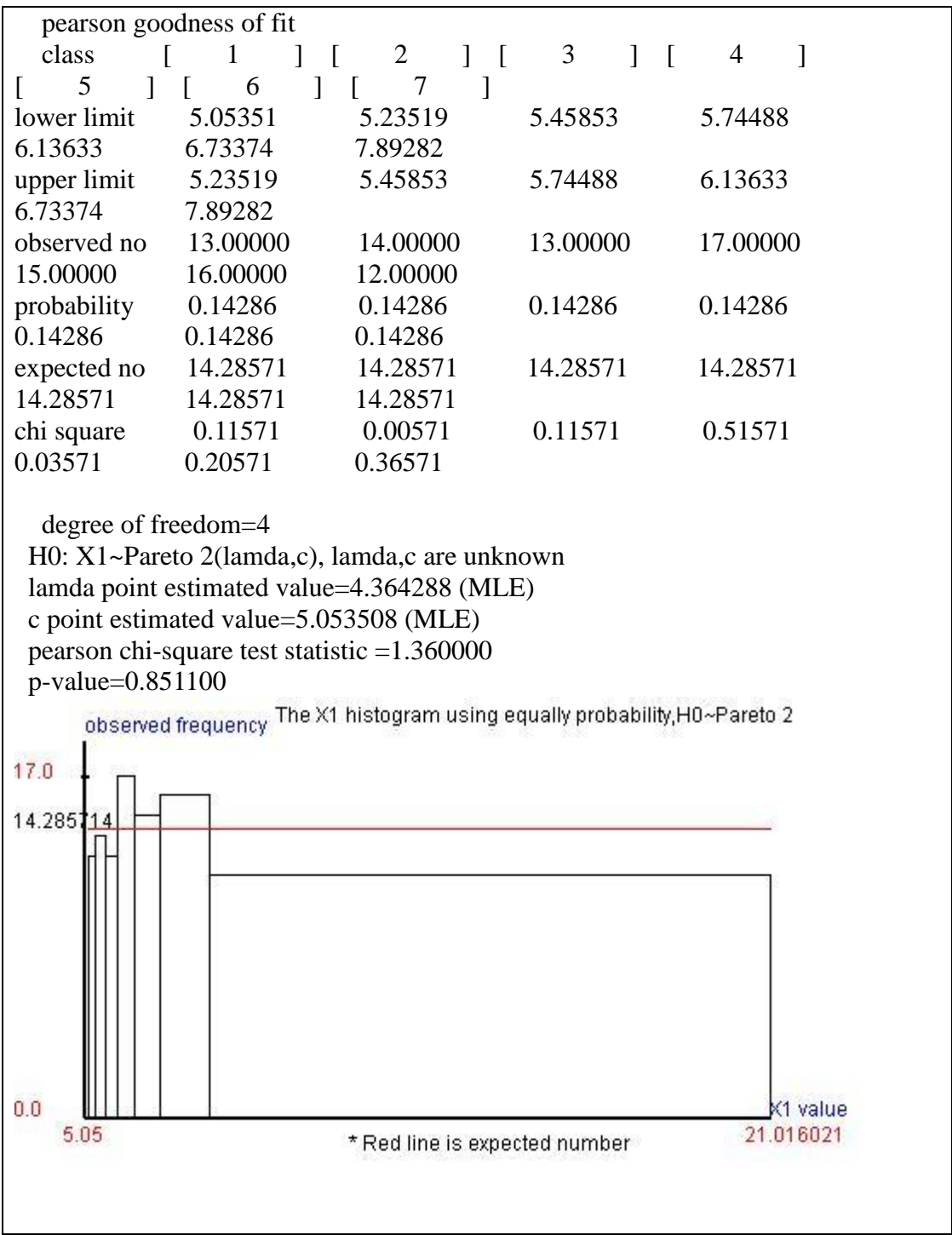
X1 is Pareto2(lamda=4.000000,c=5.000000),

X1
7.6664368458
11.0604991343
6.6765970312
5.2139686033
6.1404307158
6.9270109679
5.7449299621
5.6552526141
5.8106696686
7.6441511766
5.3504679861
6.3963955816
5.2859431863
6.1821378867
5.2654105610
7.9926247468
9.5148832684
7.6574146716
5.6167046192
5.0535079505
6.3189419163
8.8445384368
5.4777855811
6.7352549536
5.7365478619
6.2486040978
5.2528459973
5.1986129672
9.5822113469
8.9628389878
21.0160213878
7.4607189055
5.1846687547
7.0216407267
6.2486849707
5.3823289543
5.8841377935
5.8199724864
5.9542806085
5.4229302125
6.3645467495
5.3632190714
7.0942635939
6.4857574144
5.2110273863
5.8501948676
5.8629580087
8.8225790392
5.2577245034
7.4354943312
5.7018585436
9.3155436649
7.2107163510
5.4069085911
5.5229763781
5.7635785498
5.3802086203
5.2922413384
5.3968912669
5.6018481662
5.8327107576
15.5217819979
6.5254317307
5.0690373891
7.1665531182
7.1287957543
5.6027276732
5.6063365471
5.0561039336
9.5511102082
6.1474147567

6.6761508560
 5.9766648949
 6.3161943454
 5.8921531397
 7.0698707433
 5.6767100002
 9.3338752628
 5.0833103717
 5.6754681055
 6.0865072346
 5.1788683391
 5.8538468126
 5.2316255320
 7.8714301667
 6.9312138367
 5.8163662440
 7.2331145198
 6.0663331975
 5.4998702282
 6.4945055369
 5.8573725881
 5.9027069176
 5.7205386092
 6.6490143724
 5.4239486300
 5.2261062661
 5.1837567207
 5.2706165374
 5.1194346105

X1 is Pareto2(lamda=4.000000,c=5.000000),





1.4.6)The population distribution is rayleigh distribution.

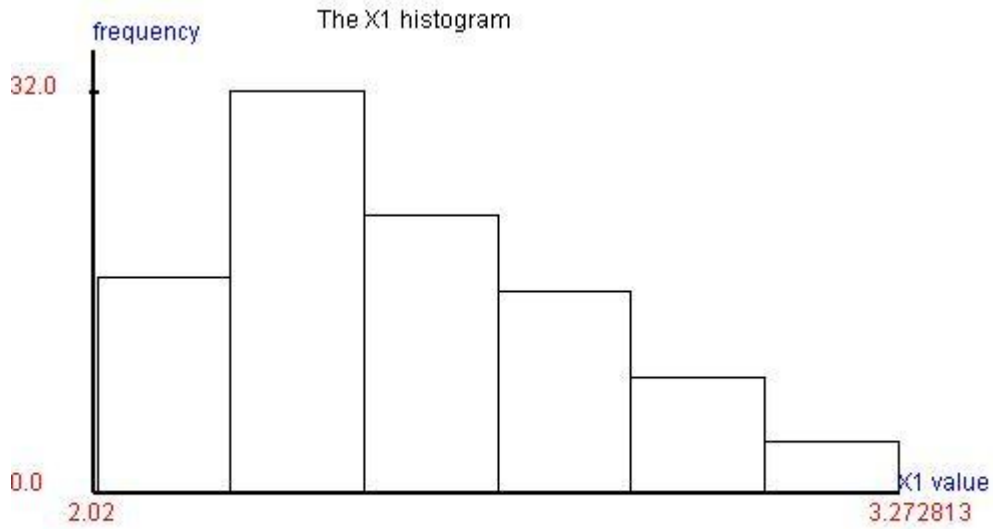
X1 is Rayleigh($\lambda=3.000000, c=2.000000$),

X1

2.7779350260
2.3019677472
2.2771156347
2.2333480452
3.0789375544
3.1202213289
2.1426234161
2.4476797879
2.3509430064
2.2004574723
2.7681856628
2.4217804415
2.6857615823
2.2617101350
2.2795094545
2.1502172052
2.7559627338
3.0126782492
2.5987922121
2.2070360712
2.8974854766
2.1160745525
2.2264583130
2.5159163069
2.4862192799
2.4381144926
2.6598982691
2.1026476181
2.2751871246
2.8810822583
2.1607873104
2.3259410045
2.5666535219
2.2800956005
2.7076326675
2.2837646897
2.2712969228
2.3399316194
2.5588871682
2.1872704551
2.8385056786
2.3138606385
2.5652449606
2.4303888279
2.5756162244
2.3157278778
2.1971583685
2.6993822779
2.8309118141
2.3303292488
2.4882903866
2.4683000227
2.7081892600
2.6809391472
2.1728120442
2.3226822555
2.4272824431
2.1605082480
2.5166725343
2.5781278613
2.7834486104
2.0272807547
2.5189600235
2.2774093331
2.9889935934
2.8822416336
2.0980636537
2.9724483063
2.3549045012
2.8712156730
2.4284555629

2.2656410306
 2.8045927509
 2.8227679252
 2.7450977728
 2.5949676846
 2.3869780180
 2.3798790718
 2.8168043997
 2.2750087665
 2.9082731057
 2.3388472723
 2.6489597475
 2.5374309779
 2.2666791209
 2.6193227604
 2.6236147546
 3.0895234523
 2.5691338950
 2.1699676163
 2.8627946277
 2.4538898723
 2.4358298313
 3.2728134633
 2.6500081928
 2.3779042459
 2.5366382452
 2.2086120164
 2.2601313427
 2.3806363864

X1 is Rayleigh(lamda=3.000000,c=2.000000),



pearson goodness of fit				
class	[1]	[2]	[3]	[4]
[5]	[6]	[7]		
lower limit	2.02728	2.24335	2.34650	2.43896
2.53384	2.64324	2.79496		
upper limit	2.24335	2.34650	2.43896	2.53384
2.64324	2.79496			
observed no	17.00000	20.00000	12.00000	8.00000
12.00000	13.00000	18.00000		
probability	0.14286	0.14286	0.14286	0.14286
0.14286	0.14286	0.14286		
expected no	14.28571	14.28571	14.28571	14.28571
14.28571	14.28571	14.28571		
chi square	0.51571	2.28571	0.36571	2.76571
0.36571	0.11571	0.96571		

degree of freedom=4
 H0: $X_1 \sim \text{Rayleigh}(\lambda, c)$, λ, c are unknown
 λ point estimated value=3.301933 (MLE)
 c point estimated value=2.027281 (MLE)
 pearson chi-square test statistic =7.380000
 p-value=0.117100

observed frequency The X1 histogram using equally probability, H0~Rayleigh

* Red line is expected number

X1 value

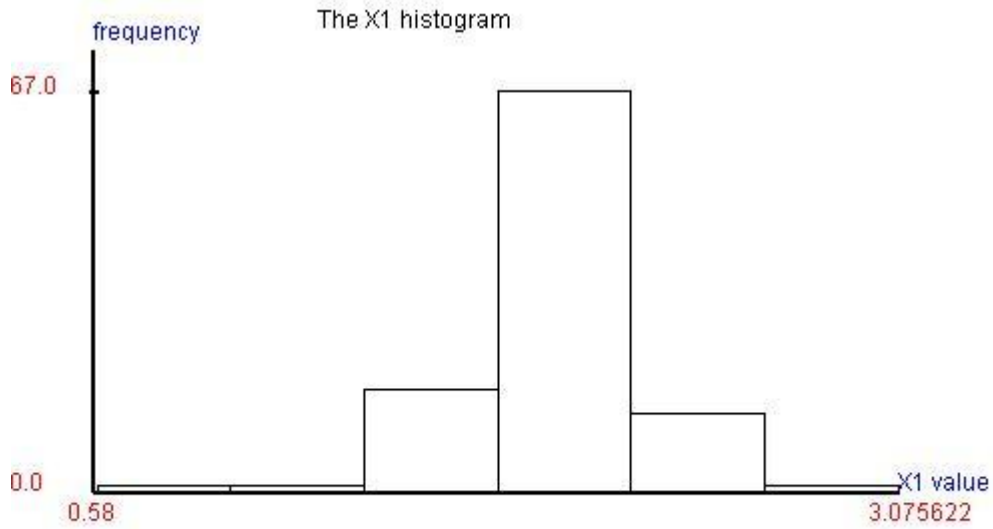
1.4.7)The population distribution is double exponential distribution.

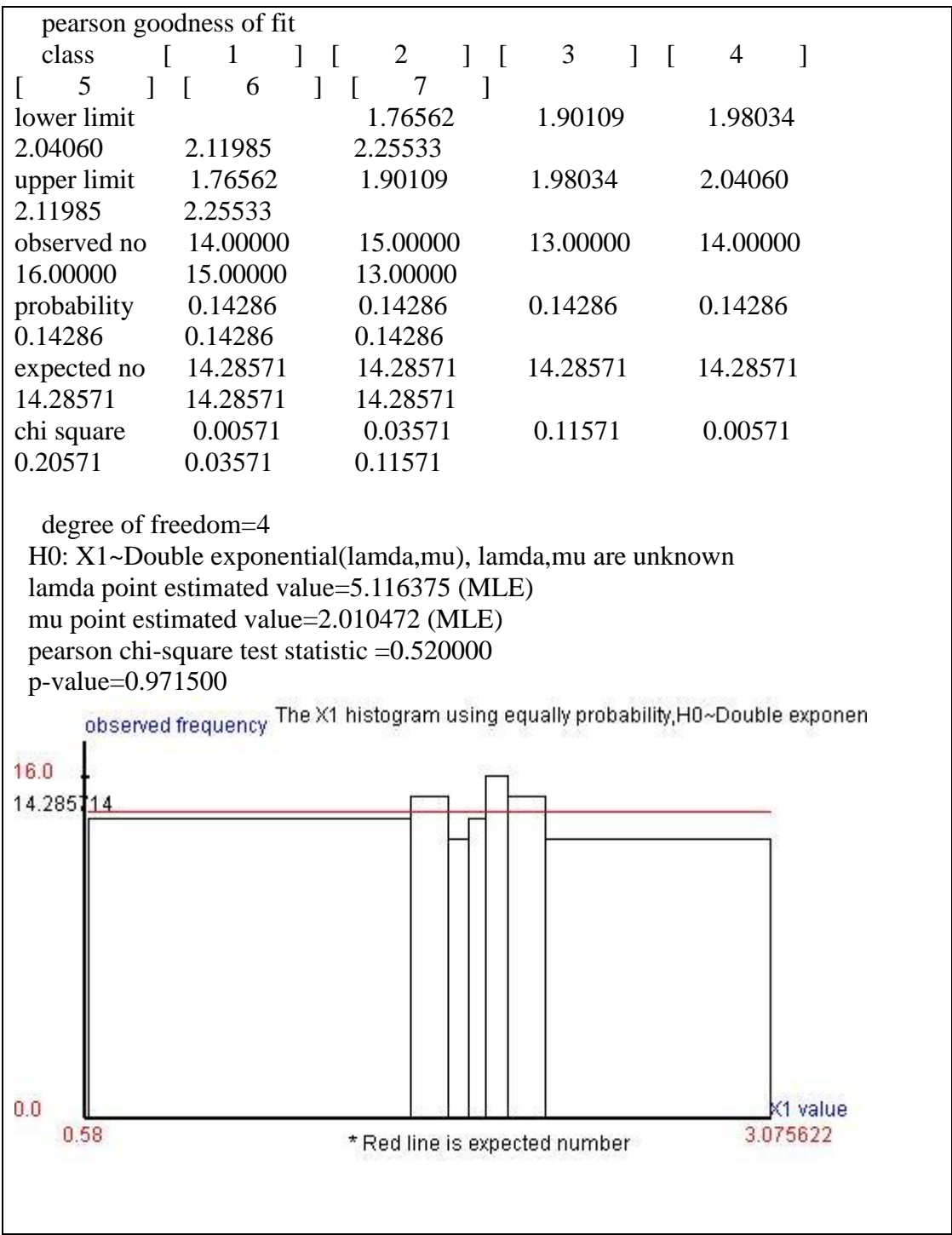
X1 is Double Exponential($\lambda=5.000000,\mu=2.000000$),

X1
1.9945619276
2.1056919562
2.1945514662
2.0400846828
1.9961652616
2.2206276674
1.8886285111
1.8693726794
0.5883767742
2.0119292800
1.9409108345
1.9898074969
2.2843436141
1.9111028012
2.3768240903
2.0992028558
2.2105056683
1.9183631081
2.2292189353
2.5449783928
1.7957578012
1.9785297423
1.9259917943
2.0067638547
2.1143066555
1.8383096343
2.0890397012
2.0141433795
2.5009498980
2.0090154538
1.9637225250
2.0629056712
2.1702492580
2.1137074383
1.7332626087
2.0131964873
2.1283966715
1.6171272437
1.5497424512
2.3694354238
3.0756229935
1.9796661171
1.5535590140
2.0063257596
2.2654244145
2.3117987196
2.0270472233
2.0281775150
1.7736433080
2.3057213008
2.2487125824
2.2595650026
2.1041295666
1.8722550365
1.7808536929
1.9506373229
1.7170064989
2.0707185106
2.1707399846
2.3162519604
1.8723897842
1.8502448772
2.0834918257
1.4684819071
1.9136466200
2.0032105912
2.0913760472
1.8562462119
1.8725059304
2.4052938574
2.0793496685

2.0428857200
 2.1588593953
 2.0556995413
 1.5389648176
 1.8646036086
 2.0836795288
 1.8200905050
 2.1695975487
 2.1276453415
 1.4096851986
 1.9775930924
 2.1688002620
 2.2186240813
 1.5599355116
 1.9655912549
 1.4387594431
 1.9455754953
 2.3910662564
 1.8415607037
 2.1207506797
 1.9468502660
 1.6820329491
 1.7631919553
 2.0654781956
 2.1170533761
 1.6506629774
 1.7670466161
 2.2456857380
 2.0001202390

X1 is Double Exponential(lamda=5.000000,mu=2.000000),





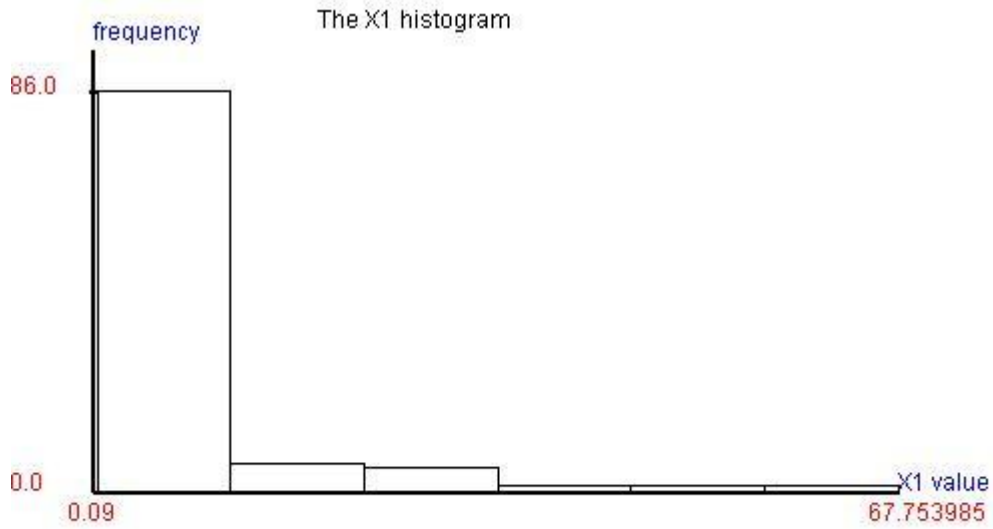
1.4.8)The population distribution is lognormal distribution.

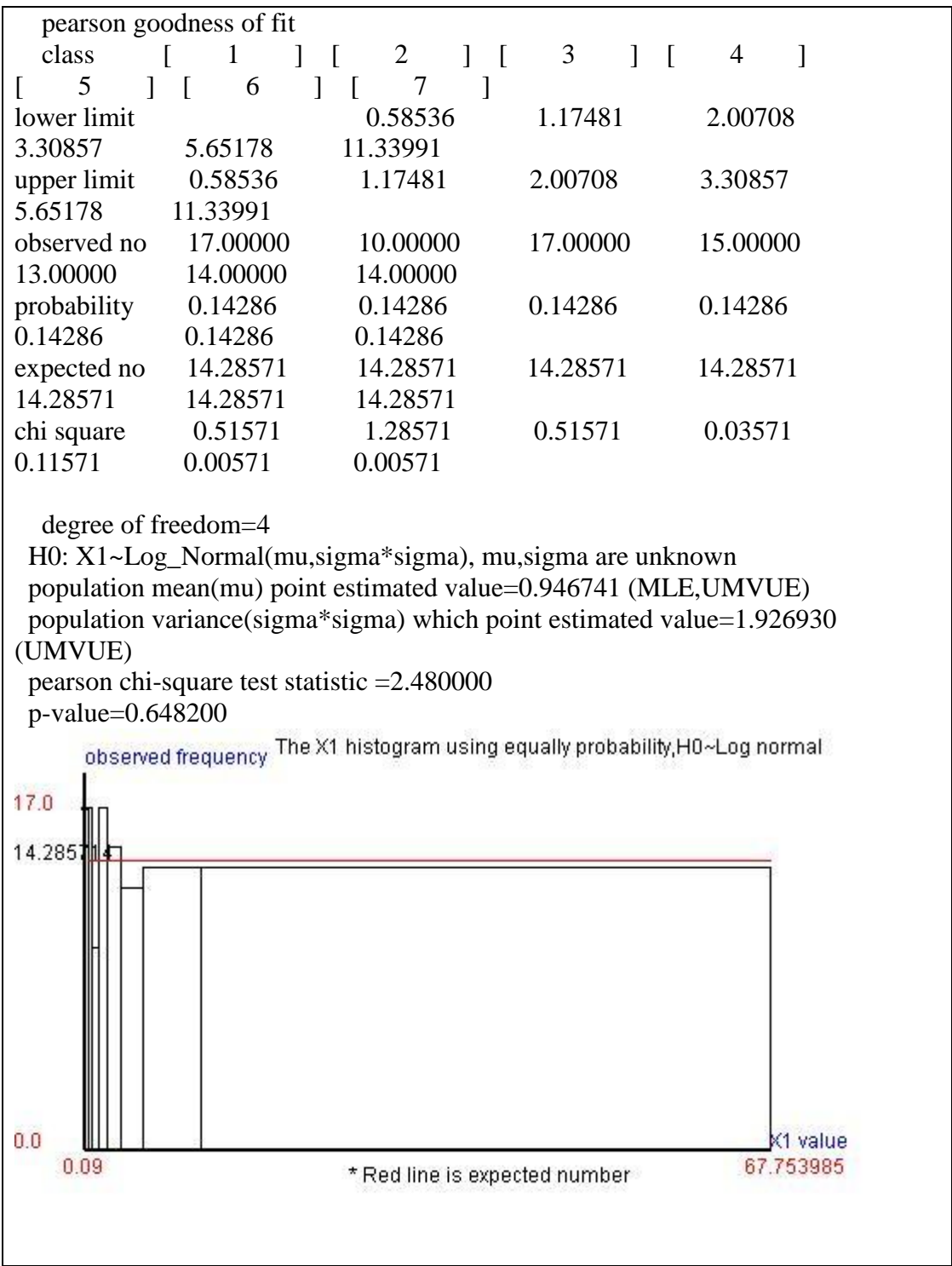
X1 is Log normal($\mu=1.000000$, $\sigma=1.500000$),

X1
24.4346188643
1.6441637932
2.0517108709
2.7586820147
4.7419130801
20.5963860430
1.6323620718
8.1061453392
9.2532203804
1.3724240658
2.0274136023
1.5391493815
2.5238756622
2.0456768925
24.8429808980
1.0980973865
2.7530096686
29.1962058069
0.1115919907
3.7636390419
1.6532860562
1.9461251849
9.9890658050
7.3301491397
4.4928196103
0.7806226446
0.4522634488
9.7169367111
0.0911755452
1.2717212057
10.8313209640
33.4949460451
1.8570802449
7.2586499051
47.1031656323
12.2971190114
0.5757388665
6.7011302515
5.3167111704
0.8320772990
10.7910035621
4.9450808773
0.6645662938
0.6897381533
11.1581833205
3.9172759846
3.2678877773
2.4177302666
0.4044753425
3.0926527169
2.8244968441
3.2084259921
1.2552047140
0.3055267714
4.3078560778
1.1636943821
19.4368006061
0.5611785289
24.4160159185
6.5064296951
0.8102685463
0.4571380202
4.9220126836
0.3299574342
1.9512243669
3.6345687677
1.2512405143
4.8438094990
13.3682135321
0.4226838012
37.6784685859

3.2297654355
 4.5642651214
 0.2910798602
 6.0366214026
 0.5759960354
 6.0680810845
 0.2823053335
 4.0001937621
 67.7539858814
 1.2274076656
 1.2203073928
 15.6130685597
 1.7045561810
 3.9454950892
 1.8935507210
 2.6042585861
 11.4860414046
 9.1280818845
 2.2510605592
 0.2142242122
 1.7451918525
 0.5285199640
 0.7299451903
 0.3419105501
 0.7419941025
 0.6351591753
 0.4689124973
 1.5236842739
 3.1374970443

X1 is Log normal($\mu=1.000000$, $\sigma=1.500000$),





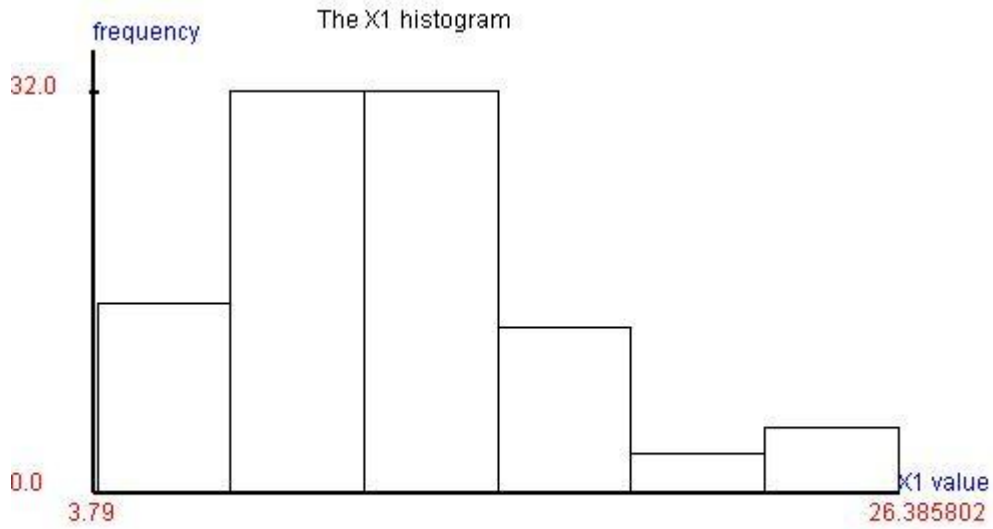
1.4.9)The population distribution is gamma distribution.

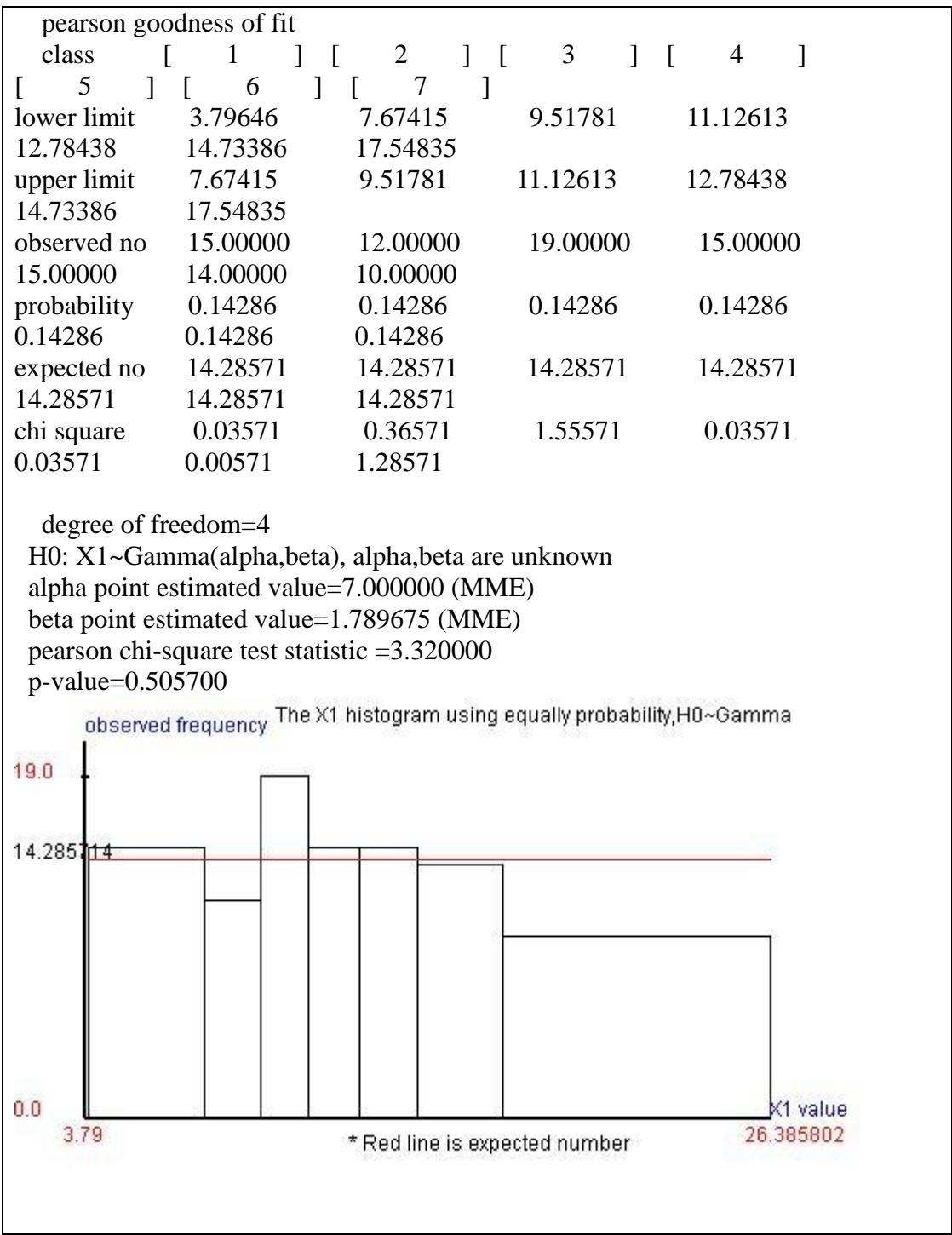
X1 is Gamma(alpha=6.000000,beta=2.000000),

X1
20.7917862390
14.9338238071
12.2034480237
12.6334794013
8.3645998060
9.1628519091
17.2627788116
5.5140865096
10.3496087279
13.3079531810
12.8328388206
5.2742318221
14.1531130799
15.2068743685
12.4667301971
13.5754718741
8.8276984224
12.5266083822
11.9776025436
5.5198272278
14.3821570912
19.8095868674
9.8734240763
13.6781063502
5.3949730740
16.5966725905
6.6466983448
14.4652429320
11.1484743667
7.5299584479
3.7964645081
8.8963413868
5.9049603451
14.0052677334
12.6306161971
10.2829424737
9.5986665123
10.6517084968
18.4461811055
16.5550021526
10.7092951201
23.1464239736
12.1770378243
24.9517151301
23.0573608656
8.2885028271
12.7663995219
15.0070981393
12.6654204158
10.4657355693
7.9999376458
11.3329527027
26.3858029609
13.1839546972
10.2883410989
8.1052593834
8.8186127766
5.4650658743
21.2902035563
6.1480142878
10.5659181325
14.5875144646
7.3482910112
17.1494970066
10.2146532111
16.9423520216
13.7641341158
6.8091704709
9.6681978089
13.4386132459
7.9436215604

22.6718362019
 17.1396527262
 12.1035547236
 15.6693925937
 14.4217852362
 9.9857538683
 5.0641213051
 12.5520793803
 15.3972622806
 8.4566522199
 16.1603616272
 12.4564173933
 5.3548998462
 9.2460422335
 18.8027944845
 10.4493482060
 10.6697974300
 14.5469560524
 10.8526463111
 14.5503689350
 10.5391555431
 10.4756992737
 8.0945212705
 15.0679254630
 4.1991363650
 9.9902908534
 12.3158172207
 11.0373642688
 17.2509544338

X1 is Gamma(alpha=6.000000,beta=2.000000),





1.4.10)The population distribution is beta distribution.

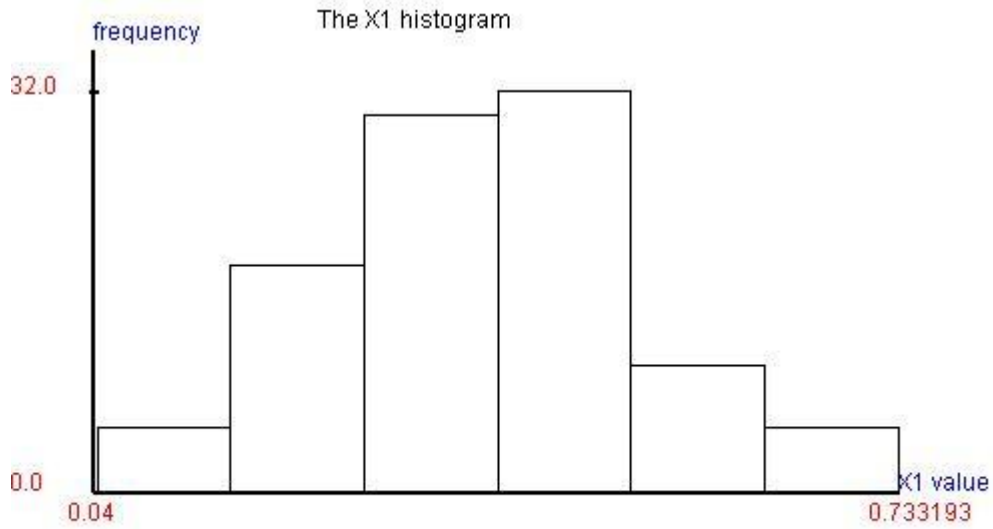
X1 is Beta(alpha=5.000000,beta=9.000000),

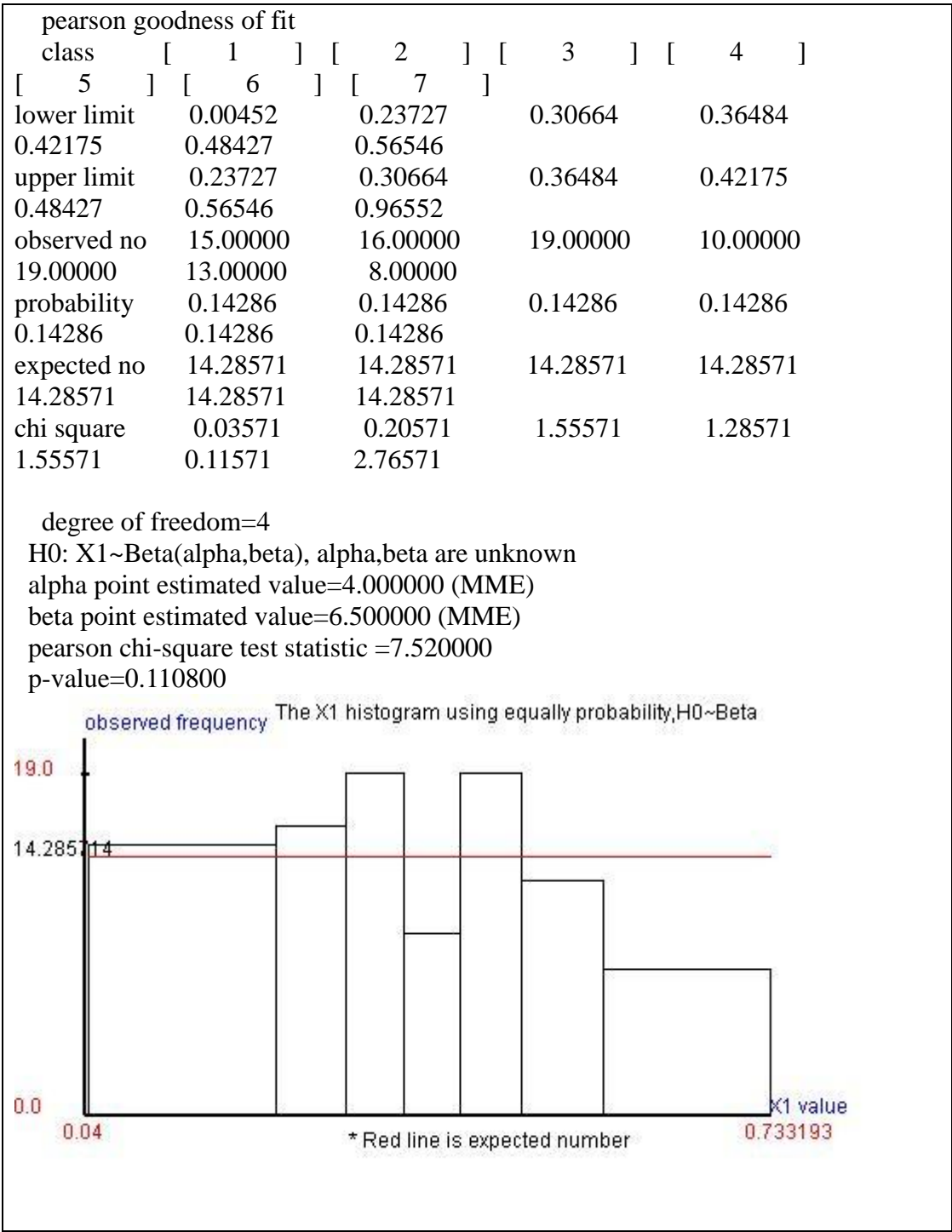
X1

0.5588880636
0.4327067646
0.5311573554
0.3590198226
0.3258200702
0.4848431136
0.3978608977
0.3904998632
0.2298528940
0.2921056724
0.4383794902
0.4626893700
0.4689726982
0.4555719325
0.2900435382
0.1232424502
0.3496299560
0.2665852189
0.1825405415
0.0469034177
0.6184360529
0.0934606164
0.3551205848
0.5239729724
0.1925452897
0.4847297867
0.3594488399
0.2042238660
0.3690036737
0.4811408651
0.3915564260
0.3766302639
0.3392691890
0.3145927200
0.3160444337
0.2782792997
0.4203263099
0.3578896222
0.4468790309
0.3068133693
0.6473516908
0.1940424181
0.6616321615
0.3402296002
0.3057229416
0.4836002992
0.3502742064
0.2671496114
0.4967829266
0.2623212757
0.2426128526
0.4339391406
0.3336933031
0.2490408063
0.4004306038
0.1701142914
0.5311046211
0.5099868143
0.2700345951
0.4790221762
0.5138628682
0.4237235248
0.6100051710
0.4641401484
0.3028305131
0.4130263392
0.1247511439
0.7081680968
0.3137495695
0.4437409355
0.2929204453

0.4414217007
0.3483680549
0.3974400638
0.4391834725
0.4636178102
0.2878917820
0.5067515608
0.1384176067
0.2596730305
0.4874255357
0.3411474388
0.5810673325
0.4797927874
0.2483758972
0.4823600150
0.2254155777
0.1882684378
0.4886082446
0.3106328821
0.3883427989
0.3644737069
0.2095267243
0.4895476834
0.7063827196
0.2834167190
0.3131420550
0.1932457136
0.4663025839
0.7331934615

X1 is Beta(alpha=5.000000,beta=9.000000),





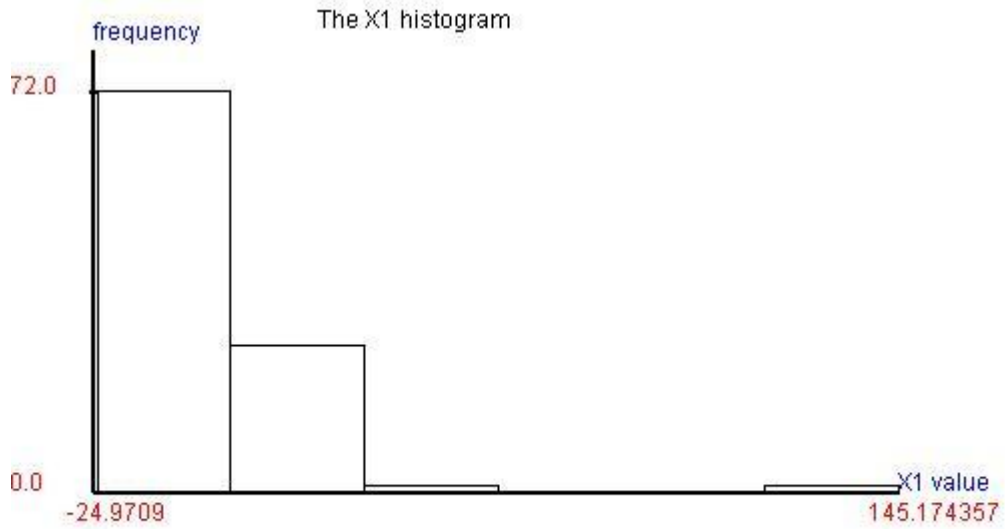
1.4.11)The population distribution is cauchy distribution.

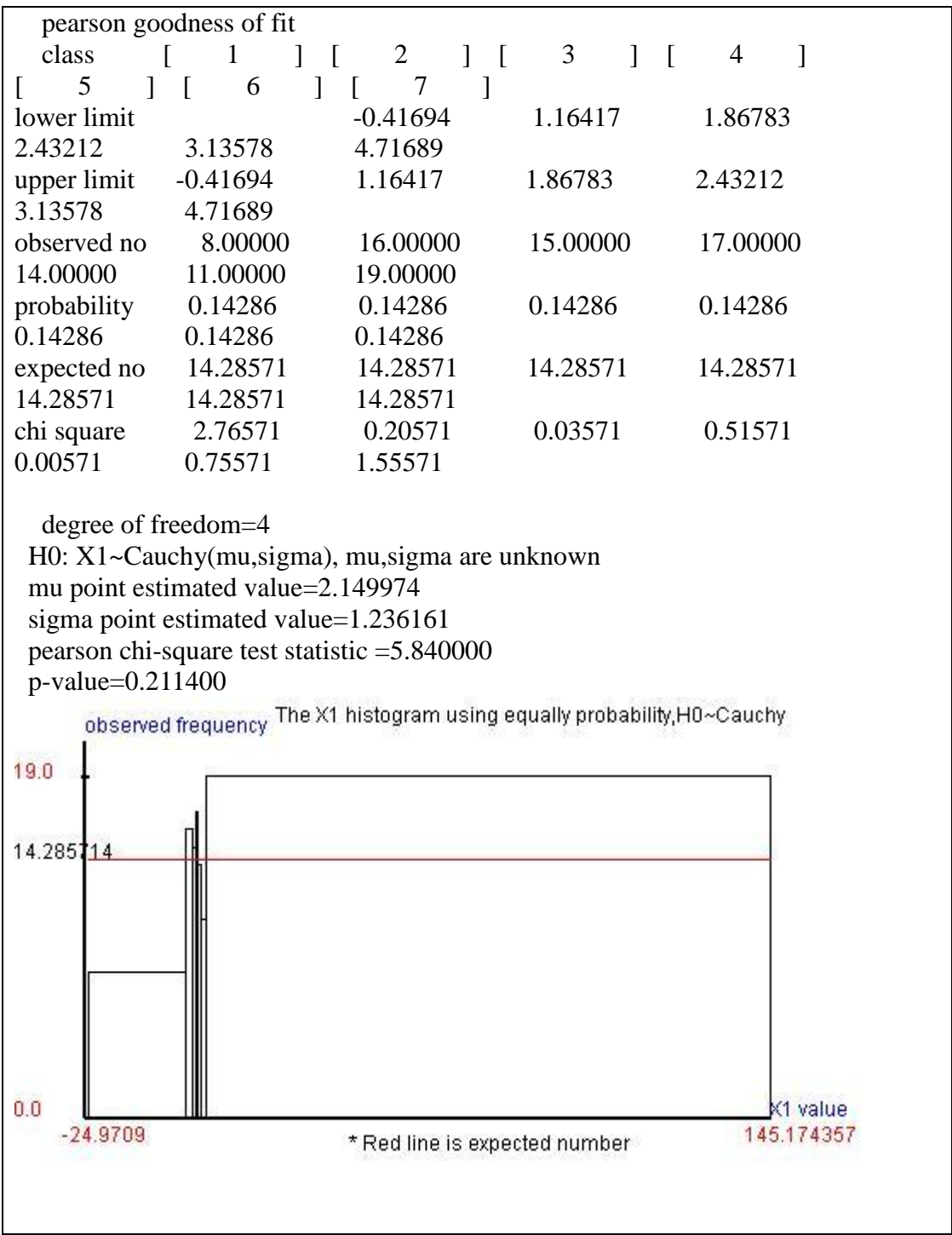
X1 is Cauchy($\mu=2.000000$, $\sigma=1.000000$),

X1
2.9553832302
0.6549396535
6.8077474306
1.8760304643
2.0013153423
2.7602720662
1.6638932341
1.7395326448
0.7883757905
10.5596704528
0.1500074041
5.1865820570
2.5428979560
1.2077708585
1.1907094059
-24.9709708387
1.4695747047
1.1485129521
1.9213210342
-2.9028298959
1.3655990623
145.1743574074
0.6943343331
2.5399015107
2.6134740035
2.1762727516
2.3226547995
1.0728785069
0.5784240731
3.5381499668
0.2064685113
1.5075940975
1.8061858243
-3.4001232400
-6.3730336099
1.7598391732
1.2768550758
4.0383089033
1.9667854492
1.2839986769
5.1491706605
2.6437178899
1.5776588568
1.1953992813
-1.2603874335
4.0482536730
51.2591623551
4.2063135190
5.3381942489
2.1961548804
15.4058763838
2.2266869463
2.1236755171
2.6711292516
3.6653768214
1.9241312292
2.6156703271
3.0969702272
10.5787142387
-0.0918669863
2.6472087987
3.3356056400
2.1060826065
4.1549075786
1.9987656391
2.1812961841
2.6055567127
6.2886418164
1.7585825719
3.4203165301
1.1162864019

1.4423470550
 -0.0657048549
 1.1075172440
 -2.3861286773
 1.8868234052
 2.9196442967
 3.6100029620
 0.7424557523
 23.6276475815
 7.7264536773
 5.3464309656
 5.4598718070
 1.8811142380
 13.9051963003
 20.1816229975
 20.0867319940
 6.8926541965
 2.4992844873
 7.6881713670
 3.2579310913
 2.2620231360
 1.1505209492
 -3.4574215475
 4.6647153760
 0.1776239509
 1.9569770568
 2.8813607779
 0.9473587077
 -1.9569089875

X1 is Cauchy(mu=2.000000,sigma=1.000000),





1.4.12)The population distribution is arcsin distribution.

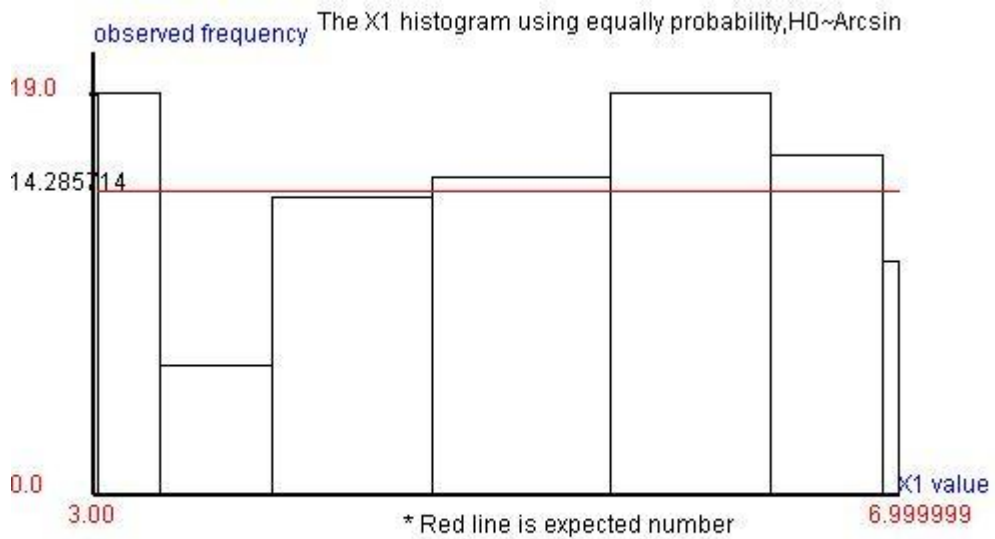
X1 is Arcsin($\mu=5.000000,c=2.000000$),

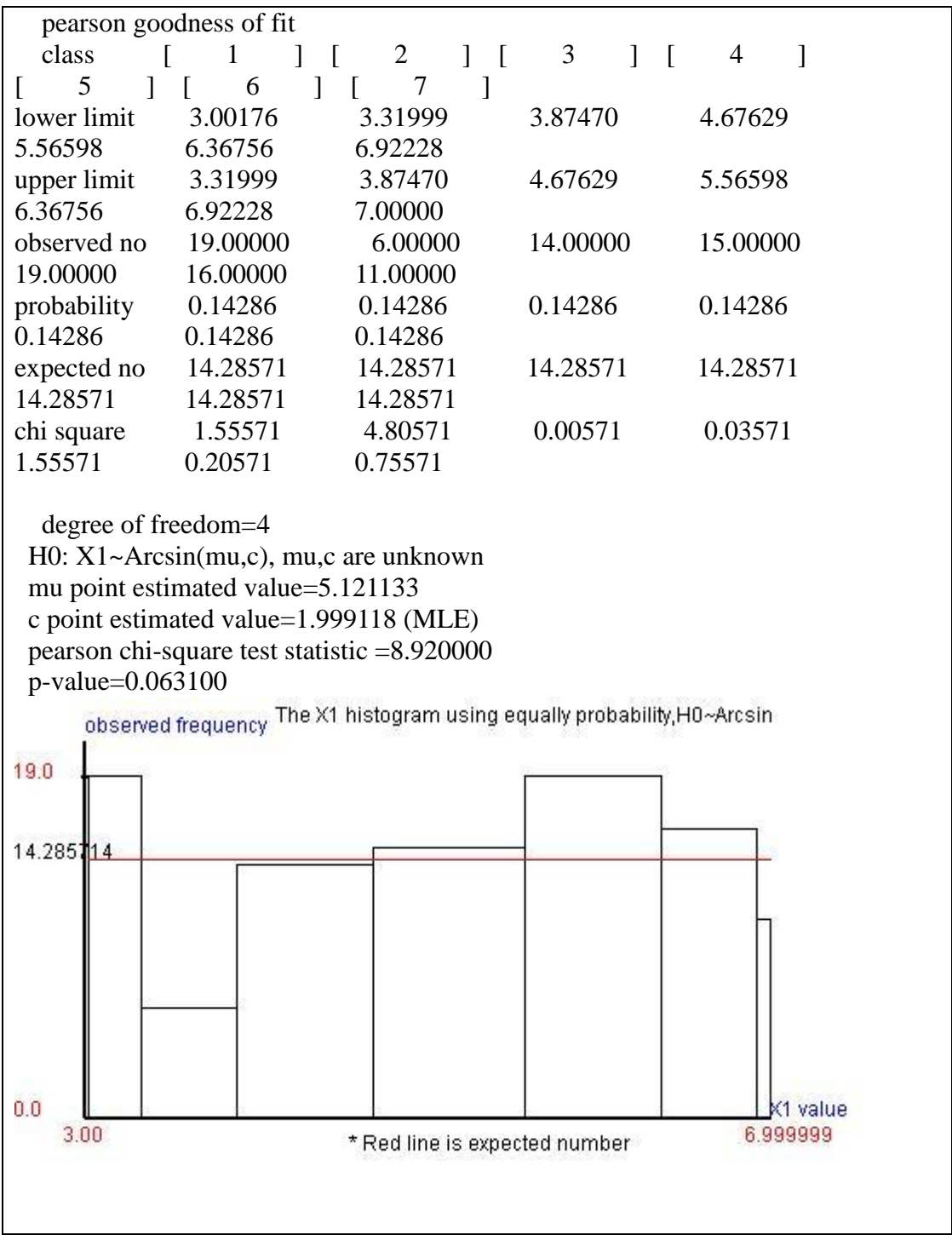
X1

4.9153253442
5.0926376307
3.1467734899
4.9166808173
3.2327469598
6.9918534558
6.9767011637
3.8999664017
6.9785601720
4.4984644724
5.7598731348
5.1286566309
6.1053365245
5.5069329324
6.9565852189
6.7537777139
4.3422609663
5.2300095218
3.1575156362
4.1078056533
4.6562574460
5.6082081605
6.6799152159
6.0191991803
3.6609916359
6.6860296517
5.1034902512
6.4363775089
6.0977707034
3.1395899583
6.6065778019
4.1569447909
6.9060190433
3.0170905559
3.0575603322
3.1015645117
3.0824717917
3.1595009971
3.0081693665
3.1844617222
5.7542382164
6.9460760354
5.7221560298
4.9790351752
6.9164034032
5.7126755908
6.4481890533
3.0017630819
6.8407853443
3.6595977333
5.6179517262
6.8947548039
3.0313317891
6.2296783786
4.6330568971
6.9352713294
4.0132214301
3.0287390990
3.6291889826
3.5245998803
3.6711401204
6.6158995693
6.8472956216
6.7068812650
3.0476367463
5.6370644499
6.3360148303
5.3410389781
3.4063075033
5.9281382724
3.9109599973

6.8378092755
 5.7570796866
 6.0430183495
 3.3153611710
 3.1392260506
 5.5501824035
 5.3432752896
 5.6964299503
 4.8971022548
 3.2675253886
 5.6279793732
 6.9320894055
 5.3228129796
 3.9056923903
 4.5003460294
 3.9465405441
 5.0506528552
 5.6874305544
 4.4712501612
 5.1696239506
 6.9757086373
 5.5853449454
 6.9428910930
 6.7655870417
 3.9808803964
 6.3751359200
 6.9892308386
 6.9999995430
 3.0033749439

X1 is Arcsin(mu=5.000000,c=2.000000).





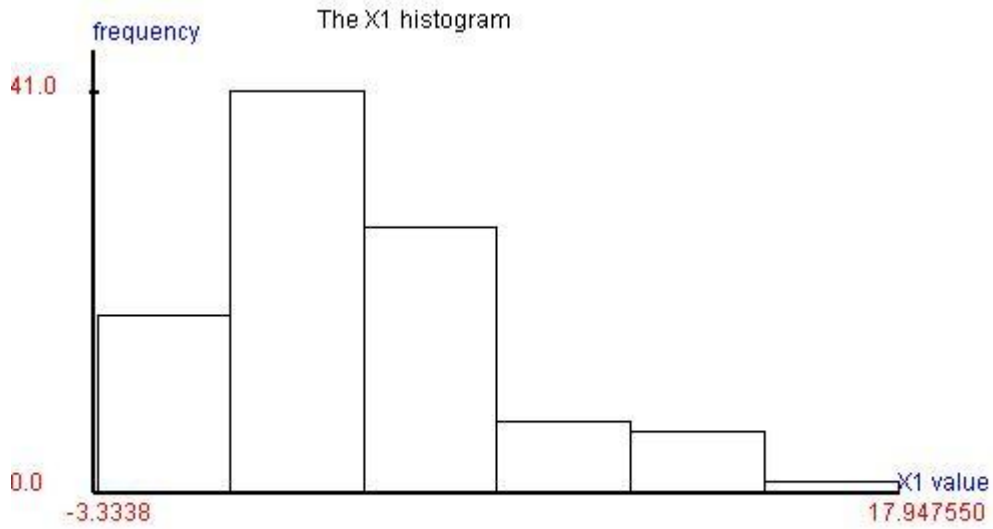
1.4.13)The population distribution is gumbel distribution.

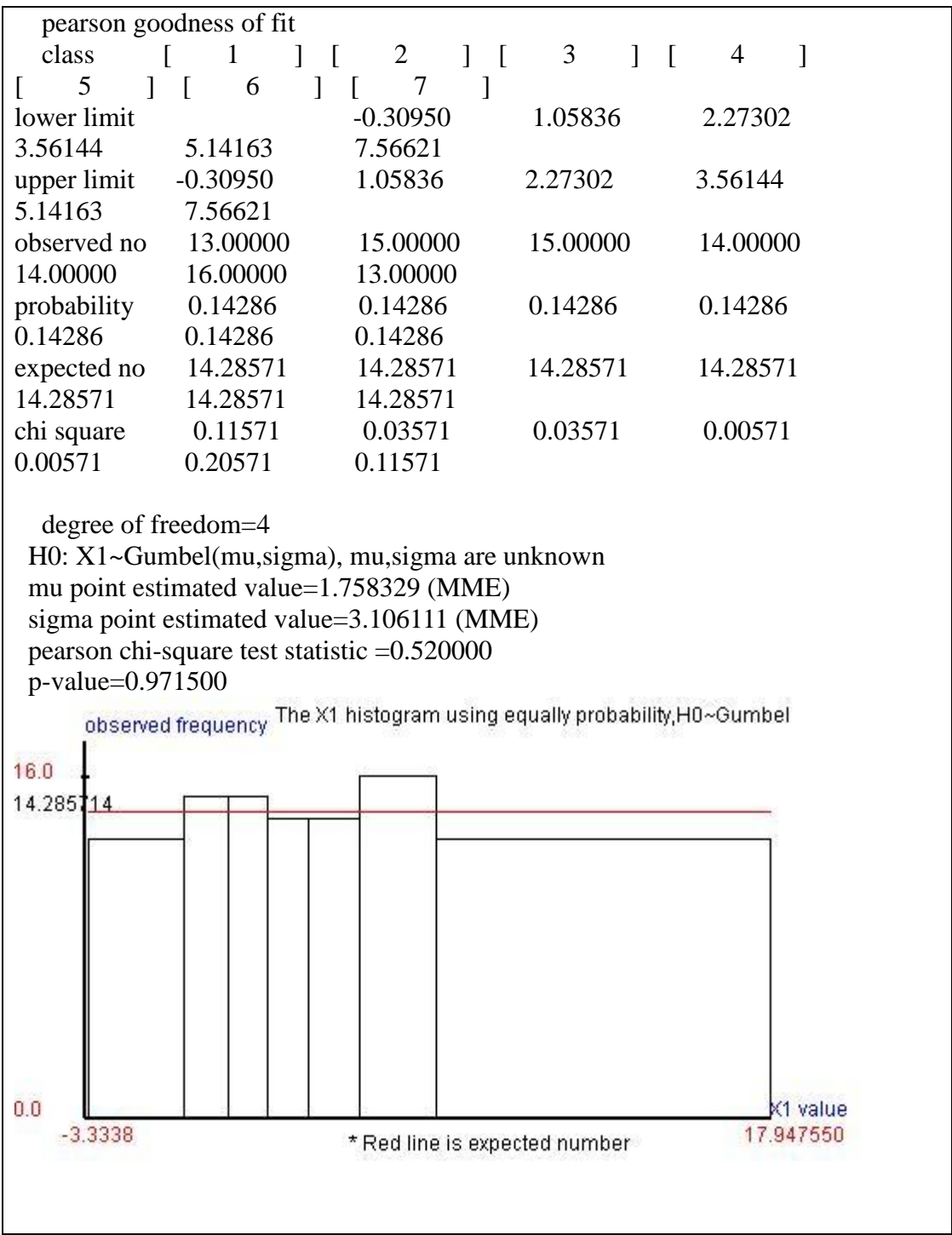
X1 is Gumbel($\mu=2.000000,\sigma=3.000000$),

X1
0.0833940241
8.5580010474
8.1416382098
6.4938567640
-0.1990518228
1.9000036632
6.2691908754
1.8509451839
1.6447302899
1.3671789811
2.7824479699
9.4311712369
-2.3820751980
9.7580922941
4.2678644075
9.6683273910
5.4292705382
0.4132143186
2.6693314329
0.3690836930
6.6099514239
2.9514533297
9.4237018136
-2.3087835228
3.6327406444
0.5573553043
6.5042206668
-1.3452878265
-1.3680413460
5.2987587861
0.8992161987
3.5099290614
2.9903482415
-0.1371182943
12.6642712239
2.6149313068
1.4502689159
3.5710568568
0.5818166635
11.4045412975
5.9096797886
-1.0456364881
-1.5283098191
7.5482385742
1.3469161770
1.1804362618
4.2198085532
1.6131625600
1.8503927463
3.0863437144
6.8815915130
2.8352631405
-3.3338522419
4.2085160643
4.0944530587
2.8282453319
-1.2354715017
2.9071599755
-2.7439427631
1.4083744190
7.1945234755
11.9481243453
5.1948635027
-0.2843134002
4.4016362793
5.1939747786
1.2774355472
1.3681524103
5.2527976466
2.1900232232
-1.9721932510

17.9475503305
 11.5653331687
 4.9685480081
 2.5981957812
 2.9553877093
 0.5578863491
 7.2621276846
 1.7390857266
 11.8065803624
 -0.7214153894
 7.1604758331
 3.4108336386
 -0.5244756263
 4.4119856012
 0.8777679161
 0.8994969660
 -0.0113456386
 2.3945295574
 4.0519699542
 13.5630733457
 4.9089056637
 1.2835028577
 3.9415972346
 0.9814647550
 -1.5553123622
 6.9228221207
 4.1577878655
 4.8704423787
 0.9054982002

X1 is Gumbel(mu=2.000000,sigma=3.000000),





1.4.14)The population distribution is triangular 1 distribution.

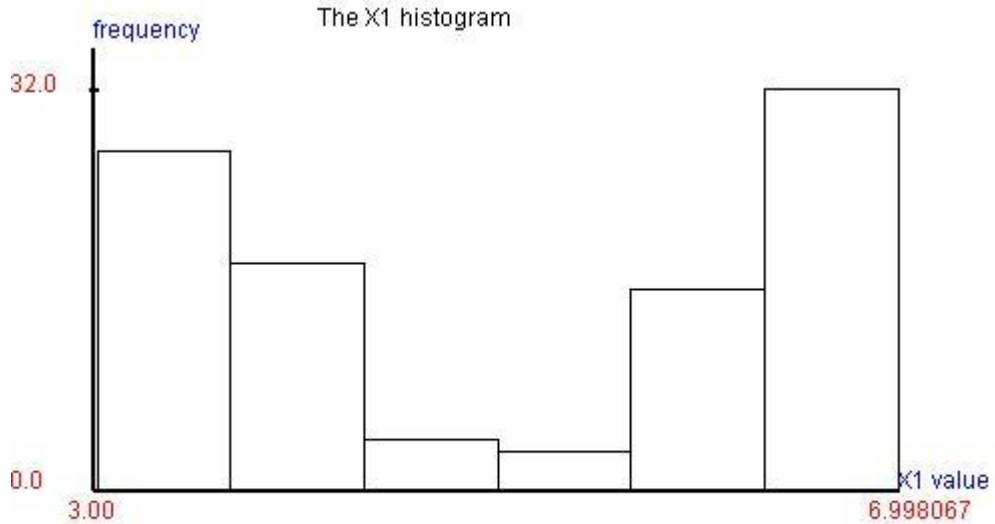
X1 is Triangular1($\mu=5.000000,c=2.000000$),

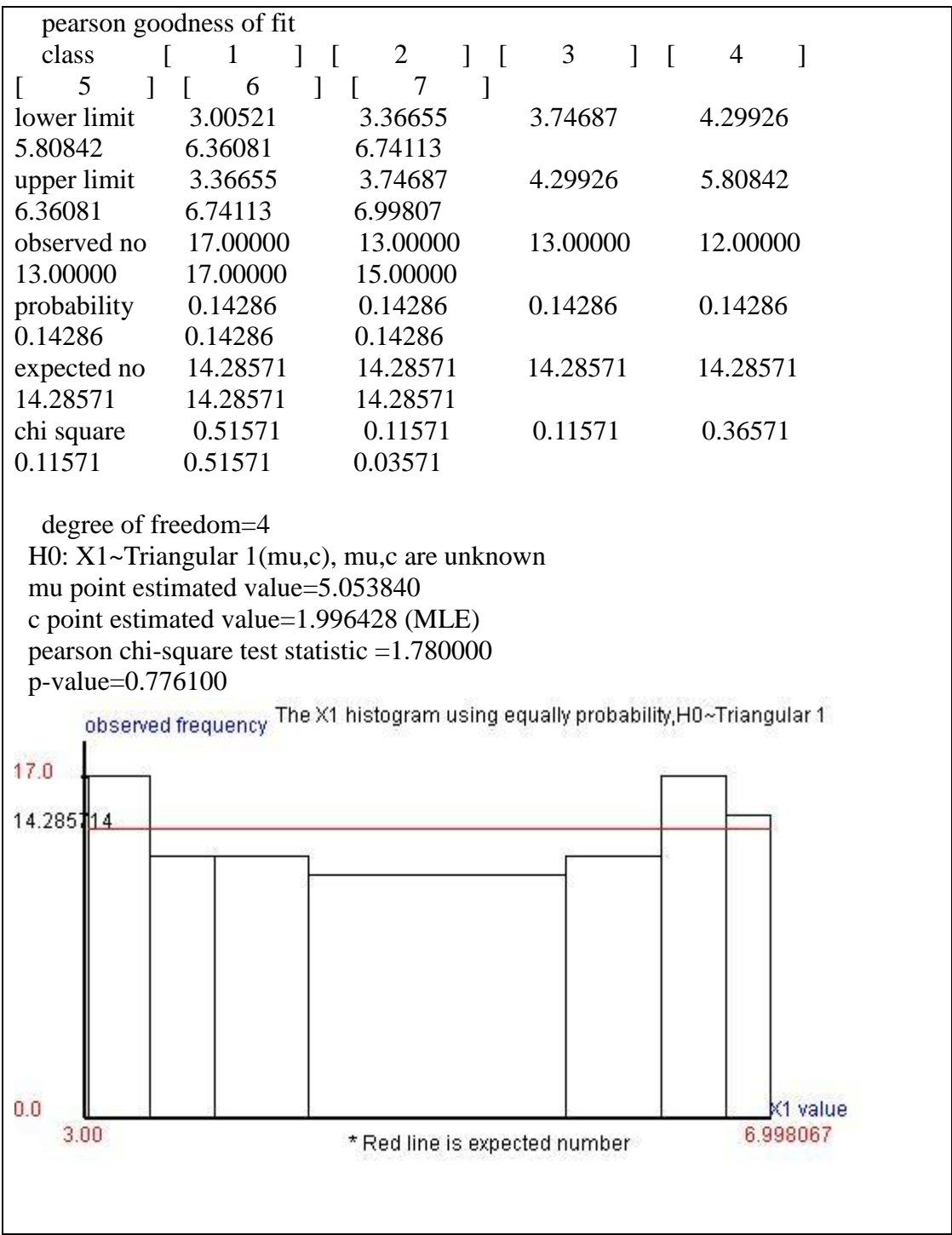
X1

3.1115152422
6.0433967327
6.6592771320
3.9236879514
6.4180777797
6.8845372428
4.2601606501
6.0425050168
6.9694746485
3.7516367697
3.0850753885
4.7365366860
3.5410434941
6.7571537450
6.6291618515
3.7832044466
6.6183569192
6.4005651021
3.5800557399
3.8981568278
6.9556309669
3.3306323374
6.7269915824
3.7668806139
4.3285999634
5.4482608580
6.7836816217
6.0948575244
6.5783756847
3.0052112611
3.5054413408
4.2397103794
4.1443499445
6.0204521379
3.3740612775
6.5933484822
6.6420847199
3.3970419839
6.6694876431
3.5441151238
3.2898378215
5.8315846424
3.2368581216
3.1980371688
5.6834583315
3.0480877040
3.3506774328
3.4083096676
6.9940852840
5.8299984966
6.4569946021
6.1693782099
3.0085169925
5.9362361853
3.2483097865
6.3673866787
4.4323451408
5.9482697769
3.7114738237
6.5766685312
6.6828004211
6.7010862574
3.9435442896
4.6232224569
3.4476891688
5.7184765919
3.2069656053
6.9468167657
5.7048694881
6.2144039581
3.8732148424

3.6876004487
 3.5259251266
 4.0309646861
 6.9115123374
 6.8815269155
 4.6383827569
 3.1780223925
 4.3057420351
 4.0262780819
 3.0519234813
 3.3825513995
 3.2956467223
 6.8905017518
 6.9980670851
 3.7348619379
 3.1965316342
 5.4284477057
 6.1154615711
 3.8435961843
 5.4587286649
 6.5191966843
 6.0546510451
 6.8300741301
 6.8363778922
 3.1980670958
 6.7796508685
 6.1848394386
 6.4334841377
 6.9330298619

X1 is Triangular1(mu=5.000000,c=2.000000),





1.4.15) The population distribution is trapezoid distribution.

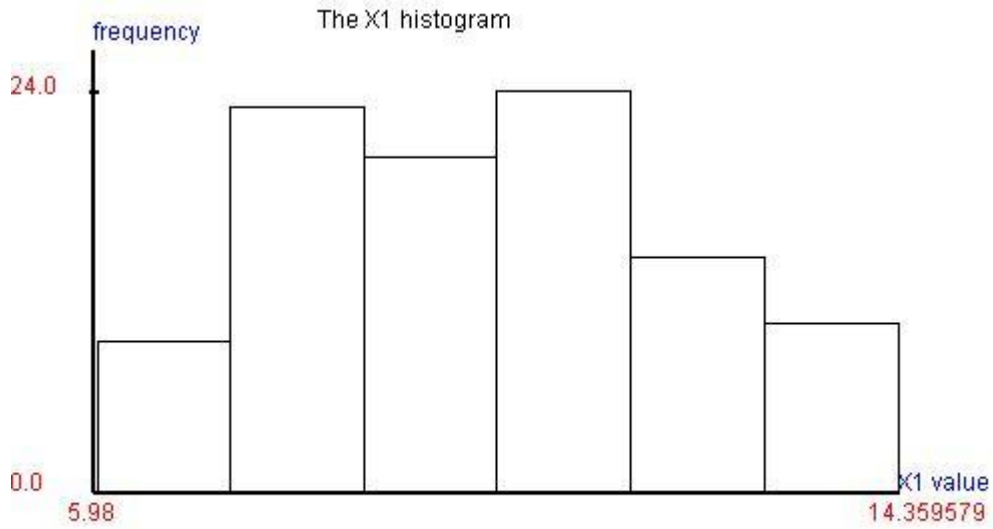
X_1 is Trapezoid($\mu=10.000000, c=3.000000$),

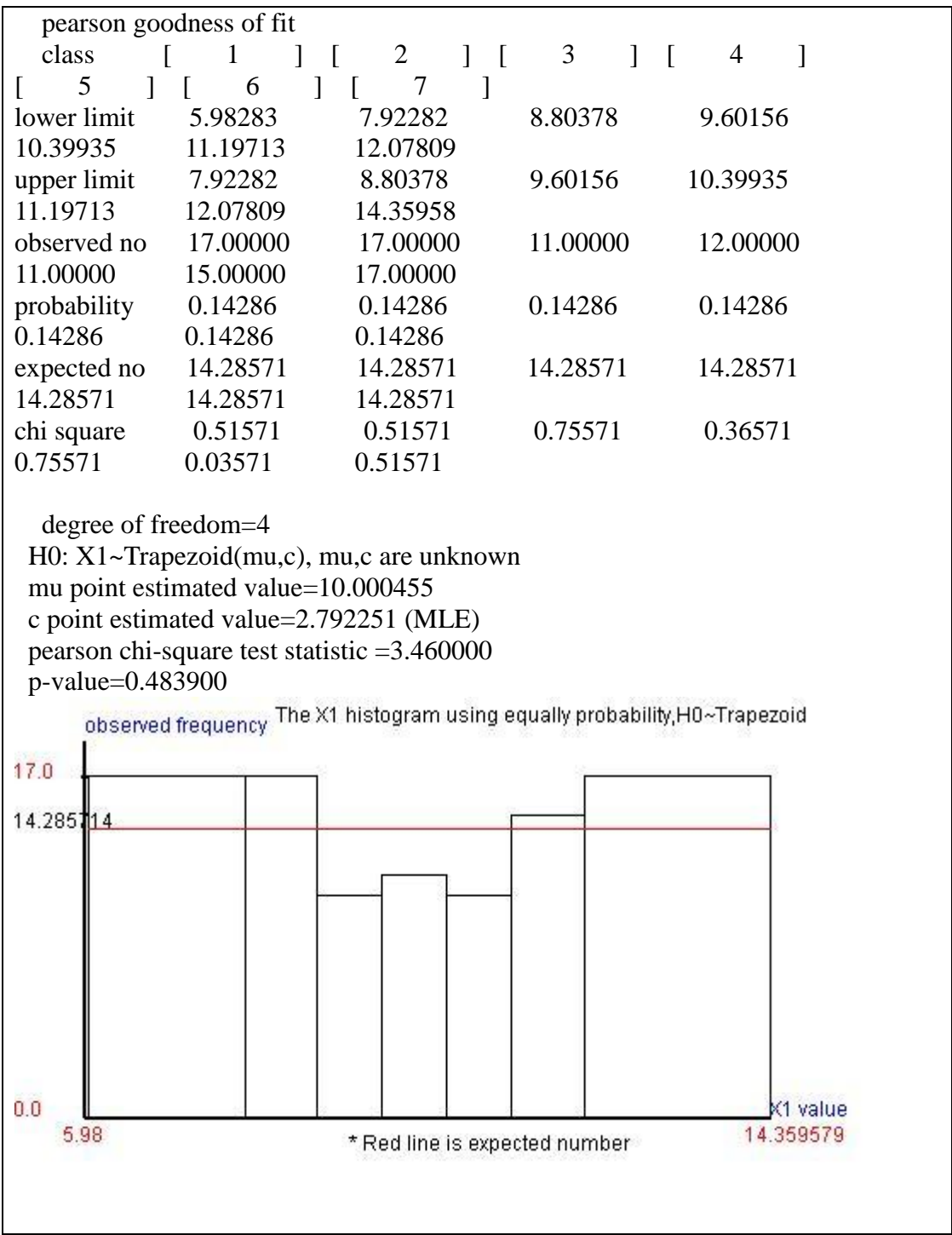
X_1

8.3971191238
10.1972625708
12.9521609042
8.6703610505
11.3853199500
7.8715491299
7.9506512509
13.0117342142
8.7106197267
12.3471125474
13.5344486951
5.9828256639
8.1967410085
10.4006287215
10.3037329969
14.3595790084
13.2138207806
12.3956583529
6.8285873431
8.7914534424
6.7425844137
13.1683686913
10.0297463820
9.8896178278
13.8487152325
9.0778769680
12.4878594331
11.6830413071
9.6415960947
6.8515536759
8.6534756930
10.9549163072
9.5136371471
10.7465990217
7.8856867942
10.1764632794
11.0587743873
9.8242003731
12.2568779465
10.4442550663
8.0870028874
8.3790019414
7.3382034994
11.2492415943
11.5274258160
8.8049216695
10.8013982816
10.9644219320
11.5725455157
9.8891328101
7.5548669362
8.0529084548
6.4161194404
11.4561171882
13.5002909349
9.2278501654
11.6727346907
7.5778516829
9.7924485962
7.7708637074
11.8319983377
11.4051728779
12.4194919625
11.8193913581
10.8895193167
12.9810234986
11.9167100104
11.4702462233
5.9878399538
10.4474942270
10.6699276666

8.8610775282
 8.1509648762
 9.3614180493
 12.6580433375
 11.3006213710
 11.6365355161
 10.2962773643
 8.8028604057
 7.8657884627
 8.6467292126
 8.5371974113
 7.9677286705
 14.0339010646
 7.2042614211
 10.3328295043
 10.4833889062
 7.4781683413
 9.3989614546
 8.2533536671
 9.0978244105
 9.0527527194
 9.3059425577
 11.3567062921
 8.2714010473
 7.2462528192
 9.2101427668
 9.9552396011
 7.6751307706
 13.6925959065

X1 is Trapezoid(mu=10.000000,c=3.000000),





1.4.16)The population distribution is U quadratic distribution.

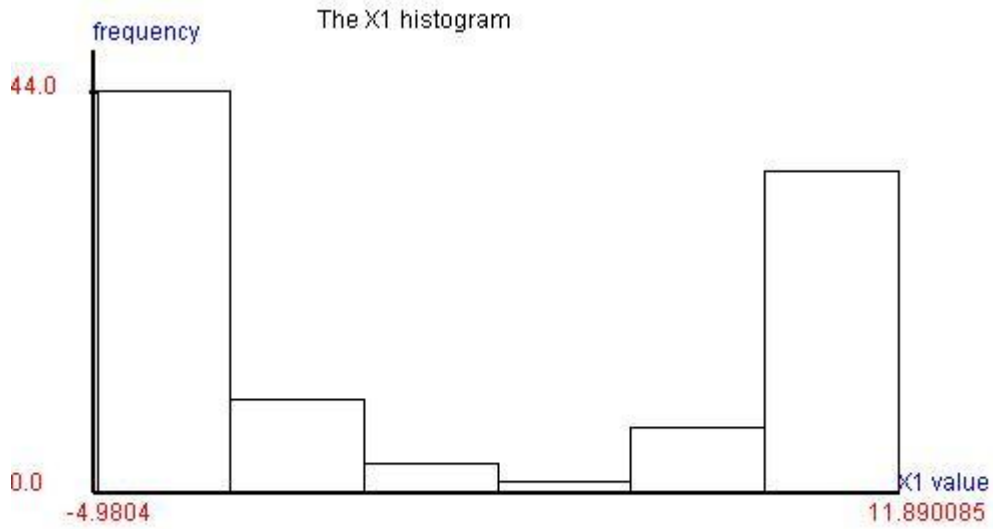
X1 is U-quadratic(a=-5.000000,b=12.000000),

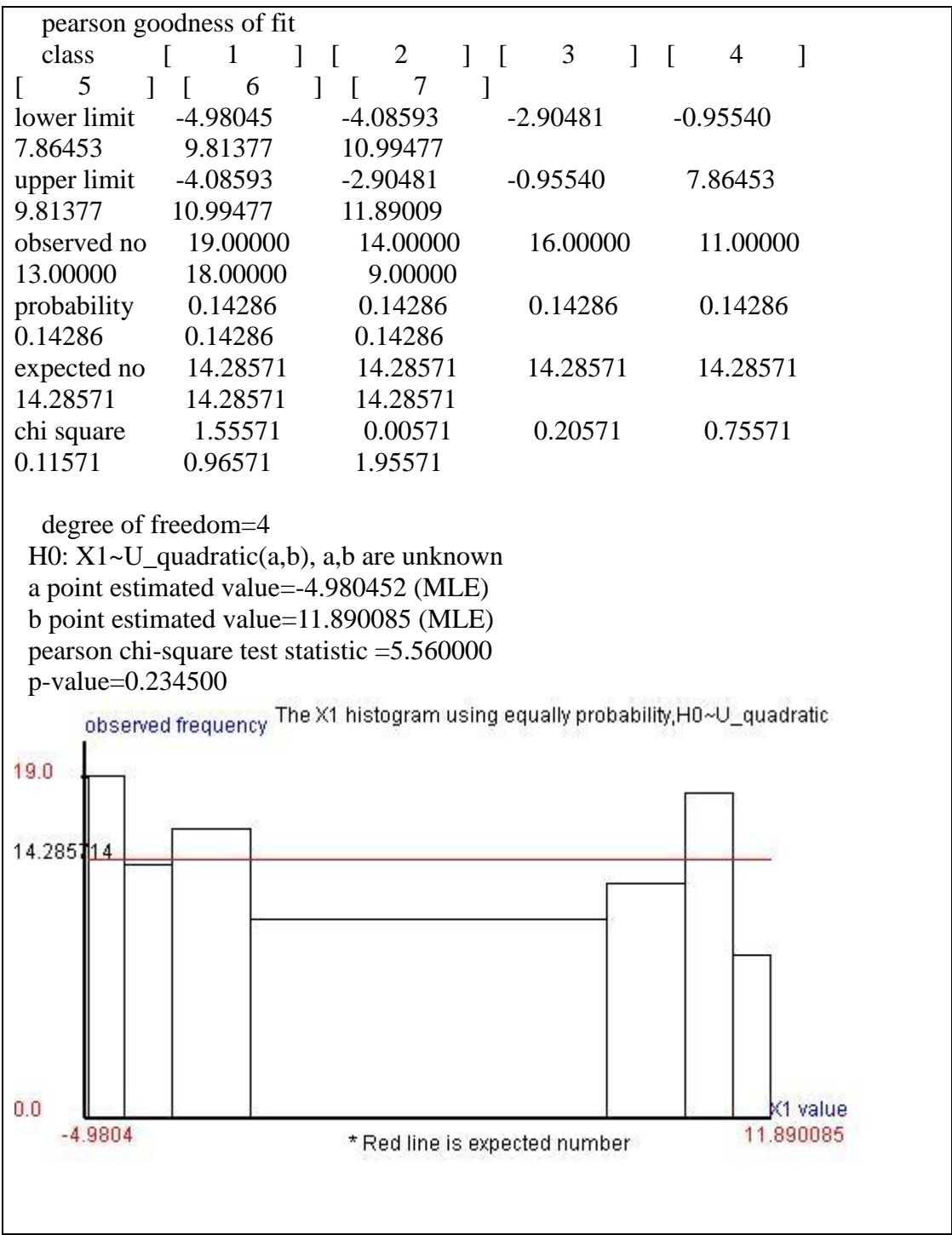
X1

-4.2396953368
-2.5268694900
-2.7573727740
-4.8766830233
-4.1206857155
8.1837411886
-4.3739864901
4.5480937227
10.9483426739
10.4613572745
10.7733654292
-0.6633002279
-2.4697943522
-2.2632124290
9.5916732703
-4.9531048477
-2.8665730486
-4.9285128137
-2.5029263002
-3.1532357691
-4.7133160819
10.4192868921
-4.6973120270
11.5970357350
10.8979937761
-2.9284548225
9.9665999082
8.7551040551
-2.6042861155
-1.0140877352
9.9754254440
-2.5795665109
-4.3542721954
9.5032269041
-4.4096860263
11.8900854851
7.7151382520
10.5808198101
11.0178539202
-3.8183047759
-3.6585548869
1.2255677039
8.2083149690
11.2547052984
9.4030626677
7.3560368047
-3.9969931600
10.3946933865
9.2097653436
-2.8270028608
-3.2789438036
-4.0009460635
-3.2379831611
11.6778956029
11.4321319121
-2.2971059694
-3.9285785007
-4.1346645639
11.7252719103
9.6227076934
10.8128799384
10.6175904072
10.9291273801
-1.0501225423
9.5314666623
8.9846543619
-2.9192447651
-4.3691873574
11.4363657272
-3.3957952577
-1.6564088684

-3.6929710780
 -4.6675632141
 11.1490614748
 -2.7658489772
 -4.2055619666
 -0.8312589328
 -4.0635705684
 10.9360700784
 -4.2072098863
 9.7579457118
 10.3408692483
 -3.2326353614
 -4.9804520494
 2.1330030475
 -1.3230421375
 8.3755369981
 9.9346785434
 -4.5115347553
 -0.3608898305
 -4.5908022896
 0.5509395082
 10.7190631150
 10.9874457434
 9.3560713353
 -1.5766111316
 1.2475326250
 10.9106548910
 -0.9098307223
 -4.1611781610

X1 is U-quadratic(a=-5.000000,b=12.000000),





1.4.17)The population distribution is semi circle distribution.

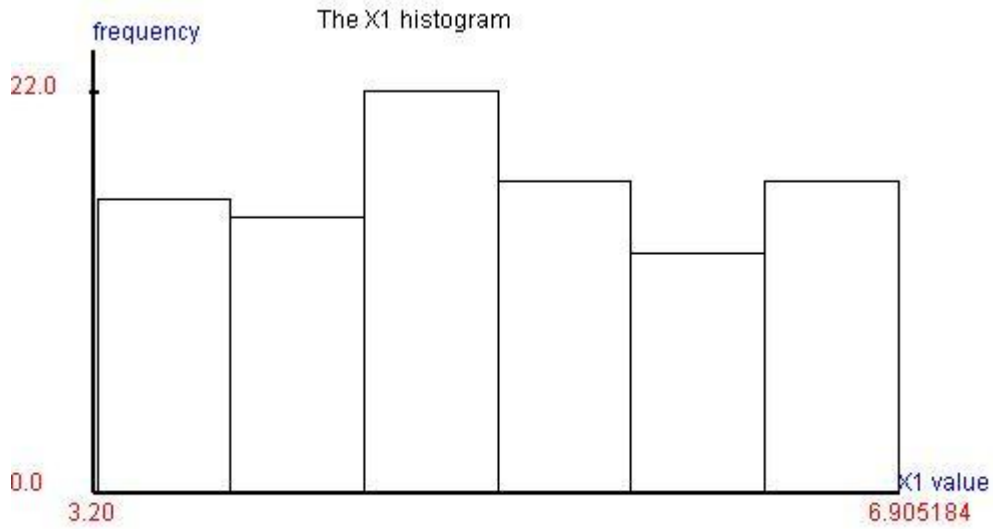
X1 is Semi-circle($\mu=5.000000,R=2.000000$),

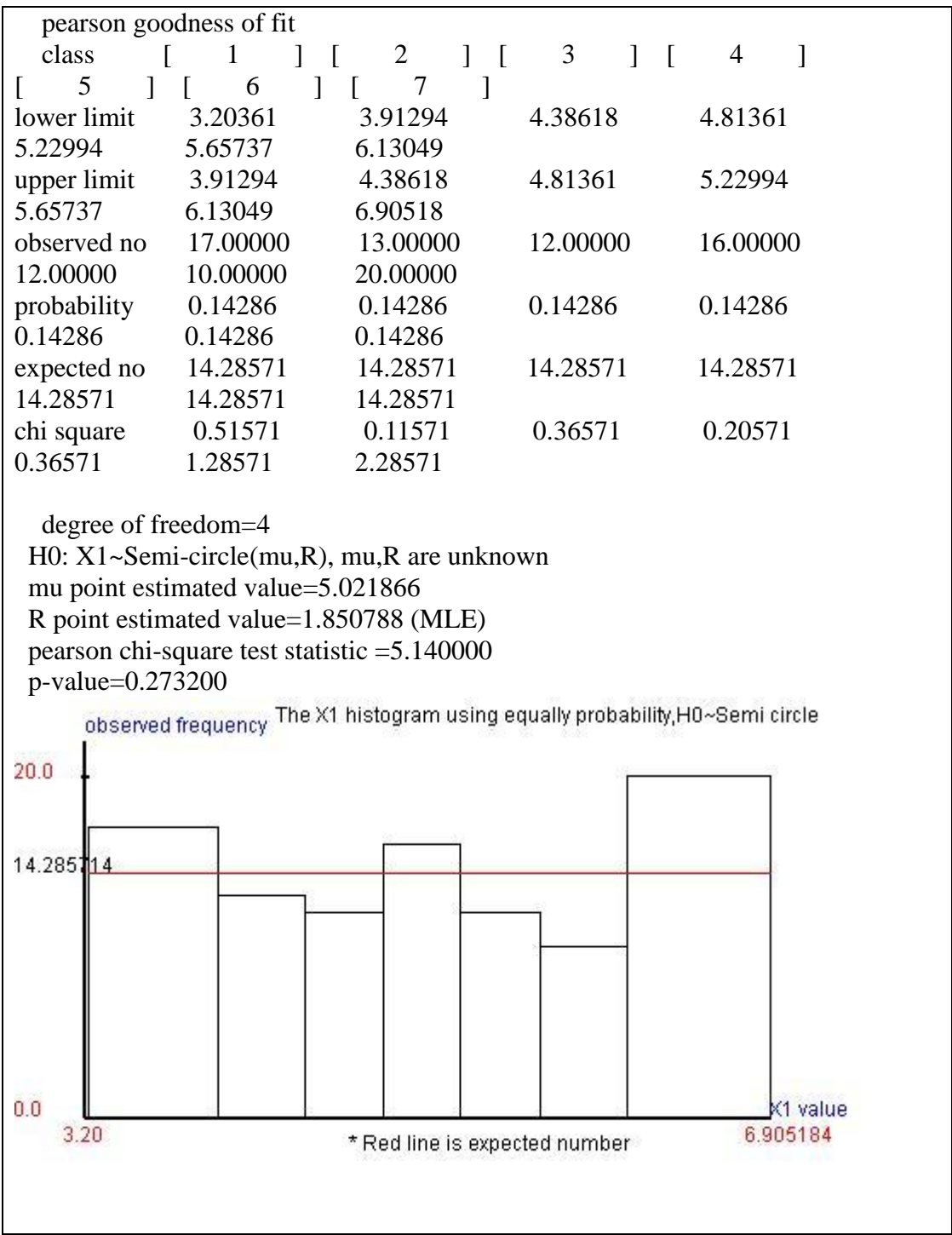
X1

3.5019828121
6.4754000158
6.1093984132
6.3318213257
4.2986399769
4.5050272314
6.2628999311
3.5875467529
3.4345327602
4.4175627508
6.9051848788
4.7546523383
6.7407044179
3.4874223947
6.5215529196
3.7406423332
6.2364950718
3.5636973846
3.9378617094
6.5236295162
4.8881965247
3.5695137621
4.0288545695
5.0005041239
5.0389218348
6.4125239774
4.2999484189
5.8802733684
5.9917972358
6.4126883086
5.2692800054
4.0255170741
5.1211045269
5.4609915787
6.7689068047
4.0238392173
3.2780459351
3.3156341443
4.4648394597
4.6640614312
6.1078388162
3.9650745844
4.2624311458
5.0514252185
6.8246365408
5.4396007695
6.3889559218
5.9073027776
5.0862342821
4.6635363872
4.9556002329
4.0659355405
4.8480771440
5.8511376423
5.6093169798
4.0222029313
4.5410005636
3.9211936549
5.7917780110
6.6222807968
6.3434741985
4.1873854753
3.5112934445
4.4920139788
5.5670461188
5.9722584985
3.4049543992
3.2350715145
3.4145921974
4.6069010927
6.2004423465

5.7511181077
 5.0543137499
 5.8007800324
 6.8663839793
 5.6068060482
 3.6359750660
 3.8160936042
 4.9667693814
 4.8884221834
 4.4582712204
 4.4842884731
 5.2832623740
 4.5156375286
 3.2036083379
 6.3397811780
 5.1364819481
 5.1227012979
 6.4406371043
 3.9895597695
 3.8697932487
 5.2982138703
 6.6747271546
 4.9087418371
 5.2139241287
 5.5776097410
 4.8166319974
 5.5610880252
 5.3275085684
 5.4643940756

X1 is Semi-circle(mu=5.000000,R=2.000000),





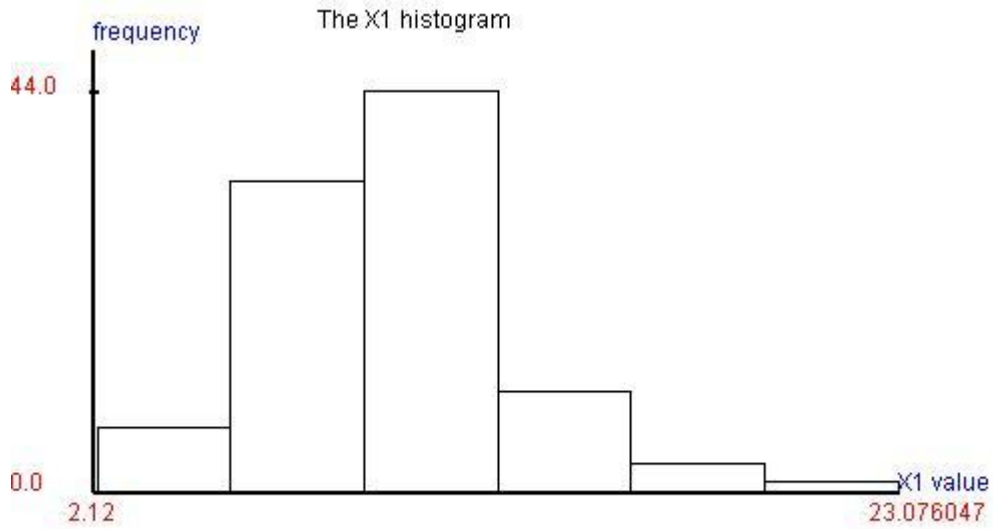
1.4.18)The population distribution is logistic distribution.

X1 is Logistic($\mu=10.000000$, $\sigma=2.000000$),

X1
7.7771736564
12.2030902336
11.0718340962
12.0934097938
9.5131929338
9.9309264444
11.0533537656
10.1976105521
7.0754929223
9.9751529699
12.6216839113
7.9718571225
10.9207045211
11.3479712262
10.3813249472
8.9203633978
8.8005670974
13.0153641267
2.1207813174
10.1889488695
14.5026688953
11.8955569592
13.9626071893
12.3602136377
4.7193163996
6.6896266386
9.0891706038
11.1580129078
10.2728606939
7.2955938937
10.3049151607
10.1302115059
10.8112950411
10.5010093828
8.4818722612
23.0760477271
18.8646286459
4.6435865758
5.9076491184
9.9042144357
12.9015401818
8.1623694543
7.4904507392
13.5341380852
11.2124461578
8.0871483035
9.9686516342
7.6464551045
12.2755430658
9.4831183700
5.9983845531
16.7569104312
9.2546142950
9.8742314955
9.9596542967
11.0542784000
8.6019260787
10.7381013371
7.9985706499
7.5102856464
8.4063624151
10.8643528136
9.7738277996
9.8140014987
13.9851803108
9.3926601074
12.6549150058
7.9310627470
6.7276287978
9.7930344483
16.4168104852

6.5813020837
 13.1274775599
 8.5807554411
 8.2075881169
 5.5651956079
 6.7558805581
 9.6064358891
 8.2708903503
 7.4849605871
 8.3381112514
 3.9269186041
 9.8736501774
 12.1838822353
 10.2730158517
 4.4423672048
 7.6369924717
 4.4518227132
 12.4227973251
 8.1415276703
 13.1310492780
 13.5989677482
 6.2119456119
 10.8243235210
 10.6655106821
 9.8593096096
 11.1861683183
 8.7384345603
 6.2580251435
 8.5134089461

X1 is Logistic(mu=10.000000,sigma=2.000000),



pearson goodness of fit				
class	[1]	[2]	[3]	[4]
[5]	[6]	[7]		
lower limit		6.74613	8.24257	9.31706
10.30053	11.37501	12.87146		
upper limit	6.74613	8.24257	9.31706	10.30053
11.37501	12.87146			
observed no	14.00000	16.00000	12.00000	20.00000
16.00000	9.00000	13.00000		
probability	0.14286	0.14286	0.14286	0.14286
0.14286	0.14286	0.14286		
expected no	14.28571	14.28571	14.28571	14.28571
14.28571	14.28571	14.28571		
chi square	0.00571	0.20571	0.36571	2.28571
0.20571	1.95571	0.11571		

degree of freedom=4

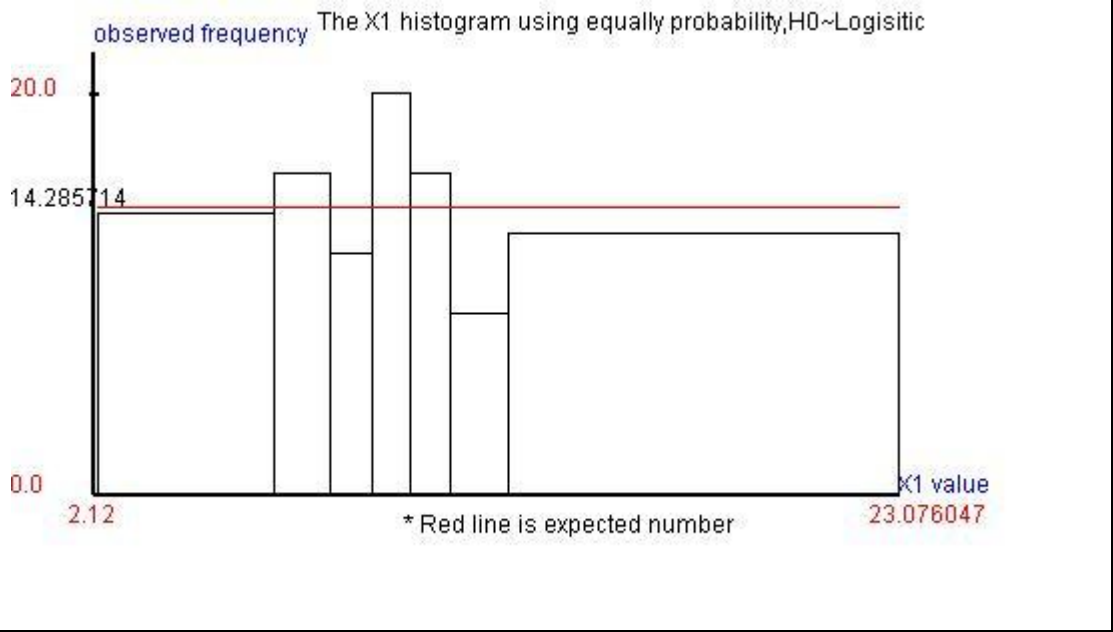
H0: X1~Logisitic(mu,sigma), mu,sigma are unknown

mu point estimated value=9.808792 (MME)

sigma point estimated value=1.709306 (MME)

pearson chi-square test statistic =5.140000

p-value=0.273200



1.4.19)The population distribution is weibull distribution.

X1 is Weibull(alpha=2.000000,beta=4.000000,gamma=2.000000),

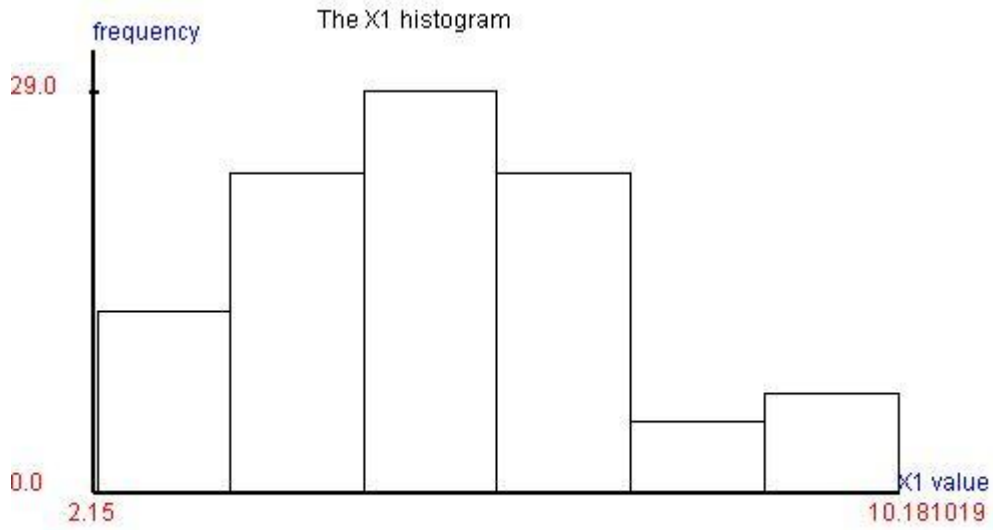
The parameter of gamma value is supposed to 2.

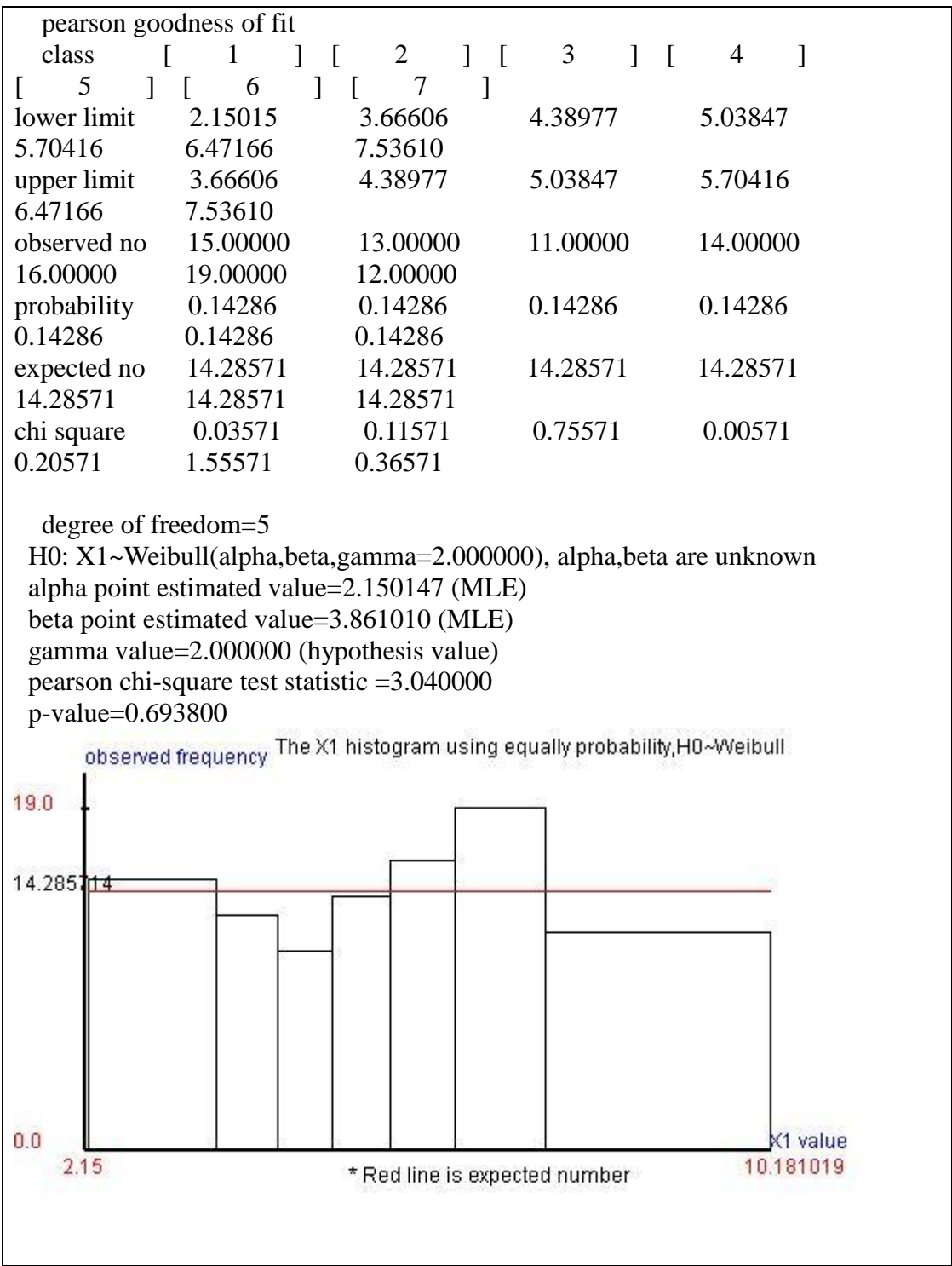
X1

6.0664551888
9.1272071944
4.7852909886
5.7593071914
4.8854974833
8.2139913772
2.5369525823
4.8896581174
6.7834400740
3.7602930113
6.4813948837
5.2526926373
3.3456509832
8.9720169292
7.0936656003
6.7260160670
6.9347976129
6.0387090356
3.8513489009
3.7198235764
3.1004133101
8.8748327453
4.5503910686
4.1694110767
3.3421656009
7.8056786692
2.7323792824
5.4057182890
5.8703046651
5.7062287983
3.6424435209
6.5932086630
5.5181391762
3.0205326798
3.8822124813
3.4980461118
5.8724033982
7.4292595473
6.6662261686
4.9690401894
6.1601851801
4.7490880593
6.8934135911
6.5674077516
8.6552664709
5.0907807817
7.9153968175
4.6794827190
3.9452414396
4.8149127992
4.6234422157
5.3536200943
7.0717728404
3.7935091001
3.0085298379
6.7064853570
5.1122558272
5.0495117688
4.7496633951
5.3965437968
3.4125548406
4.2546525783
7.0723905050
4.5581811215
5.6350709023
2.1501465147
5.6367954964
2.8224049539
8.1175562391

6.9140252190
 3.0938649405
 6.9822451519
 10.1810191599
 2.9980734919
 4.0736527207
 9.9910261941
 6.0371307800
 9.2531248674
 6.1513004176
 3.0744082487
 7.2918816617
 5.3018590973
 6.5304163153
 3.9424481361
 6.0730207672
 6.1543228612
 5.4645918522
 7.1821372722
 3.9956042226
 5.0593088808
 6.5331839376
 9.1937958227
 5.1491720367
 6.1458514449
 3.9096274134
 6.3704608194
 6.4193940187
 6.3027247646
 6.2969991148
 3.8841277620

X1 is Weibull(alpha=2.000000,beta=4.000000,gamma=2.000000),





1.4.20)The population distribution is pareto 3 distribution.

X1 is Pareto3(lamda=5.000000,c=3.000000),

X1

0.4108817395
0.1124102593
0.1445468863
0.0230333005
1.5636072225
0.0382753052
0.7529734787
0.4641464035
0.2850142014
0.0160688795
0.0309209240
0.7895569291
0.5169916332
0.4102077337
0.4191212435
0.0282555007
0.5836775905
0.3842308184
1.0419484055
0.4495994527
0.8447248492
0.8027879823
0.3916006944
0.5897683707
0.3646115650
0.2761320306
0.9612883625
0.0515279675
0.0222636566
0.4347582794
0.6367218974
1.3355879850
0.3621379587
0.8944089841
0.1240408109
0.7749387236
1.2702549940
1.7130119839
1.0446863264
0.1966438670
0.0924501245
0.6349513610
0.4518938196
0.0661805169
0.6315320457
0.1492052624
1.5330078949
0.2296450285
0.1396759771
1.2446677648
0.3425232732
1.6365857651
0.3033524275
0.5675576350
0.2833099329
0.2360844250
0.0042936256
1.3879455543
0.4754383831
0.0749122986
0.3847138640
0.2069773844
0.3413844894
0.4716967189
0.9300171875
0.0160502104
0.1325503348
0.3131398747
0.0081964190
0.1671028622
0.7030707921

0.6134065214
0.2845689210
0.5300549307
0.4457178216
0.2559240956
0.0494989503
1.5216935044
0.3457575867
0.0249438253
0.0350819457
0.8458182963
0.1169298374
0.2279840004
0.1530987026
0.6751795588
0.9375930175
0.1038765310
0.2463390856
0.5324956709
0.2374557536
0.0207975798
0.8161182871
1.1344302180
0.4110219878
0.0303101896
0.1750697647
0.3303849933
0.2511936699
1.0332413196

X1 is Pareto3(lamda=5.000000,c=3.000000),

