2)Two independent normal population variances test and confidence interval when two population variances σ_1^2, σ_2^2 are known.

2.1)*two-tailed test* (*two-sided test*)：

(i) $H_0 : \frac{\sigma_1^2}{\sigma_2^2} = c, H_1 : \frac{\sigma_1^2}{\sigma_2^2} \neq c, (c > 0)$, c is known value.

(ii) $\alpha = P\left(reject \quad H_0 \middle| H_0 : \frac{\sigma_1^2}{\sigma_2^2} = c \right)$

(iii) $F_{n_1-1, n_2-1} = \frac{S_1^2 / S_2^2}{c}, \overline{X} = \sum_{i=1}^{n_1} X_i \Big/ n_1, \overline{Y} = \sum_{j=1}^{n_2} Y_j \Big/ n_2$,

$$S_1^2 = \frac{\sum_{i=1}^{n_1} \left(X_i - \overline{X} \right)^2}{n_1 - 1}, S_2^2 = \frac{\sum_{j=1}^{n_2} \left(Y_j - \overline{Y} \right)^2}{n_2 - 1},$$

(iv)test statistic：

F test statistic：

$$F_{n_1-1, n_2-1} = \frac{S_1^2 / S_2^2}{c} < F_{1-\alpha/2, n_1-1, n_2-1} = \frac{1}{F_{\alpha/2, n_2-1, n_1-1}},$$

$$F_{n_1-1, n_2-1} = \frac{S_1^2 / S_2^2}{c} > F_{\alpha/2, n_1-1, n_2-1} \Rightarrow reject \quad H_0,$$

v) $(1-\alpha) \times 100\% \quad C.I. \quad for \quad \frac{\sigma_1^2}{\sigma_2^2}$

$$\frac{S_1^2 / S_2^2}{F_{\alpha/2, n_1-1, n_2-1}} \leq \frac{\sigma_1^2}{\sigma_2^2} \leq \frac{S_1^2 / S_2^2}{F_{1-\alpha/2, n_1-1, n_2-1}} = S_1^2 / S_2^2 \times F_{\alpha/2, n_2-1, n_1-1},$$

2.2)*right-tailed test* (*right-sided test*)：

(i) $H_0 : \frac{\sigma_1^2}{\sigma_2^2} \leq c, H_1 : \frac{\sigma_1^2}{\sigma_2^2} > c, (c > 0)$, c is known value.

(ii) $\alpha = P\left(reject \quad H_0 \middle| H_0 : \frac{\sigma_1^2}{\sigma_2^2} = c \right)$

(iii) $F_{n_1-1, n_2-1} = \frac{S_1^2 / S_2^2}{c}, \overline{X} = \sum_{i=1}^{n_1} X_i \Big/ n_1, \overline{Y} = \sum_{j=1}^{n_2} Y_j \Big/ n_2$,

$$S_1^2 = \frac{\sum_{i=1}^{n_1} \left(X_i - \overline{X} \right)^2}{n_1 - 1}, S_2^2 = \frac{\sum_{j=1}^{n_2} \left(Y_j - \overline{Y} \right)^2}{n_2 - 1},$$

(iv)test statistic：

F test statistic：

$$F_{n_1-1, n_2-1} = \frac{S_1^2 / S_2^2}{c} > F_{\alpha, n_1-1, n_2-1} \Rightarrow reject H_0,$$

2.3)*left-tailed test* (*left-sided test*)：

(i) $H_0:\dfrac{\sigma_1^2}{\sigma_2^2}\ge c, H_1:\dfrac{\sigma_1^2}{\sigma_2^2}<c,(c>0),\quad c$ is known value.

(ii) $\alpha = P\left(reject \quad H_0 \middle| H_0 : \dfrac{\sigma_1^2}{\sigma_2^2} = c \right)$

(iii) $F_{n_1-1,n_2-1} = \dfrac{S_1^2 / S_2^2}{c}, \overline{X} = \sum_{i=1}^{n_1} X_i \Big/ n_1, \overline{Y} = \sum_{j=1}^{n_2} Y_j \Big/ n_2,$

$$S_1^2 = \frac{\sum_{i=1}^{n_1}\left(X_i - \overline{X}\right)^2}{n_1 - 1}, S_2^2 = \frac{\sum_{j=1}^{n_2}\left(Y_j - \overline{Y}\right)^2}{n_2 - 1},$$

(iv)test statistic：

F test statistic：

$$F_{n_1-1,n_2-1} = \frac{S_1^2 / S_2^2}{c} < F_{1-\alpha,n_1-1,n_2-1} = \frac{1}{F_{\alpha,n_2-1,n_1-1}},$$

2.4) Two populations are independent.
The first population is Nomral(mu=10,sigma*sigma=25) and
the second population is Nomral(mu=15,sigma*sigma=25).
The sample size of two populations are 20.

Please input the sample size of the following random variables

1.X1	20
2.X2	20

X1	X2
3.6811009053	17.4635583605
14.3527448590	0.5558189789
7.0656989420	9.6149054007
15.2717935267	11.3534956947
12.9672877863	7.9187491647
11.3406473110	18.5217731382
17.9801565782	8.4724573887
14.8571650640	22.9986298860
4.2835929465	15.0544788101
4.3136970335	15.9864965627
10.0945417116	28.3863063729
15.8200217683	10.7583153139
14.3523828063	22.5956321536
10.6192019417	14.6773514604
10.3002772134	19.7164316749
12.4074756704	16.9260904550
7.5411981018	6.7147492994
9.1756416163	12.8233560777
6.2831401647	18.1859026532
3.3919787967	22.8413229949

X1 is Normal(mu=10.000000,sigma*sigma=25.000000),
X2 is Normal(mu=15.000000,sigma*sigma=25.000000),

X1 is mean= 10.3049872372, s.d.= 4.4698139748, variance= 19.9792369697,
skewed coefficient= -0.0992816722, kurtosis coefficient= 1.6640462925, MAD= 3.6919004940,
Q1= 6.6744195534, median= 10.4597395775, Q3= 14.3527448590,
MIN= 3.3919787967, MAX= 17.9801565782, Range= 14.5881777814,
Mid-Range= 10.6860676875, C.V.= 0.4337524998, sample size=20

X2 is mean= 15.0782910921, s.d.= 6.7001336103, variance= 44.8917903954,
skewed coefficient= -0.1117675702, kurtosis coefficient= 2.4371215169, MAD= 5.2839233331,
Q1= 10.1866103573, median= 15.5204876864, Q3= 19.7164316749,
MIN= 0.5558189789, MAX= 28.3863063729, Range= 27.8304873939,
Mid-Range= 14.4710626759, C.V.= 0.4443562980, sample size=20

after storing the sample data is below

	X1	X2
1	3.3919787967	0.5558189789
2	3.6811009053	6.7147492994
3	4.2835929465	7.9187491647
4	4.3136970335	8.4724573887
5	6.2831401647	9.6149054007
6	7.0656989420	10.7583153139
7	7.5411981018	11.3534956947
8	9.1756416163	12.8233560777
9	10.0945417116	14.6773514604
10	10.3002772134	15.0544788101
11	10.6192019417	15.9864965627
12	11.3406473110	16.9260904550
13	12.4074756704	17.4635583605
14	12.9672877863	18.1859026532
15	14.3523828063	18.5217731382
16	14.3527448590	19.7164316749
17	14.8571650640	22.5956321536
18	15.2717935267	22.8413229949
19	15.8200217683	22.9986298860
20	17.9801565782	28.3863063729

==

The sample data rank is below

	X1	rank(X1)	X2	rank(X2)
1	3.6811009053	2.000	17.4635583605	13.000
2	14.3527448590	16.000	0.5558189789	1.000
3	7.0656989420	6.000	9.6149054007	5.000
4	15.2717935267	18.000	11.3534956947	7.000
5	12.9672877863	14.000	7.9187491647	3.000
6	11.3406473110	12.000	18.5217731382	15.000
7	17.9801565782	20.000	8.4724573887	4.000
8	14.8571650640	17.000	22.9986298860	19.000
9	4.2835929465	3.000	15.0544788101	10.000
10	4.3136970335	4.000	15.9864965627	11.000
11	10.0945417116	9.000	28.3863063729	20.000
12	15.8200217683	19.000	10.7583153139	6.000
13	14.3523828063	15.000	22.5956321536	17.000
14	10.6192019417	11.000	14.6773514604	9.000
15	10.3002772134	10.000	19.7164316749	16.000
16	12.4074756704	13.000	16.9260904550	12.000
17	7.5411981018	7.000	6.7147492994	2.000
18	9.1756416163	8.000	12.8233560777	8.000
19	6.2831401647	5.000	18.1859026532	14.000
20	3.3919787967	1.000	22.8413229949	18.000

--------------- inference statistiscs ------------------

* Suppose two population distributions are the normal distribution.

population	X1	X2
sample size	20	20
sample mean	10.30499	15.07829
sample Variance	19.97924	44.89179
sample s.d.	4.46981	6.70013

A. X1 and X2 are independent random samples

two population variances ratio test and confidence interval when population means are unknown
H0: sigma(X1)=sigma(X2) , sigma(X1),sigma(X2) are population sigma
F(19,19)=0.445053 which formula is F=sample variance(X1)/sample variance(X2)
left tail test p-value= 0.0428
right tail test p-value= 0.9572
two tailes test p-value= 0.0856
The significant level =1.914400 , two population variances are equal.
Spool =((19*S(X1)^2+19*S(X2)^2)/38)^0.5=5.695218,
the degree of freedom=38
The Variance(X1)=Variance(X2)=population variance

The common population standard deviation and variance confidence interval
90% confidence interval for population variance
[23.088395 , 49.531806]
90% confidence interval for population standard deviation
[4.805038 , 7.037884]
95% confidence interval for population variance
[21.664164 , 53.872925]
95% confidence interval for population standard deviation
[4.654478 , 7.339818]
99% confidence interval for population variance
[19.203628 , 63.903223]
99% confidence interval for population standard deviation
[4.382194 , 7.993949]
two population means test and confidence interval when populations sigma is unknown
H0: mu(X1)=mu(X2) , mu(X1),mu(X2) are population means
Suppose the population variances are equal. sigma(X1)=sigma(X2)
t(df=38)=-2.650383 which formula is t=(X1 sample mean-X2 sample mean)/standard error
Spool =((19*S(X1)^2+19*S(X2)^2)/38)^0.5=5.695218,
the standard error=Spool*(1/20+1/20)^0.5=1.800986
X1 sample size=20, X2 sample size=20
the degree of freedom=38
left tail test p-value= 0.0059
right tail test p-value= 0.9941
two tailes test p-value= 0.0118
90% confidence interval for mu(X1)-mu(X2)
[-7.810178 , -1.736429]
95% confidence interval for mu(X1)-mu(X2)
[-8.419780 , -1.126828]
99% confidence interval for mu(X1)-mu(X2)
[-9.657660 , 0.111052]

B. X1 and X2 are paired samples, and rho(X1,X2) is not 0.

two population means test and confidence interval when populations sigma is unknown
H0: mu(X1)=mu(X2) , mu(X1),mu(X2) are population means
t(df=19)=-2.386550 which formula is t=(X1 sample mean-X2 sample mean)/standard error
standard error=((S(X1-X2)^2/20)^0.5,
df=19
left tail test p-value= 0.0138
right tail test p-value= 0.9862
two tailes test p-value= 0.0276
90% confidence interval for mu(X1)-mu(X2)
[-8.231228 , -1.315379]
95% confidence interval for mu(X1)-mu(X2)
[-8.958887 , -0.587720]
99% confidence interval for mu(X1)-mu(X2)
[-10.496803 , 0.950195]

Sd=S(X1-X2)=8.944652
the degree of freedom=19
The Variance(X1-X2)=population variance
90% confidence interval for population variance
[50.428517 , 150.251365]
90% confidence interval for population standard deviation
[7.101304 , 12.257706]
95% confidence interval for population variance
[46.271580 , 170.676156]
95% confidence interval for population standard deviation
[6.802322 , 13.064308]
99% confidence interval for population variance
[39.398008 , 222.110905]
99% confidence interval for population standard deviation
[6.276783 , 14.903386]

two populations correlation coefficient test
H0: rho(X1,X2)=0
r(X1,X2)=-0.252698 ,n=20
Zr(X1,X2)=-1.064968 ,n=20
left tail test p-value= 0.1435
right tail test p-value= 0.8565
two tailes test p-value= 0.2870
90% confidence interval for rho(X1,X2)

[-0.576524 ,   0.139735]
95% confidence interval for rho(X1,X2)
[-0.625302 ,   0.213732]
99% confidence interval for rho(X1,X2)
[-0.707969 ,   0.350939]

5. Two population variances test when the population means are unknwon,

select 5:

The two population sigmas ratio test ,H0 sigma(X1)/sigma(X2)=c

c= 1

two population variances ratio test and confidence interval when population means are unknown
H0: sigma(X1)/sigma(X2)=1.000000 , sigma(X1),sigma(X2) are population sigma
F(19,19)=0.445053 which formula is F=(sample variance(X1)/sample variance(X2))/1.000000
left tail test p-value= 0.0428
right tail test p-value= 0.9572
two tailes test p-value= 0.0856
The significant level =1.914400 , two population variances ratio are 1.000000.
Spool =((19*S(X1)^2/1.000000+19*S(X2)^2)/38)^0.5=5.695218,
the degree of freedom=38
The Variance(X1)=1.000000*Variance(X2),Variance(X2)=population variance
The common population standard deviation and variance confidence interval
90% confidence interval for population variance
[23.088395 ,   49.531806]
90% confidence interval for population standard deviation
[4.805038 ,   7.037884]
95% confidence interval for population variance
[21.664164 ,   53.872925]
95% confidence interval for population standard deviation
[4.654478 ,   7.339818]
99% confidence interval for population variance
[19.203628 ,   63.903223]
99% confidence interval for population standard deviation
[4.382194 ,   7.993949]