5)Latin square.

5.1)model:

$$X_{ijk} = \mu + \alpha_i + \beta_j + \gamma_k + \varepsilon_{ijk}, i = 1,..,n,\ j = 1,...,n, k = 1,..,n,$$

$$n_T = n^2,$$

$$\varepsilon_{ijk} \overset{iid}{\sim} N(0,\sigma^2), \sum_{i=1}^{n} \alpha_i = 0, \sum_{j=1}^{n} \beta_j = 0, \sum_{k=1}^{n} \gamma_k = 0$$

Note : The error distribution is normal distribution only.

5.2)The analysis:

$$X_{ijk} = \hat{\mu} + \hat{\alpha}_i + \hat{\beta}_j + \hat{\gamma}_k + e_{ijk}, n_T = n^2$$

$$SST = \sum_i \sum_j \sum_k \left(X_{ijk}\right)^2 - n_T \times \left(\overline{X}\,\bullet\bullet\bullet\right)^2, df = n_T - 1,,$$

$$SSA = \sum_i \sum_j \sum_k \left(\hat{\alpha}_i\right)^2, df = n-1, SSB = \sum_i \sum_j \sum_k \left(\hat{\beta}_j\right)^2, df = n-1,$$

$$SSC = \sum_i \sum_j \sum_k \left(\gamma_k\right)^2, df = n-1,$$

$$SSE = SST - SSA - SSB - SSC, df = (n-1)(n-2)$$

ANOVA

Soruce	df	SS	MS
Factor A	$n-1$	SSA	MSA=SSA/ (n-1)
Factor B	$n-1$	SSB	MSB=SSB/ (n-1)
Factor C	$n-1$	SSC	MSC=SSB/ (n-1)
Error	$(n-1)(n-2)$	SSE	MSE=SSE/$((n-1)(n-2))$
Total	n^2-1	SST	

$$H_0 : \alpha_1 = ... = \alpha_n = 0, H_1 : \text{against} \quad H_0$$

$$test \quad statistic = \frac{SSA/(n-1)}{SSE/((n-1)(n-2))},$$

$$H_0 : \beta_1 = ... = \beta_n = 0, H_1 : \text{against} \quad H_0$$

$$test \quad statistic = \frac{SSB/(n-1)}{SSE/((n-1)(n-2))},$$

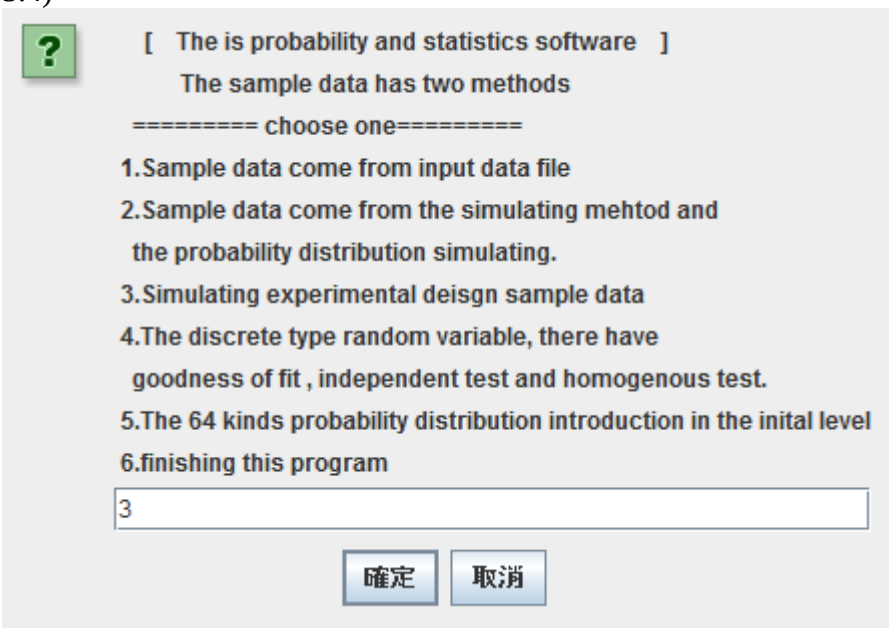
$$H_0 : \gamma_1 = ... = \gamma_n = 0, H_1 : \text{against} \quad H_0$$

$$test \quad statistic = \frac{SSC/(n-1)}{SSE/((n-1)(n-2))}$$

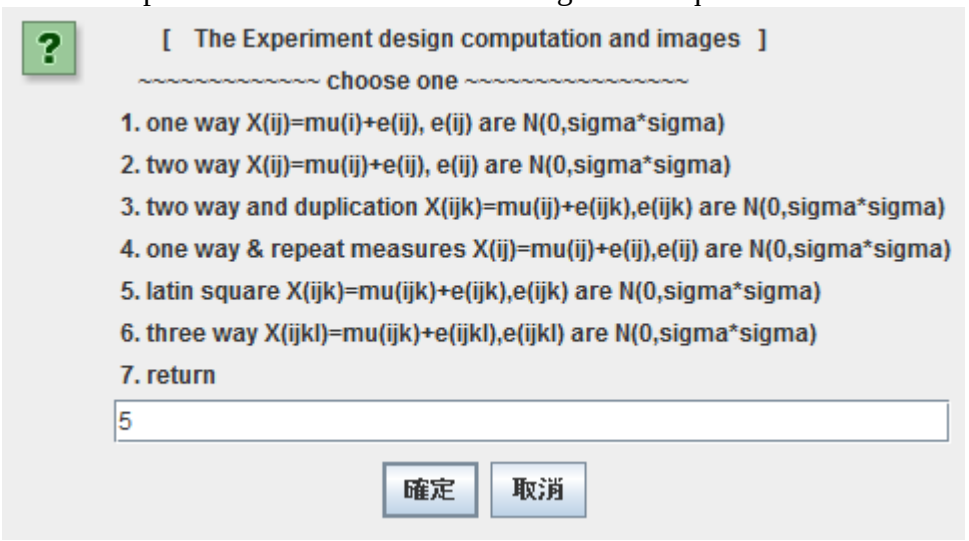
5.3)sampling form, for example n=5,

	A1	A2	A3	A4	A5
B1|	C1	C2	C3	C4	C5
B2|	C2	C3	C4	C5	C1
B3|	C3	C4	C5	C1	C2
B4|	C4	C5	C1	C2	C3
B5|	C5	C1	C2	C3	C4

5.4)



The sample data is simulated in according to the requirements.



5.4.1)The number of factor A is 4 and the number of factor B is 3.

(n=5). $X_{ijk} = \mu + \alpha_i + \beta_j + \gamma_k + \varepsilon_{ijk}, i = 1,..,n,\ j = 1,...,n, k = 1,..,n,$

5.4.1.1) (n=5). $X_{ijk} = \mu + \alpha_i + \beta_j + \gamma_k + \varepsilon_{ijk}, i = 1,..,n,\ j = 1,...,n, k = 1,..,n,$

$\alpha_1 = \alpha_2 = ... = \alpha_n = 0,\ \beta_1 = \beta_2 = ... = \beta_n = 0,\ \gamma_1 = \gamma_2 = ... = \gamma_n = 0,$

$\mu = -10,\ \varepsilon_{ij}$ ~ Normal distribution, $E(\varepsilon_{ij}) = 0, Var(\varepsilon_{ij}) = 25,$

X(1,1,1)~Normal(mu=-10.000000,sigma*sigma=25.000000), sample size=1
X(1,2,2)~Normal(mu=-10.000000,sigma*sigma=25.000000), sample size=1
X(1,3,3)~Normal(mu=-10.000000,sigma*sigma=25.000000), sample size=1
X(1,4,4)~Normal(mu=-10.000000,sigma*sigma=25.000000), sample size=1
X(1,5,5)~Normal(mu=-10.000000,sigma*sigma=25.000000), sample size=1
X(2,1,2)~Normal(mu=-10.000000,sigma*sigma=25.000000), sample size=1
X(2,2,3)~Normal(mu=-10.000000,sigma*sigma=25.000000), sample size=1
X(2,3,4)~Normal(mu=-10.000000,sigma*sigma=25.000000), sample size=1
X(2,4,5)~Normal(mu=-10.000000,sigma*sigma=25.000000), sample size=1
X(2,5,1)~Normal(mu=-10.000000,sigma*sigma=25.000000), sample size=1
X(3,1,3)~Normal(mu=-10.000000,sigma*sigma=25.000000), sample size=1
X(3,2,4)~Normal(mu=-10.000000,sigma*sigma=25.000000), sample size=1
X(3,3,5)~Normal(mu=-10.000000,sigma*sigma=25.000000), sample size=1
X(3,4,1)~Normal(mu=-10.000000,sigma*sigma=25.000000), sample size=1
X(3,5,2)~Normal(mu=-10.000000,sigma*sigma=25.000000), sample size=1
X(4,1,4)~Normal(mu=-10.000000,sigma*sigma=25.000000), sample size=1
X(4,2,5)~Normal(mu=-10.000000,sigma*sigma=25.000000), sample size=1
X(4,3,1)~Normal(mu=-10.000000,sigma*sigma=25.000000), sample size=1
X(4,4,2)~Normal(mu=-10.000000,sigma*sigma=25.000000), sample size=1
X(4,5,3)~Normal(mu=-10.000000,sigma*sigma=25.000000), sample size=1
X(5,1,5)~Normal(mu=-10.000000,sigma*sigma=25.000000), sample size=1
X(5,2,1)~Normal(mu=-10.000000,sigma*sigma=25.000000), sample size=1
X(5,3,2)~Normal(mu=-10.000000,sigma*sigma=25.000000), sample size=1
X(5,4,3)~Normal(mu=-10.000000,sigma*sigma=25.000000), sample size=1
X(5,5,4)~Normal(mu=-10.000000,sigma*sigma=25.000000), sample size=1

X(ijk)=mu+alpha(i)+beta(j)+gamma(k)+e(ijk)
i=1,..,5, j=1,..,5,k=1,..,5, total sample size=25

mu=-10.000000,
alpha(1)=0.000000, alpha(2)=0.000000, alpha(3)=0.000000, alpha(4)=0.000000, alpha(5)=0.000000,
beta(1)=0.000000, beta(2)=0.000000, beta(3)=0.000000, beta(4)=0.000000, beta(5)=0.000000,
gamma(1)=0.000000, gamma(2)=0.000000, gamma(3)=0.000000, gamma(4)=0.000000, gamma(5)=0.000000,
eijk iid ~Normal(0,sigma*sigma=5.000000)

	A1	A2	A3	A4	A5
B1	C1	C2	C3	C4	C5
B2	C2	C3	C4	C5	C1
B3	C3	C4	C5	C1	C2
B4	C4	C5	C1	C2	C3
B5	C5	C1	C2	C3	C4

	A1	A2	A3	A4	A5
B1	-14.2546236044	-9.5413822721	-8.8874897539	-9.1538288938	-8.9831354544
B2	-11.2999499365	-11.5223795892	-9.2687797031	-9.0283903982	-11.0181333517
B3	-4.9592137350	-7.5203399751	-10.1070632654	-11.3035183573	-12.2201177933
B4	-11.7738420993	-10.8063552504	-8.6315999449	-7.7689968688	-13.1270147198
B5	-8.8008822782	-6.6086093518	-12.2809454771	-10.3173933675	-5.5255247342

Latin square model

X(ijk)=mu+alpha(i)+beta(j)+gamma(k)+e(ijk),
i=1,2...,5, j=1,2...,5, k=1,2...,5, total sample size=25

	A1	A2	A3	A4	A5
factor A sample mean	-10.21770	-9.19981	-9.83518	-9.51443	-10.17479
alpha estimate value	-0.42932	0.58857	-0.04680	0.27395	-0.38640
	B1	B2	B3	B4	B5
factor B sample mean	-10.16409	-10.42753	-9.22205	-10.42156	-8.70667
beta estimate value	-0.37571	-0.63915	0.56633	-0.63318	1.08171
	C1	C2	C3	C4	C5
factor C sample mean	-10.36330	-10.62228	-9.76270	-8.64846	-9.54517
gamma estimate value	-0.57492	-0.83390	0.02568	1.13992	0.24322

Total sample size=25 , grand mean=-9.788380

summation of alpha(i)=0.000000
summation of beta(j)=0.000000
summation of gamma(k)=0.000000

ANOVA

Source	df	SS	MS	F
Factor A	4	3.7863914097	0.9465978524	0.1187324167
Factor B	4	12.2070513394	3.0517628349	0.3827847016
Factor C	4	11.9256981798	2.9814245450	0.3739621217
Error	12	95.6703699714	7.9725308309	
Total	24	123.5895109003		

H0:alpha(1)=...=alpha(5)=0
F(4,12) test value=0.118732

The F test p value=0.973300

H0:beta(1)=...=beta(5)=0
F(4,12) test value=0.382785

The F test p value=0.816800

H0:gamma(1)=...=gamma(5)=0
F(4,12) test value=0.373962

The F test p value=0.822900

class	[1]	[2]	[3]	[4]	[5]
lower limit		-2.37635	-0.71525	0.71528	2.37618
upper limit	-2.37635	-0.71525	0.71528	2.37618	
observed no	2.00000	8.00000	6.00000	5.00000	4.00000
probability	0.20000	0.20000	0.20000	0.20000	0.20000
expected no	5.00000	5.00000	5.00000	5.00000	5.00000
chi square	1.80000	1.80000	0.20000	0.00000	0.20000

degree of freedom=3

H0: residual~Normal(0,sigma(error)*sigma(error)), sigma(error) are unknown
pearson chi-square test statistic =4.000000
p-value=0.261400

H0: Variances are equal
Max(residual(ij)*residual(ij)/SSE=test value=0.227615
p value=0.276968

~~~~~ The run test of residual~~~~~~~~~~~~~~~
number of the negative of residual=12
number of the positive ofresidual=13
Run=15
H0: residualis random , H1: Increasing line or decreasing line
Z=0.622115, p-value=0.733100
H0: residual is random , H1: Oscillation
Z=0.622115, p-value=0.266900
H0: residual is random , H1: Increasing line or decreasing line or Oscillation
Z=0.622115, p-value=0.533800

multiple comparison of population means

Factor A ,there has 5 cateogries
. LSD(least significant difference)
The confidence coefficietn=0.95
95% C.I. for mu(1)-mu(2)
[-4.9085276209, 2.87274953500]
mu(1)=mu(2)

95% C.I. for mu(1)-mu(3)
[-4.2731652798, 3.50811187620]
~~~~~

mu(1)=mu(3)

95% C.I. for mu(1)-mu(4)
[-4.5939153315, 3.18736182440]
mu(1)=mu(4)

95% C.I. for mu(1)-mu(5)
[-3.9335556980, 3.84772145800]
mu(1)=mu(5)

95% C.I. for mu(2)-mu(3)
[-3.2552762368, 4.52600091910]
mu(2)=mu(3)

95% C.I. for mu(2)-mu(4)
[-3.5760262886, 4.20525086730]
mu(2)=mu(4)

95% C.I. for mu(2)-mu(5)
[-2.9156666550, 4.86561050090]
mu(2)=mu(5)

95% C.I. for mu(3)-mu(4)
[-4.2113886297, 3.56988852620]
mu(3)=mu(4)

95% C.I. for mu(3)-mu(5)
[-3.5510289962, 4.23024815980]
mu(3)=mu(5)

95% C.I. for mu(4)-mu(5)
[-3.2302789444, 4.55099821150]
mu(4)=mu(5)

Factor B ,there has 5 cateogries
. LSD(least significant difference)
The confidence coefficietn=0.95
95% C.I. for mu(1)-mu(2)
[-3.6272039780, 4.15407317800]
mu(1)=mu(2)

95% C.I. for mu(1)-mu(3)
[-4.8326799485, 2.94859720750]
mu(1)=mu(3)

95% C.I. for mu(1)-mu(4)
[-3.6331687971, 4.14810835890]
mu(1)=mu(4)

95% C.I. for mu(1)-mu(5)
[-5.3480595319, 2.43321762400]
mu(1)=mu(5)

95% C.I. for mu(2)-mu(3)
[-5.0961145485, 2.68516260750]
mu(2)=mu(3)

95% C.I. for mu(2)-mu(4)
[-3.8966033971, 3.88467375890]
mu(2)=mu(4)

95% C.I. for mu(2)-mu(5)
[-5.6114941319, 2.16978302400]
mu(2)=mu(5)

95% C.I. for mu(3)-mu(4)
[-2.6911274266, 5.09014972940]
mu(3)=mu(4)

95% C.I. for mu(3)-mu(5)
[-4.4060181614, 3.37525899450]
mu(3)=mu(5)

95% C.I. for mu(4)-mu(5)
[-5.6055293128, 2.17574784310]
mu(4)=mu(5)

Factor C ,there has 5 cateogries
. LSD(least significant difference)
The confidence coefficietn=0.95
95% C.I. for mu(1)-mu(2)

[-3.4324521041, 4.34882505180]
mu(1)=mu(2)

95% C.I. for mu(1)-mu(3)
[-4.2920323406, 3.48924481540]
mu(1)=mu(3)

95% C.I. for mu(1)-mu(4)
[-5.4062674926, 2.37500966330]
mu(1)=mu(4)

95% C.I. for mu(1)-mu(5)
[-4.5095652444, 3.27171191150]
mu(1)=mu(5)

95% C.I. for mu(2)-mu(3)
[-4.5554669406, 3.22581021540]
mu(2)=mu(3)

95% C.I. for mu(2)-mu(4)
[-5.6697020926, 2.11157506330]
mu(2)=mu(4)

95% C.I. for mu(2)-mu(5)
[-4.7729998444, 3.00827731150]
mu(2)=mu(5)

95% C.I. for mu(3)-mu(4)
[-4.4642261221, 3.31705103380]
mu(3)=mu(4)

95% C.I. for mu(3)-mu(5)
[-3.5675238739, 4.21375328200]
mu(3)=mu(5)

95% C.I. for mu(4)-mu(5)
[-4.7670350253, 3.01424213060]
mu(4)=mu(5)

~~~~~~~~~~~~ error ~~~~~~~~~~~~
~~~~~~~~~~~~

-3.08629	0.86804	1.29772	-0.40361	1.32415
0.39080	-1.70910	0.06562	0.88197	0.37071
4.66648	-0.02677	-1.08143	-1.78051	-1.77776
-2.06288	-1.21658	2.41167	3.21251	-2.34473
0.09190	2.08441	-2.69358	-1.91036	2.42763

The common population standard deviation and variance confidence interval
90% confidence interval for population variance
[4.550101 , 18.306665]
90% confidence interval for population standard deviation
[2.133097 , 4.278629]
95% confidence interval for population variance
[4.099511 , 21.724543]
95% confidence interval for population standard deviation
[2.024725 , 4.660959]
99% confidence interval for population variance
[3.380537 , 31.123722]
99% confidence interval for population standard deviation
[1.838624 , 5.578864]

5.4.1.2) (n=5). $X_{ijk} = \mu + \alpha_i + \beta_j + \gamma_k + \varepsilon_{ijk}, i = 1,..,n,\ j = 1,...,n, k = 1,..,n,$

$\alpha_1 = 0, \alpha_2 = 5, \alpha_3 = 10, \alpha_4 = 15, \alpha_5 = 20,$

$\beta_1 = \beta_2 = ... = \beta_n = 0,\ \gamma_1 = \gamma_2 = ... = \gamma_n = 0,$

$\mu = -10,\ \varepsilon_{ij}$ ~Normal distribution, $E(\varepsilon_{ij}) = 0, Var(\varepsilon_{ij}) = 25,$

X(1,1,1)~Normal(mu=-10.000000,sigma*sigma=25.000000), sample size=1
X(1,2,2)~Normal(mu=-10.000000,sigma*sigma=25.000000), sample size=1
X(1,3,3)~Normal(mu=-10.000000,sigma*sigma=25.000000), sample size=1
X(1,4,4)~Normal(mu=-10.000000,sigma*sigma=25.000000), sample size=1
X(1,5,5)~Normal(mu=-10.000000,sigma*sigma=25.000000), sample size=1
X(2,1,2)~Normal(mu=-10.000000,sigma*sigma=25.000000), sample size=1
X(2,2,3)~Normal(mu=-10.000000,sigma*sigma=25.000000), sample size=1
X(2,3,4)~Normal(mu=-10.000000,sigma*sigma=25.000000), sample size=1
X(2,4,5)~Normal(mu=-10.000000,sigma*sigma=25.000000), sample size=1
X(2,5,1)~Normal(mu=-10.000000,sigma*sigma=25.000000), sample size=1
X(3,1,3)~Normal(mu=-10.000000,sigma*sigma=25.000000), sample size=1
X(3,2,4)~Normal(mu=-10.000000,sigma*sigma=25.000000), sample size=1
X(3,3,5)~Normal(mu=-10.000000,sigma*sigma=25.000000), sample size=1
X(3,4,1)~Normal(mu=-10.000000,sigma*sigma=25.000000), sample size=1
X(3,5,2)~Normal(mu=-10.000000,sigma*sigma=25.000000), sample size=1
X(4,1,4)~Normal(mu=-10.000000,sigma*sigma=25.000000), sample size=1
X(4,2,5)~Normal(mu=-10.000000,sigma*sigma=25.000000), sample size=1
X(4,3,1)~Normal(mu=-10.000000,sigma*sigma=25.000000), sample size=1
X(4,4,2)~Normal(mu=-10.000000,sigma*sigma=25.000000), sample size=1
X(4,5,3)~Normal(mu=-10.000000,sigma*sigma=25.000000), sample size=1
X(5,1,5)~Normal(mu=-10.000000,sigma*sigma=25.000000), sample size=1
X(5,2,1)~Normal(mu=-10.000000,sigma*sigma=25.000000), sample size=1
X(5,3,2)~Normal(mu=-10.000000,sigma*sigma=25.000000), sample size=1
X(5,4,3)~Normal(mu=-10.000000,sigma*sigma=25.000000), sample size=1
X(5,5,4)~Normal(mu=-10.000000,sigma*sigma=25.000000), sample size=1

X(ijk)=mu+alpha(i)+beta(j)+gamma(k)+e(ijk)
i=1,..,5, j=1,..,5,k=1,..,5, total sample size=25

mu=-10.000000,
alpha(1)=0.000000, alpha(2)=0.000000, alpha(3)=0.000000, alpha(4)=0.000000, alpha(5)=0.000000,
beta(1)=0.000000, beta(2)=0.000000, beta(3)=0.000000, beta(4)=0.000000, beta(5)=0.000000,
gamma(1)=0.000000, gamma(2)=0.000000, gamma(3)=0.000000, gamma(4)=0.000000, gamma(5)=0.000000,
eijk iid ~Normal(0,sigma*sigma=5.000000)

	A1	A2	A3	A4	A5
B1	C1	C2	C3	C4	C5
B2	C2	C3	C4	C5	C1
B3	C3	C4	C5	C1	C2

B4| C4 C5 C1 C2 C3
B5| C5 C1 C2 C3 C4

	A1	A2	A3	A4	A5
B1 |	-9.6705719975	-10.9845552667	-11.0769320970	-6.1752435002	-14.3728315696
B2 |	-7.8801962723	-9.9796208731	-13.1053905828	-8.4973618035	-6.0503822298
B3 |	-11.2943464329	-11.9904108811	-4.0499197134	-7.6452628593	-10.0086095852
B4 |	-10.0374821971	-9.5267014771	-8.8707238205	-7.7523832919	-11.6610021242
B5 |	-7.0195841583	-11.4175600463	-7.6414977696	-9.1699139827	-7.8931779929

Latin square model

X(ijk)=mu+alpha(i)+beta(j)+gamma(k)+e(ijk),
i=1,2...,5, j=1,2...,5, k=1,2...,5, total sample size=25

	A1	A2	A3	A4	A5
factor A sample mean	-9.18044	-10.77977	-8.94889	-7.84803	-9.99720
alpha estimate value	0.17043	-1.42890	0.40197	1.50283	-0.64633
	B1	B2	B3	B4	B5
factor B sample mean	-10.45603	-9.10259	-8.99771	-9.56966	-8.62835
beta estimate value	-1.10516	0.24828	0.35316	-0.21879	0.72252
	C1	C2	C3	C4	C5
factor C sample mean	-8.73090	-8.85345	-10.63636	-9.84034	-8.69328
gamma estimate value	0.61997	0.49742	-1.28550	-0.48947	0.65759

Total sample size=25 , grand mean=-9.350867

summation of alpha(i)=0.000000
summation of beta(j)=0.000000
summation of gamma(k)=0.000000

ANOVA

Source	df	SS	MS	F
Factor A	4	24.5432494288	6.1358123572	0.8590062565
Factor B	4	9.8882240970	2.4720560243	0.3460848324
Factor C	4	14.7814506260	3.6953626565	0.5173462709
Error	12	85.7150314450	7.1429192871	
Total	24	134.9279555969		

H0:alpha(1)=...=alpha(5)=0
F(4,12) test value=0.859006

The F test p value=0.515600

H0:beta(1)=...=beta(5)=0
F(4,12) test value=0.346085

The F test p value=0.841800

H0:gamma(1)=...=gamma(5)=0
F(4,12) test value=0.517346

The F test p value=0.724800

class	[1]	[2]	[3]	[4]	[5]
lower limit		-2.24932	-0.67702	0.67704	2.24915
upper limit	-2.24932	-0.67702	0.67704	2.24915	
observed no	2.00000	7.00000	9.00000	4.00000	3.00000
probability	0.20000	0.20000	0.20000	0.20000	0.20000
expected no	5.00000	5.00000	5.00000	5.00000	5.00000
chi square	1.80000	0.80000	3.20000	0.20000	0.80000

degree of freedom=3

H0: residual~Normal(0,sigma(error)*sigma(error)), sigma(error) are unknown
pearson chi-square test statistic =6.800000
p-value=0.078500

H0: Variances are equal
Max(residual(ij)*residual(ij)/SSE=test value=0.180011
p value=0.602193

~~~~~ The run test of residual~~~~~~~~~~~~~~
number of the negative of residual=14
number of the positive ofresidual=11
Run=15
H0: residualis random , H1: Increasing line or decreasing line
Z=0.696925, p-value=0.757100
H0: residual is random , H1: Oscillation
Z=0.696925, p-value=0.242900
H0: residual is random , H1: Increasing line or decreasing line or Oscillation
Z=0.696925, p-value=0.485800

multiple comparison of population means

Factor A ,there has 5 cateogries
. LSD(least significant difference)
The confidence coefficietn=0.95
95% C.I. for mu(1)-mu(2)
[-2.0833182927, 5.28198528710]
mu(1)=mu(2)

95% C.I. for mu(1)-mu(3)
[-3.9141952048, 3.45110837490]
mu(1)=mu(3)

95% C.I. for mu(1)-mu(4)
[-5.0150549140, 2.35024866580]
mu(1)=mu(4)
~~~~~

95% C.I. for mu(1)-mu(5)
[-2.8658873012, 4.49941627860]
mu(1)=mu(5)

95% C.I. for mu(2)-mu(3)
[-5.5135287021, 1.85177487770]
mu(2)=mu(3)

95% C.I. for mu(2)-mu(4)
[-6.6143884112, 0.75091516860]
mu(2)=mu(4)

95% C.I. for mu(2)-mu(5)
[-4.4652207984, 2.90008278140]
mu(2)=mu(5)

95% C.I. for mu(3)-mu(4)
[-4.7835114990, 2.58179208080]
mu(3)=mu(4)

95% C.I. for mu(3)-mu(5)
[-2.6343438862, 4.73095969350]
mu(3)=mu(5)

95% C.I. for mu(4)-mu(5)
[-1.5334841771, 5.83181940270]
mu(4)=mu(5)

Factor B ,there has 5 cateogries
. LSD(least significant difference)
The confidence coefficietn=0.95
95% C.I. for mu(1)-mu(2)
[-5.0360883238, 2.32921525600]
mu(1)=mu(2)

95% C.I. for mu(1)-mu(3)
[-5.1409687817, 2.22433479810]
mu(1)=mu(3)

95% C.I. for mu(1)-mu(4)
[-4.5690200939, 2.79628348590]

mu(1)=mu(4)

95% C.I. for mu(1)-mu(5)
[-5.5103318861, 1.85497169370]
mu(1)=mu(5)

95% C.I. for mu(2)-mu(3)
[-3.7875322478, 3.57777133200]
mu(2)=mu(3)

95% C.I. for mu(2)-mu(4)
[-3.2155835600, 4.14972001980]
mu(2)=mu(4)

95% C.I. for mu(2)-mu(5)
[-4.1568953522, 3.20840822760]
mu(2)=mu(5)

95% C.I. for mu(3)-mu(4)
[-3.1107031021, 4.25460047770]
mu(3)=mu(4)

95% C.I. for mu(3)-mu(5)
[-4.0520148943, 3.31328868550]
mu(3)=mu(5)

95% C.I. for mu(4)-mu(5)
[-4.6239635821, 2.74133999770]
mu(4)=mu(5)

Factor C ,there has 5 cateogries
. LSD(least significant difference)
The confidence coefficietn=0.95
95% C.I. for mu(1)-mu(2)
[-5.2852302389, 2.08007334080]
mu(1)=mu(2)

95% C.I. for mu(1)-mu(3)
[-3.5023155741, 3.86298800570]
mu(1)=mu(3)

95% C.I. for mu(1)-mu(4)
[-4.2983376452, 3.06696593450]
mu(1)=mu(4)

95% C.I. for mu(1)-mu(5)
[-5.4453989317, 1.91990464810]
mu(1)=mu(5)

95% C.I. for mu(2)-mu(3)
[-2.1488790402, 5.21642453960]
mu(2)=mu(3)

95% C.I. for mu(2)-mu(4)
[-2.9449011114, 4.42040246840]
mu(2)=mu(4)

95% C.I. for mu(2)-mu(5)
[-4.0919623978, 3.27334118200]
mu(2)=mu(5)

95% C.I. for mu(3)-mu(4)
[-2.8400206534, 4.52528292630]
mu(3)=mu(4)

95% C.I. for mu(3)-mu(5)
[-3.9870819399, 3.37822163990]
mu(3)=mu(5)

95% C.I. for mu(4)-mu(5)
[-4.5590306277, 2.80627295210]
mu(4)=mu(5)

~~~~~~~~~~~~ error ~~~~~~~~~~~~

-0.00494	0.40296	0.26262	3.26742	-3.92806
0.55455	1.83737	-3.91530	-1.55519	3.07858
-1.18157	-1.07432	3.88823	-0.77035	-0.86198
-0.14878	0.81427	-0.32301	-0.18298	-0.15951
0.78075	-1.98028	0.08746	-0.75890	1.87098

The common population standard deviation and variance confidence interval
~~~~~~~~~~~~

90% confidence interval for population variance
[4.076623 , 16.401696]
90% confidence interval for population standard deviation
[2.019065 , 4.049901]
95% confidence interval for population variance
[3.672921 , 19.463914]
95% confidence interval for population standard deviation
[1.916487 , 4.411793]
99% confidence interval for population variance
[3.028762 , 27.885027]
99% confidence interval for population standard deviation
[1.740334 , 5.280628]

5.4.1.3) (n=5). $X_{ijk} = \mu + \alpha_i + \beta_j + \gamma_k + \varepsilon_{ijk}, i = 1,..,n,\ j = 1,...,n, k = 1,..,n,$

$\alpha_1 = \alpha_2 = ... = \alpha_n = 0,$

$\beta_1 = 2, \beta_2 = 4, \beta_3 = 6, \beta_4 = 8, \beta_5 = 10,$

$\gamma_1 = \gamma_2 = ... = \gamma_n = 0,$

$\mu = -10,\ \varepsilon_{ij} \sim$ Normal distribution, $E(\varepsilon_{ij}) = 0, Var(\varepsilon_{ij}) = 25,$

X(1,1,1)~Normal(mu=-8.000000,sigma*sigma=25.000000), sample size=1
X(1,2,2)~Normal(mu=-6.000000,sigma*sigma=25.000000), sample size=1
X(1,3,3)~Normal(mu=-4.000000,sigma*sigma=25.000000), sample size=1
X(1,4,4)~Normal(mu=-2.000000,sigma*sigma=25.000000), sample size=1
X(1,5,5)~Normal(mu=0.000000,sigma*sigma=25.000000), sample size=1
X(2,1,2)~Normal(mu=-8.000000,sigma*sigma=25.000000), sample size=1
X(2,2,3)~Normal(mu=-6.000000,sigma*sigma=25.000000), sample size=1
X(2,3,4)~Normal(mu=-4.000000,sigma*sigma=25.000000), sample size=1
X(2,4,5)~Normal(mu=-2.000000,sigma*sigma=25.000000), sample size=1
X(2,5,1)~Normal(mu=0.000000,sigma*sigma=25.000000), sample size=1
X(3,1,3)~Normal(mu=-8.000000,sigma*sigma=25.000000), sample size=1
X(3,2,4)~Normal(mu=-6.000000,sigma*sigma=25.000000), sample size=1
X(3,3,5)~Normal(mu=-4.000000,sigma*sigma=25.000000), sample size=1
X(3,4,1)~Normal(mu=-2.000000,sigma*sigma=25.000000), sample size=1
X(3,5,2)~Normal(mu=0.000000,sigma*sigma=25.000000), sample size=1
X(4,1,4)~Normal(mu=-8.000000,sigma*sigma=25.000000), sample size=1
X(4,2,5)~Normal(mu=-6.000000,sigma*sigma=25.000000), sample size=1
X(4,3,1)~Normal(mu=-4.000000,sigma*sigma=25.000000), sample size=1
X(4,4,2)~Normal(mu=-2.000000,sigma*sigma=25.000000), sample size=1
X(4,5,3)~Normal(mu=0.000000,sigma*sigma=25.000000), sample size=1
X(5,1,5)~Normal(mu=-8.000000,sigma*sigma=25.000000), sample size=1
X(5,2,1)~Normal(mu=-6.000000,sigma*sigma=25.000000), sample size=1
X(5,3,2)~Normal(mu=-4.000000,sigma*sigma=25.000000), sample size=1
X(5,4,3)~Normal(mu=-2.000000,sigma*sigma=25.000000), sample size=1
X(5,5,4)~Normal(mu=0.000000,sigma*sigma=25.000000), sample size=1

X(ijk)=mu+alpha(i)+beta(j)+gamma(k)+e(ijk)
i=1,..,5, j=1,..,5,k=1,..,5, total sample size=25

mu=-10.000000,
alpha(1)=0.000000, alpha(2)=0.000000, alpha(3)=0.000000, alpha(4)=0.000000, alpha(5)=0.000000,
beta(1)=2.000000, beta(2)=4.000000, beta(3)=6.000000, beta(4)=8.000000, beta(5)=10.000000,
gamma(1)=0.000000, gamma(2)=0.000000, gamma(3)=0.000000, gamma(4)=0.000000, gamma(5)=0.000000,
eijk iid ~Normal(0,sigma*sigma=5.000000)

	A1	A2	A3	A4	A5
B1	C1	C2	C3	C4	C5
B2	C2	C3	C4	C5	C1

B3| C3 C4 C5 C1 C2
B4| C4 C5 C1 C2 C3
B5| C5 C1 C2 C3 C4

	A1	A2	A3	A4	A5
B1 |	-7.1641494308	-6.1240020403	-6.8547517049	-10.5633782652	-11.3511213225
B2 |	-4.7685913779	-5.6201739581	-5.7892148418	-6.9586910832	-6.3252445730
B3 |	-7.2422784754	-1.9078208732	-6.3436842840	-1.2286136575	-5.1477972791
B4 |	-0.1313373424	-0.1058233687	-5.2325772078	2.1143798716	2.8363774303
B5 |	1.9389508247	1.5096558331	-6.0966855575	0.1403936496	-0.6846251662

Latin square model

X(ijk)=mu+alpha(i)+beta(j)+gamma(k)+e(ijk),
i=1,2...,5, j=1,2...,5, k=1,2...,5, total sample size=25

	A1	A2	A3	A4	A5
factor A sample mean	-3.47348	-2.44963	-6.06338	-3.29918	-4.13448
alpha estimate value	0.41055	1.43440	-2.17935	0.58485	-0.25045
	B1	B2	B3	B4	B5
factor B sample mean	-8.41148	-5.89238	-4.37404	-0.10380	-0.63846
beta estimate value	-4.52745	-2.00835	-0.49001	3.78024	3.24557
	C1	C2	C3	C4	C5
factor C sample mean	-3.68819	-4.00454	-3.34809	-3.81528	-4.56407
gamma estimate value	0.19585	-0.12051	0.53595	0.06876	-0.68004

Total sample size=25 , grand mean=-3.884032

summation of alpha(i)=0.000000
summation of beta(j)=-0.000000
summation of gamma(k)=0.000000

ANOVA

Source	df	SS	MS	F
Factor A	4	36.9019865863	9.2254966466	1.1723981399
Factor B	4	247.9763947477	61.9940986869	7.8783580735
Factor C	4	4.0364979583	1.0091244896	0.1282419494
Error	12	94.4269322753	7.8689110229	
Total	24	383.3418115676		

H0:alpha(1)=...=alpha(5)=0
F(4,12) test value=1.172398

The F test p value=0.371100

H0:beta(1)=...=beta(5)=0
F(4,12) test value=7.878358

The F test p value=0.002400

H0:gamma(1)=...=gamma(5)=0
F(4,12) test value=0.128242

The F test p value=0.969300

class	[1]	[2]	[3]	[4]	[5]
lower limit		-2.36086	-0.71059	0.71062	2.36068
upper limit	-2.36086	-0.71059	0.71062	2.36068	
observed no	4.00000	4.00000	7.00000	6.00000	4.00000
probability	0.20000	0.20000	0.20000	0.20000	0.20000
expected no	5.00000	5.00000	5.00000	5.00000	5.00000
chi square	0.20000	0.20000	0.80000	0.20000	0.20000

degree of freedom=3

H0: residual~Normal(0,sigma(error)*sigma(error)), sigma(error) are unknown
pearson chi-square test statistic =1.600000
p-value=0.659400

H0: Variances are equal
Max(residual(ij)*residual(ij)/SSE=test value=0.154111
p value=0.808975

~~~~~ The run test of residual~~~~~~~~~~~~~~
number of the negative of residual=12
number of the positive ofresidual=13
Run=11
H0: residualis random , H1: Increasing line or decreasing line
Z=-1.015030, p-value=0.155100
H0: residual is random , H1: Oscillation
Z=-1.015030, p-value=0.844900
H0: residual is random , H1: Increasing line or decreasing line or Oscillation
Z=-1.015030, p-value=0.310200

multiple comparison of population means

Factor A ,there has 5 cateogries
. LSD(least significant difference)
The confidence coefficietn=0.95
95% C.I. for mu(1)-mu(2)
[-4.8891206495, 2.84142409160]
mu(1)=mu(2)

95% C.I. for mu(1)-mu(3)
[-1.2753708117, 6.45517392940]
mu(1)=mu(3)

95% C.I. for mu(1)-mu(4)
[-4.0395716340, 3.69097310710]
mu(1)=mu(4)
~~~~~

95% C.I. for mu(1)-mu(5)
[-3.2042713488, 4.52627339230]
mu(1)=mu(5)

95% C.I. for mu(2)-mu(3)
[-0.2515225328, 7.47902220830]
mu(2)=mu(3)

95% C.I. for mu(2)-mu(4)
[-3.0157233551, 4.71482138600]
mu(2)=mu(4)

95% C.I. for mu(2)-mu(5)
[-2.1804230699, 5.55012167120]
mu(2)=mu(5)

95% C.I. for mu(3)-mu(4)
[-6.6294731928, 1.10107154830]
mu(3)=mu(4)

95% C.I. for mu(3)-mu(5)
[-5.7941729076, 1.93637183340]
mu(3)=mu(5)

95% C.I. for mu(4)-mu(5)
[-3.0299720854, 4.70057265570]
mu(4)=mu(5)

Factor B ,there has 5 cateogries
. LSD(least significant difference)
The confidence coefficietn=0.95
95% C.I. for mu(1)-mu(2)
[-6.3843697565, 1.34617498460]
mu(1)=mu(2)

95% C.I. for mu(1)-mu(3)
[-7.9027140095, -0.17216926840]
mu(1)<mu(3)
95% C.I. for mu(1)-mu(4)
[-12.1729567999, -4.44241205880]
mu(1)<mu(4)

95% C.I. for mu(1)-mu(5)
[-11.6382908400, -3.90774609890]
mu(1)<mu(5)
95% C.I. for mu(2)-mu(3)
[-5.3836166235, 2.34692811760]
mu(2)=mu(3)

95% C.I. for mu(2)-mu(4)
[-9.6538594139, -1.92331467280]
mu(2)<mu(4)
95% C.I. for mu(2)-mu(5)
[-9.1191934541, -1.38864871300]
mu(2)<mu(5)
95% C.I. for mu(3)-mu(4)
[-8.1355151610, -0.40497041990]
mu(3)<mu(4)
95% C.I. for mu(3)-mu(5)
[-7.6008492011, 0.12969554000]
mu(3)=mu(5)

95% C.I. for mu(4)-mu(5)
[-3.3306064107, 4.39993833040]
mu(4)=mu(5)

Factor C ,there has 5 cateogries
. LSD(least significant difference)
The confidence coefficietn=0.95
95% C.I. for mu(1)-mu(2)
[-8.2722136466, -0.54166890550]
mu(1)<mu(2)
95% C.I. for mu(1)-mu(3)
[-8.9286663116, -1.19812157050]
mu(1)<mu(3)
95% C.I. for mu(1)-mu(4)
[-8.4614776255, -0.73093288440]
mu(1)<mu(4)
95% C.I. for mu(1)-mu(5)
[-7.7126790765, 0.01786566460]
mu(1)=mu(5)

95% C.I. for mu(2)-mu(3)
[-6.4095689257, 1.32097581540]
mu(2)=mu(3)

95% C.I. for mu(2)-mu(4)

[-5.9423802396, 1.78816450150]
mu(2)=mu(4)

95% C.I. for mu(2)-mu(5)
[-5.1935816906, 2.53696305050]
mu(2)=mu(5)

95% C.I. for mu(3)-mu(4)
[-4.4240359866, 3.30650875450]
mu(3)=mu(4)

95% C.I. for mu(3)-mu(5)
[-3.6752374376, 4.05530730350]
mu(3)=mu(5)

95% C.I. for mu(4)-mu(5)
[0.5950053528, 8.32555009390]
mu(4)>mu(5)

~~~~~~~~~~~~ error ~~~~~~~~~~~~

0.64093	0.97359	3.20013	-2.80550	-2.00915
0.83375	-1.69814	2.21376	-0.97112	-0.37826
-3.81474	0.96306	0.88975	2.36473	-0.40280
-0.50685	-0.75638	-3.14528	1.75383	2.65468
2.84690	0.51787	-3.15837	-0.34194	0.13553

The common population standard deviation and variance confidence interval
90% confidence interval for population variance
[4.490963 , 18.068731]
90% confidence interval for population standard deviation
[2.119189 , 4.250733]
95% confidence interval for population variance
[4.046229 , 21.442187]
95% confidence interval for population standard deviation
[2.011524 , 4.630571]
99% confidence interval for population variance
[3.336599 , 30.719204]
99% confidence interval for population standard deviation
[1.826636 , 5.542491]
~~~~~~~~~~~~

5.4.1.4) (n=5). $X_{ijk} = \mu + \alpha_i + \beta_j + \gamma_k + \varepsilon_{ijk}, i = 1,..,n,\ j = 1,...,n, k = 1,..,n,$

$\alpha_1 = \alpha_2 = ... = \alpha_n = 0,$

$\beta_1 = \beta_2 = ... = \beta_n = 0,$

$\gamma_1 = 2, \gamma_2 = 4, \gamma_3 = 6, \gamma_4 = 8, \gamma_5 = 10,$

$\mu = -10,\ \varepsilon_{ij} \sim$ Normal distribution, $E(\varepsilon_{ij}) = 0, Var(\varepsilon_{ij}) = 25,$

X(1,1,1)~Normal(mu=-8.000000,sigma*sigma=625.000000), sample size=1
X(1,2,2)~Normal(mu=-6.000000,sigma*sigma=625.000000), sample size=1
X(1,3,3)~Normal(mu=-4.000000,sigma*sigma=625.000000), sample size=1
X(1,4,4)~Normal(mu=-2.000000,sigma*sigma=625.000000), sample size=1
X(1,5,5)~Normal(mu=0.000000,sigma*sigma=625.000000), sample size=1
X(2,1,2)~Normal(mu=-6.000000,sigma*sigma=625.000000), sample size=1
X(2,2,3)~Normal(mu=-4.000000,sigma*sigma=625.000000), sample size=1
X(2,3,4)~Normal(mu=-2.000000,sigma*sigma=625.000000), sample size=1
X(2,4,5)~Normal(mu=0.000000,sigma*sigma=625.000000), sample size=1
X(2,5,1)~Normal(mu=-8.000000,sigma*sigma=625.000000), sample size=1
X(3,1,3)~Normal(mu=-4.000000,sigma*sigma=625.000000), sample size=1
X(3,2,4)~Normal(mu=-2.000000,sigma*sigma=625.000000), sample size=1
X(3,3,5)~Normal(mu=0.000000,sigma*sigma=625.000000), sample size=1
X(3,4,1)~Normal(mu=-8.000000,sigma*sigma=625.000000), sample size=1
X(3,5,2)~Normal(mu=-6.000000,sigma*sigma=625.000000), sample size=1
X(4,1,4)~Normal(mu=-2.000000,sigma*sigma=625.000000), sample size=1
X(4,2,5)~Normal(mu=0.000000,sigma*sigma=625.000000), sample size=1
X(4,3,1)~Normal(mu=-8.000000,sigma*sigma=625.000000), sample size=1
X(4,4,2)~Normal(mu=-6.000000,sigma*sigma=625.000000), sample size=1
X(4,5,3)~Normal(mu=-4.000000,sigma*sigma=625.000000), sample size=1
X(5,1,5)~Normal(mu=0.000000,sigma*sigma=625.000000), sample size=1
X(5,2,1)~Normal(mu=-8.000000,sigma*sigma=625.000000), sample size=1
X(5,3,2)~Normal(mu=-6.000000,sigma*sigma=625.000000), sample size=1
X(5,4,3)~Normal(mu=-4.000000,sigma*sigma=625.000000), sample size=1
X(5,5,4)~Normal(mu=-2.000000,sigma*sigma=625.000000), sample size=1
X(ijk)=mu+alpha(i)+beta(j)+gamma(k)+e(ijk)
i=1,..,5, j=1,..,5,k=1,..,5, total sample size=25
mu=-10.000000,
alpha(1)=0.000000, alpha(2)=0.000000, alpha(3)=0.000000, alpha(4)=0.000000, alpha(5)=0.000000,
beta(1)=0.000000, beta(2)=0.000000, beta(3)=0.000000, beta(4)=0.000000, beta(5)=0.000000,
gamma(1)=2.000000, gamma(2)=4.000000, gamma(3)=6.000000, gamma(4)=8.000000, gamma(5)=10.000000,
eijk iid ~Normal(0,sigma*sigma=25.000000)

	A1	A2	A3	A4	A5
B1	C1	C2	C3	C4	C5
B2	C2	C3	C4	C5	C1
B3	C3	C4	C5	C1	C2
B4	C4	C5	C1	C2	C3

B5| C5 C1 C2 C3 C4

	A1	A2	A3	A4	A5
B1 |	-10.6011227502	-4.3497629301	-4.9515050584	-3.9824955193	4.2560074123
B2 |	-7.6309625739	-8.9046329346	-3.9362096867	-10.4278957726	-5.3566753482
B3 |	-4.6460832748	0.7548389662	2.2113987603	-2.8795878662	-5.4301715015
B4 |	-1.4753288787	0.4423328953	-8.8075833942	-10.3123601458	-6.5893966448
B5 |	1.9098186448	-8.6700637801	-2.4365407637	-3.5382889906	5.0360273598

Latin square model

X(ijk)=mu+alpha(i)+beta(j)+gamma(k)+e(ijk),
i=1,2...,5, j=1,2...,5, k=1,2...,5, total sample size=25

	A1	A2	A3	A4	A5
factor A sample mean	-4.48874	-4.14546	-3.58409	-6.22813	-1.61684
alpha estimate value	-0.47609	-0.13281	0.42856	-2.21548	2.39581
	B1	B2	B3	B4	B5
factor B sample mean	-3.92578	-7.25128	-1.99792	-5.34847	-1.53981
beta estimate value	0.08687	-3.23863	2.01473	-1.33582	2.47284
	C1	C2	C3	C4	C5
factor C sample mean	-7.26301	-6.03196	-5.72598	-0.72063	-0.32167
gamma estimate value	-3.25036	-2.01931	-1.71333	3.29202	3.69098

Total sample size=25 , grand mean=-4.012650

summation of alpha(i)=0.000000
summation of beta(j)=0.000000
summation of gamma(k)=0.000000

ANOVA

Source	df	SS	MS	F
Factor A	4	55.3809523007	13.8452380752	1.4195709958
Factor B	4	112.2736076461	28.0684019115	2.8778912312
Factor C	4	210.1932855194	52.5483213798	5.3878505015
Error	12	117.0373707257	9.7531142271	
Total	24	494.8852161919		

H0:alpha(1)=...=alpha(5)=0
F(4,12) test value=1.419571

The F test p value=0.286400

H0:beta(1)=...=beta(5)=0
F(4,12) test value=2.877891

The F test p value=0.069700

H0:gamma(1)=...=gamma(5)=0
F(4,12) test value=5.387851

The F test p value=0.010200

class	[1]	[2]	[3]	[4]	[5]
lower limit		-2.62836	-0.79111	0.79113	2.62816
upper limit	-2.62836	-0.79111	0.79113	2.62816	
observed no	4.00000	2.00000	10.00000	7.00000	2.00000
probability	0.20000	0.20000	0.20000	0.20000	0.20000
expected no	5.00000	5.00000	5.00000	5.00000	5.00000
chi square	0.20000	1.80000	5.00000	0.80000	1.80000

degree of freedom=3

H0: residual~Normal(0,sigma(error)*sigma(error)), sigma(error) are unknown
pearson chi-square test statistic =9.600000
p-value=0.022200

H0: Variances are equal
Max(residual(ij)*residual(ij)/SSE=test value=0.184918
p value=0.562398

~~~~~ The run test of residual~~~~~~~~~~~~~~
number of the negative of residual=11
number of the positive ofresidual=14
Run=17
H0: residualis random , H1: Increasing line or decreasing line
Z=1.526599, p-value=0.936600
H0: residual is random , H1: Oscillation
Z=1.526599, p-value=0.063400
H0: residual is random , H1: Increasing line or decreasing line or Oscillation
Z=1.526599, p-value=0.126800

multiple comparison of population means

Factor A ,there has 5 cateogries
. LSD(least significant difference)
The confidence coefficietn=0.95
95% C.I. for mu(1)-mu(2)
[-4.6465070349, 3.95995061510]
mu(1)=mu(2)

95% C.I. for mu(1)-mu(3)
[-5.2078765630, 3.39858108690]
mu(1)=mu(3)

95% C.I. for mu(1)-mu(4)
[-2.5638389326, 6.04261871730]
mu(1)=mu(4)

95% C.I. for mu(1)-mu(5)
~~~~~

[-7.1751228471, 1.43133480290]
mu(1)=mu(5)

95% C.I. for mu(2)-mu(3)
[-4.8645983531, 3.74185929680]
mu(2)=mu(3)

95% C.I. for mu(2)-mu(4)
[-2.2205607227, 6.38589692720]
mu(2)=mu(4)

95% C.I. for mu(2)-mu(5)
[-6.8318446372, 1.77461301280]
mu(2)=mu(5)

95% C.I. for mu(3)-mu(4)
[-1.6591911946, 6.94726645540]
mu(3)=mu(4)

95% C.I. for mu(3)-mu(5)
[-6.2704751090, 2.33598254090]
mu(3)=mu(5)

95% C.I. for mu(4)-mu(5)
[-8.9145127394, -0.30805508950]
mu(4)<mu(5)

Factor B ,there has 5 cateogries
. LSD(least significant difference)
The confidence coefficietn=0.95

95% C.I. for mu(1)-mu(2)
[-0.9777293309, 7.62872831900]
mu(1)=mu(2)

95% C.I. for mu(1)-mu(3)
[-6.2310836109, 2.37537403900]
mu(1)=mu(3)

95% C.I. for mu(1)-mu(4)
[-2.8805373605, 5.72592028940]
mu(1)=mu(4)

95% C.I. for mu(1)-mu(5)
[-6.6891950882, 1.91726256180]
mu(1)=mu(5)

95% C.I. for mu(2)-mu(3)
[-9.5565831050, -0.95012545500]
mu(2)<mu(3)
95% C.I. for mu(2)-mu(4)
[-6.2060368545, 2.40042079540]
mu(2)=mu(4)

95% C.I. for mu(2)-mu(5)
[-10.0146945822, -1.40823693230]
mu(2)<mu(5)
95% C.I. for mu(3)-mu(4)
[-0.9526825745, 7.65377507540]
mu(3)=mu(4)

95% C.I. for mu(3)-mu(5)
[-4.7613403022, 3.84511734770]
mu(3)=mu(5)

95% C.I. for mu(4)-mu(5)
[-8.1118865527, 0.49457109730]
mu(4)=mu(5)

Factor C ,there has 5 cateogries
. LSD(least significant difference)
The confidence coefficietn=0.95
95% C.I. for mu(1)-mu(2)
[-2.1970450111, 6.40941263880]
mu(1)=mu(2)

95% C.I. for mu(1)-mu(3)
[-2.5030232135, 6.10343443640]
mu(1)=mu(3)

95% C.I. for mu(1)-mu(4)
[-7.5083710424, 1.09808660750]
mu(1)=mu(4)

95% C.I. for mu(1)-mu(5)

[-7.9073369821, 0.69912066780]
mu(1)=mu(5)

95% C.I. for mu(2)-mu(3)
[-5.8285227075, 2.77793494240]
mu(2)=mu(3)

95% C.I. for mu(2)-mu(4)
[-10.8338705364, -2.22741288650]
mu(2)<mu(4)
95% C.I. for mu(2)-mu(5)
[-11.2328364762, -2.62637882620]
mu(2)<mu(5)
95% C.I. for mu(3)-mu(4)
[-5.5805162564, 3.02594139350]
mu(3)=mu(4)

95% C.I. for mu(3)-mu(5)
[-5.9794821962, 2.62697545380]
mu(3)=mu(5)

95% C.I. for mu(4)-mu(5)
[-9.3300284466, -0.72357079670]
mu(4)<mu(5)
~~~~~~~~~~~~ error ~~~~~~~~~~~~

-2.94890	1.72813	0.25904	-1.13326	2.09499
2.11571	0.19278	-0.40551	-4.65213	2.74915
-0.45874	-0.40645	0.08978	4.58417	-3.80875
1.05721	2.23263	-0.63732	-0.72911	-1.92341
0.23473	-3.74709	0.69402	1.93033	0.88801

The common population standard deviation and variance confidence interval
90% confidence interval for population variance
[5.566320 , 22.395272]
90% confidence interval for population standard deviation
[2.359305 , 4.732364]
95% confidence interval for population variance
[5.015094 , 26.576498]
95% confidence interval for population standard deviation
[2.239441 , 5.155240]
99% confidence interval for population variance
[4.135545 , 38.074888]
99% confidence interval for population standard deviation
[2.033604 , 6.170485]
~~~~~~~~~~~~

5.4.1.5) (n=5). $X_{ijk} = \mu + \alpha_i + \beta_j + \gamma_k + \varepsilon_{ijk}, i = 1,..,n,\ j = 1,...,n, k = 1,..,n,$

$\alpha_1 = -10, \alpha_2 = -5, \alpha_3 = 0, \alpha_4 = 5, \alpha_5 = 10,$

$\beta_1 = 2, \beta_2 = 4, \beta_3 = 6, \beta_4 = 8, \beta_5 = 10,$

$\gamma_1 = 12, \gamma_2 = 8, \gamma_3 = 6, \gamma_4 = 4, \gamma_5 = 0,$

$\mu = -10,\ \varepsilon_{ij}$ ~Normal distribution, $E(\varepsilon_{ij}) = 0, Var(\varepsilon_{ij}) = 25,$

X(1,1,1)~Normal(mu=-6.000000,sigma*sigma=625.000000), sample size=1
X(1,2,2)~Normal(mu=-8.000000,sigma*sigma=625.000000), sample size=1
X(1,3,3)~Normal(mu=-8.000000,sigma*sigma=625.000000), sample size=1
X(1,4,4)~Normal(mu=-8.000000,sigma*sigma=625.000000), sample size=1
X(1,5,5)~Normal(mu=-10.000000,sigma*sigma=625.000000), sample size=1
X(2,1,2)~Normal(mu=-5.000000,sigma*sigma=625.000000), sample size=1
X(2,2,3)~Normal(mu=-5.000000,sigma*sigma=625.000000), sample size=1
X(2,3,4)~Normal(mu=-5.000000,sigma*sigma=625.000000), sample size=1
X(2,4,5)~Normal(mu=-7.000000,sigma*sigma=625.000000), sample size=1
X(2,5,1)~Normal(mu=7.000000,sigma*sigma=625.000000), sample size=1
X(3,1,3)~Normal(mu=-2.000000,sigma*sigma=625.000000), sample size=1
X(3,2,4)~Normal(mu=-2.000000,sigma*sigma=625.000000), sample size=1
X(3,3,5)~Normal(mu=-4.000000,sigma*sigma=625.000000), sample size=1
X(3,4,1)~Normal(mu=10.000000,sigma*sigma=625.000000), sample size=1
X(3,5,2)~Normal(mu=8.000000,sigma*sigma=625.000000), sample size=1
X(4,1,4)~Normal(mu=1.000000,sigma*sigma=625.000000), sample size=1
X(4,2,5)~Normal(mu=-1.000000,sigma*sigma=625.000000), sample size=1
X(4,3,1)~Normal(mu=13.000000,sigma*sigma=625.000000), sample size=1
X(4,4,2)~Normal(mu=11.000000,sigma*sigma=625.000000), sample size=1
X(4,5,3)~Normal(mu=11.000000,sigma*sigma=625.000000), sample size=1
X(5,1,5)~Normal(mu=2.000000,sigma*sigma=625.000000), sample size=1
X(5,2,1)~Normal(mu=16.000000,sigma*sigma=625.000000), sample size=1
X(5,3,2)~Normal(mu=14.000000,sigma*sigma=625.000000), sample size=1
X(5,4,3)~Normal(mu=14.000000,sigma*sigma=625.000000), sample size=1
X(5,5,4)~Normal(mu=14.000000,sigma*sigma=625.000000), sample size=1

X(ijk)=mu+alpha(i)+beta(j)+gamma(k)+e(ijk)
i=1,..,5, j=1,..,5,k=1,..,5, total sample size=25

mu=-10.000000,
alpha(1)=-10.000000, alpha(2)=-5.000000, alpha(3)=0.000000,
alpha(4)=5.000000, alpha(5)=10.000000,
beta(1)=2.000000, beta(2)=4.000000, beta(3)=6.000000, beta(4)=8.000000,
beta(5)=10.000000,
gamma(1)=12.000000, gamma(2)=8.000000, gamma(3)=6.000000,
gamma(4)=4.000000, gamma(5)=0.000000,
eijk iid ~Normal(0,sigma*sigma=25.000000)

	A1	A2	A3	A4	A5
B1	C1	C2	C3	C4	C5
B2	C2	C3	C4	C5	C1
B3	C3	C4	C5	C1	C2
B4	C4	C5	C1	C2	C3
B5	C5	C1	C2	C3	C4

	A1	A2	A3	A4	A5
B1	-7.7999911951	1.5581867073	-4.7127719441	3.3973643756	2.6882589861
B2	-10.9929717313	-9.4975635215	-5.4704015818	-5.8943031039	23.2905485497
B3	1.0352210206	-14.0883215643	-8.4101567961	12.1081838697	14.1878038753
B4	-3.5457972962	-3.6080354156	15.4267846208	16.5061584430	11.0985735971
B5	-5.4824691664	0.7192748744	6.2055512746	11.8579792195	15.9629766286

Latin square model

X(ijk)=mu+alpha(i)+beta(j)+gamma(k)+e(ijk),
i=1,2...,5, j=1,2...,5, k=1,2...,5, total sample size=25

	A1	A2	A3	A4	A5
factor A sample mean	-5.35720	-4.98329	0.60780	7.59508	13.44563
alpha estimate value	-7.61880	-7.24490	-1.65380	5.33347	11.18403
	B1	B2	B3	B4	B5
factor B sample mean	-0.97379	-1.71294	0.96655	7.17554	5.85266
beta estimate value	-3.23539	-3.97454	-1.29506	4.91393	3.59106
	C1	C2	C3	C4	C5
factor C sample mean	8.74896	5.49295	1.95629	-0.74884	-4.14134
gamma estimate value	6.48736	3.23134	-0.30532	-3.01044	-6.40294

Total sample size=25 , grand mean=2.261603

summation of alpha(i)=-0.000000
summation of beta(j)=-0.000000
summation of gamma(k)=-0.000000

ANOVA

Source	df	SS	MS	F
Factor A	4	1333.9909899050	333.4977474763	10.9905222223
Factor B	4	324.9218836387	81.2304709097	2.6769754891
Factor C	4	513.4051566390	128.3512891597	4.2298567425
Error	12	364.1294643475	30.3441220290	
Total	24	2536.4474945303		

H0:alpha(1)=...=alpha(5)=0
F(4,12) test value=10.990522

The F test p value=0.000600

H0:beta(1)=...=beta(5)=0
F(4,12) test value=2.676975

The F test p value=0.083400

H0:gamma(1)=...=gamma(5)=0
F(4,12) test value=4.229857

The F test p value=0.023100

class	[1]	[2]	[3]	[4]	[5]
lower limit		-4.63607	-1.39540	1.39546	4.63573
upper limit	-4.63607	-1.39540	1.39546	4.63573	
observed no	4.00000	3.00000	10.00000	5.00000	3.00000
probability	0.20000	0.20000	0.20000	0.20000	0.20000
expected no	5.00000	5.00000	5.00000	5.00000	5.00000
chi square	0.20000	0.80000	5.00000	0.00000	0.80000

degree of freedom=3

H0: residual~Normal(0,sigma(error)*sigma(error)), sigma(error) are unknown
pearson chi-square test statistic =6.800000
p-value=0.078500

H0: Variances are equal
Max(residual(ij)*residual(ij)/SSE=test value=0.175445
p value=0.639308

~~~~~ The run test of residual~~~~~~~~~~~~~~
number of the negative of residual=14
number of the positive ofresidual=11
Run=17
H0: residualis random , H1: Increasing line or decreasing line
Z=1.526599, p-value=0.936600
H0: residual is random , H1: Oscillation
Z=1.526599, p-value=0.063400
H0: residual is random , H1: Increasing line or decreasing line or Oscillation
Z=1.526599, p-value=0.126800

multiple comparison of population means

Factor A ,there has 5 cateogries
. LSD(least significant difference)
The confidence coefficietn=0.95
95% C.I. for mu(1)-mu(2)
[-7.9642296710, 7.21640989150]
mu(1)=mu(2)

95% C.I. for mu(1)-mu(3)
~~~~~

[-13.5553225697, 1.62531699290]
mu(1)=mu(3)

95% C.I. for mu(1)-mu(4)
[-20.5425980157, -5.36195845320]
mu(1)<mu(4)
95% C.I. for mu(1)-mu(5)
[-26.3931537823, -11.21251421980]
mu(1)<mu(5)
95% C.I. for mu(2)-mu(3)
[-13.1814126799, 1.99922688260]
mu(2)=mu(3)

95% C.I. for mu(2)-mu(4)
[-20.1686881260, -4.98804856340]
mu(2)<mu(4)
95% C.I. for mu(2)-mu(5)
[-26.0192438926, -10.83860433000]
mu(2)<mu(5)
95% C.I. for mu(3)-mu(4)
[-14.5775952273, 0.60304433520]
mu(3)=mu(4)

95% C.I. for mu(3)-mu(5)
[-20.4281509939, -5.24751143140]
mu(3)<mu(5)
95% C.I. for mu(4)-mu(5)
[-13.4408755479, 1.73976401470]
mu(4)=mu(5)

Factor B ,there has 5 cateogries
. LSD(least significant difference)
The confidence coefficietn=0.95
95% C.I. for mu(1)-mu(2)
[-6.8511721175, 8.32946744500]
mu(1)=mu(2)

95% C.I. for mu(1)-mu(3)
[-9.5306564763, 5.64998308620]
mu(1)=mu(3)

95% C.I. for mu(1)-mu(4)
[-15.7396471851, -0.55900762260]
mu(1)<mu(4)

95% C.I. for mu(1)-mu(5)
[-14.4167729614, 0.76386660110]
mu(1)=mu(5)

95% C.I. for mu(2)-mu(3)
[-10.2698041401, 4.91083542250]
mu(2)=mu(3)

95% C.I. for mu(2)-mu(4)
[-16.4787948488, -1.29815528630]
mu(2)<mu(4)

95% C.I. for mu(2)-mu(5)
[-15.1559206252, 0.02471893740]
mu(2)=mu(5)

95% C.I. for mu(3)-mu(4)
[-13.7993104900, 1.38132907250]
mu(3)=mu(4)

95% C.I. for mu(3)-mu(5)
[-12.4764362664, 2.70420329620]
mu(3)=mu(5)

95% C.I. for mu(4)-mu(5)
[-6.2674455576, 8.91319400490]
mu(4)=mu(5)

Factor C ,there has 5 cateogries
. LSD(least significant difference)
The confidence coefficietn=0.95
95% C.I. for mu(1)-mu(2)
[-14.0570561091, 1.12358345350]
mu(1)=mu(2)

95% C.I. for mu(1)-mu(3)
[-10.5203980696, 4.66024149290]
mu(1)=mu(3)

95% C.I. for mu(1)-mu(4)
[-7.8152745077, 7.36536505480]
mu(1)=mu(4)

95% C.I. for mu(1)-mu(5)
[-4.4227692961, 10.75787026640]
mu(1)=mu(5)

95% C.I. for mu(2)-mu(3)
[-11.2595457333, 3.92109382920]
mu(2)=mu(3)

95% C.I. for mu(2)-mu(4)
[-8.5544221714, 6.62621739110]
mu(2)=mu(4)

95% C.I. for mu(2)-mu(5)
[-5.1619169598, 10.01872260270]
mu(2)=mu(5)

95% C.I. for mu(3)-mu(4)
[-5.8749378126, 9.30570174990]
mu(3)=mu(4)

95% C.I. for mu(3)-mu(5)
[-2.4824326010, 12.69820696150]
mu(3)=mu(5)

95% C.I. for mu(4)-mu(5)
[3.7265581077, 18.90719767020]
mu(4)>mu(5)
~~~~~~~~~~~~ error ~~~~~~~~~~~~

-5.69475	6.54553	-1.77986	2.04812	-1.11904
-4.89257	-0.23441	0.90678	-3.11189	7.33210
7.99280	-4.79953	-1.31996	-0.67919	-1.19411
-0.09209	2.86427	3.41769	0.76581	-6.95568
2.68662	-4.37585	-1.22465	0.97716	1.93672

The common population standard deviation and variance confidence interval
90% confidence interval for population variance
[17.318067 , 69.676704]
90% confidence interval for population standard deviation
[4.161498 , 8.347257]
95% confidence interval for population variance
[15.603082 , 82.685434]
95% confidence interval for population standard deviation
~~~~~~~~~~~~

[3.950074 , 9.093153]
99% confidence interval for population variance
[12.866606 , 118.459502]
99% confidence interval for population standard deviation
[3.587005 , 10.883910]