Chapter two

1)The test and confidence interval of one normal population mean and sigma:

1.1)Hypothesis.

The goal of hypothesis is the special parameter value of the normal distribution.

1.2)The classification of hypothesis.

(a)Null Hypothesis： H_0 is the symbol and the value of H_0 is always wrong.

(b)Alternative Hypothesis： H_1 or H_a is the symbol and the value of H_1 is always right.

1.3)The form of hypothesis.

The model $\overline{X} = \mu + \overline{\varepsilon}, \overline{\varepsilon}$ is sampling error, $E(\overline{\varepsilon}) = 0, Var(\overline{\varepsilon}) = \sigma^2/n$.

(a) Two tailed test(Two sided test)

$H_0 : \mu = \mu_0, H_1 : \mu \neq \mu_0$, μ_0 is known value.

(b) (Right tailed test(Right sided test)

$H_0 : \mu \leq \mu_0, H_1 : \mu > \mu_0$, the reject region is right sided.

(c) Left tailed test(Left sided test)

$H_0 : \mu \geq \mu_0, H_1 : \mu < \mu_0$, the reject region is left sided.

1.4)type I error, powerful test, type II error.

a)type I error=P(reject hypothesis. | Null Hypothesis), the risk of Null Hypothesis.

b)significant level=Max(type I error)= α ,

c)type II error= β =P(fail to reject hypothesis. | Alternative Hypothesis), the risk of Alternative Hypothesis.

d)powerful test= $1-\beta = P($reject hypothesis.| Alternative Hypothesis$)$.

1.5) The population distribution is normal distribution $N(\mu,\sigma^2)$, and the sample size is n. The sample data is $X_1,....,X_n$.

1.5.1)The normal population mean test and confidence interval when population variance σ^2 is knwon.

1.5.1.1)*two-tailed test* (*two-sided test*)：

(i) $H_0:\mu=\mu_0, H_1:\mu\neq\mu_0$, μ_0 is known value.

(ii) $\alpha=P(reject \quad H_0|H_0:\mu=\mu_0)$

(iii) $Z=\dfrac{\overline{X}-\mu}{\sigma/\sqrt{n}}, \overline{X}=\sum_{i=1}^{n}X_i\Big/n,$

(iv)test statistic：

a)Z test statistic：

$$Z=\frac{\overline{X}-\mu_0}{\sigma/\sqrt{n}},\quad \left|\frac{\overline{X}-\mu_0}{\sigma/\sqrt{n}}\right|>Z_{\alpha/2}\Rightarrow reject \quad H_0,$$

b)P-value：From sample value to get Z test statistic value= $\dfrac{\overline{X}-\mu_0}{\sigma/\sqrt{n}}=Z^*$,

$$P(Z>|Z^*|)=\frac{P-value}{2}, P-value<\alpha\Rightarrow reject \quad H_0.$$

c) Critical value)：

$$|\overline{X}-\mu_0|>Z_{\alpha/2}\,\sigma/\sqrt{n} \text{ is reject } H_0,$$

$$\Rightarrow \overline{X}>\mu_0+Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}=C_1, \overline{X}<\mu_0-Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}=C_2 \text{ is reject } H_0.$$

v) type II error and the powerful test：

type II error $=\beta$

$$=P(C_2\leq\overline{X}\leq C_1|H_1:\mu=\mu_1(\neq\mu_0))=P(\overline{X}\leq C_1|\mu=\mu_1)-P(\overline{X}\leq C_2|\mu=\mu_1)$$

$$=P\left(Z=\frac{\overline{X}-\mu_1}{\sigma/\sqrt{n}}\leq\frac{C_1-\mu_1}{\sigma/\sqrt{n}}\right)-P\left(Z\leq\frac{C_2-\mu_1}{\sigma/\sqrt{n}}\right)$$

powerful test $=1-\beta$

$$=1-P(\overline{X}\leq C_1|\mu=\mu_1)+P(\overline{X}\leq C_2|\mu=\mu_1)$$

$$=1-P\left(Z=\frac{\overline{X}-\mu_1}{\sigma/\sqrt{n}}\leq\frac{C_1-\mu_1}{\sigma/\sqrt{n}}\right)+P\left(Z\leq\frac{C_2-\mu_1}{\sigma/\sqrt{n}}\right)$$

vi) $(1-\alpha)\times 100\% \quad C.I. \quad for \quad \mu$

$$1-\alpha=P\left(|\overline{X}-\mu|\leq Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right)$$

$$\overline{X}-Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\leq\mu\leq\overline{X}+Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

1.5.1.2)*right-tailed test* (*right-sided test*)：

(i) $H_0 : \mu \le \mu_0, H_1 : \mu > \mu_0$, μ_0 is known value.

(ii) $\alpha = P(reject \quad H_0 | H_0 : \mu = \mu_0)$

(iii) $Z = \dfrac{\overline{X} - \mu}{\sigma / \sqrt{n}}, \overline{X} = \sum_{i=1}^{n} X_i \Big/ n,$

(iv)test statistic：

a)Z test statistic：

$$Z = \frac{\overline{X} - \mu_0}{\sigma / \sqrt{n}}, \quad \frac{\overline{X} - \mu_0}{\sigma / \sqrt{n}} > Z_\alpha \Rightarrow reject \quad H_0 ,$$

b)P-value：From sample value to get Z test statistic value= $\dfrac{\overline{X} - \mu_0}{\sigma / \sqrt{n}} = Z^*$,

$$P(Z > Z^*) = P - value , P - value < \alpha \Rightarrow reject \quad H_0 .$$

c) Critical value)：

$\dfrac{\overline{X} - \mu_0}{\sigma / \sqrt{n}} > Z_\alpha$ is reject H_0 ,

$\Rightarrow \overline{X} > \mu_0 + Z_\alpha \dfrac{\sigma}{\sqrt{n}} = C$ is reject H_0.

v) type II error and the powerful test：

type II error $= \beta$

$$= P(\overline{X} \le C | H_1 : \mu = \mu_1 (> \mu_0)) = P\left(Z = \frac{\overline{X} - \mu_1}{\sigma / \sqrt{n}} \le \frac{C - \mu_1}{\sigma / \sqrt{n}} \right)$$

powerful test $= 1 - \beta$

$$= P(\overline{X} > C | \mu = \mu_1) = P\left(Z = \frac{\overline{X} - \mu_1}{\sigma / \sqrt{n}} > \frac{C - \mu_1}{\sigma / \sqrt{n}} \right)$$

vi) $(1 - \alpha) \times 100\% \quad C.I. \quad for \quad \mu$

$$1 - \alpha = P\left(\overline{X} - \mu \le Z_\alpha \frac{\sigma}{\sqrt{n}} \right)$$

$$\overline{X} - Z_\alpha \frac{\sigma}{\sqrt{n}} \le \mu < \infty$$

1.5.1.3)*left-tailed test* (*left-sided test*)：

(i) $H_0 : \mu \geq \mu_0, H_1 : \mu < \mu_0$, μ_0 is known value.

(ii) $\alpha = P\left(reject \quad H_0 \middle| H_0 : \mu = \mu_0\right)$

(iii) $Z = \dfrac{\overline{X} - \mu}{\sigma / \sqrt{n}}, \overline{X} = \sum_{i=1}^{n} X_i \Big/ n,$

(iv)test statistic：

a)Z test statistic：

$$Z = \frac{\overline{X} - \mu_0}{\sigma / \sqrt{n}}, \quad \frac{\overline{X} - \mu_0}{\sigma / \sqrt{n}} < -Z_\alpha \Rightarrow reject \quad H_0 ,$$

b)P-value：From sample value to get Z test statistic value= $\dfrac{\overline{X} - \mu_0}{\sigma / \sqrt{n}} = Z^*$,

$$P\left(Z < Z^*\right) = P - value, P - value < \alpha \Rightarrow reject \quad H_0 .$$

c) Critical value)：

$$\frac{\overline{X} - \mu_0}{\sigma / \sqrt{n}} < -Z_\alpha \text{ is reject } H_0 ,$$

$$\Rightarrow \overline{X} < \mu_0 - Z_\alpha \frac{\sigma}{\sqrt{n}} = C \text{ is reject } H_0 .$$

v) type II error and the powerful test：

type II error $= \beta$

$$= P\left(\overline{X} \geq C \middle| H_1 : \mu = \mu_1 (< \mu_0)\right) = P\left(Z = \frac{\overline{X} - \mu_1}{\sigma / \sqrt{n}} \geq \frac{C - \mu_1}{\sigma / \sqrt{n}} \right)$$

powerful test $= 1 - \beta$

$$= P\left(\overline{X} < C \middle| \mu = \mu_1\right) = P\left(Z = \frac{\overline{X} - \mu_1}{\sigma / \sqrt{n}} < \frac{C - \mu_1}{\sigma / \sqrt{n}} \right)$$

vi) $(1 - \alpha) \times 100\% \quad C.I. \quad for \quad \mu$

$$1 - \alpha = P\left(-Z_\alpha \frac{\sigma}{\sqrt{n}} \leq \overline{X} - \mu \right)$$

$$-\infty < \mu \leq \overline{X} + Z_\alpha \frac{\sigma}{\sqrt{n}}$$

1.5.2)The normal population mean test and confidence interval when population variance σ^2 is unknown.

1.5.2.1)*two-tailed test* (*two-sided test*)：

(i) $H_0 : \mu = \mu_0, H_1 : \mu \neq \mu_0$, μ_0 is known value.

(ii) $\alpha = P\left(reject \quad H_0 \middle| H_0 : \mu = \mu_0\right)$

(iii) $t_{n-1} = \dfrac{\overline{X} - \mu}{S/\sqrt{n}}, \overline{X} = \sum_{i=1}^{n} X_i \Big/ n, S = \dfrac{\sum_{i=1}^{n}\left(X_i - \overline{X}\right)^2}{n-1}$

(iv)test statistic：

a)t test statistic：

$$t_{n-1} = \frac{\overline{X} - \mu_0}{S/\sqrt{n}}, \quad \left|\frac{\overline{X} - \mu_0}{S/\sqrt{n}}\right| > t_{\alpha/2,n-1} \Rightarrow reject \quad H_0 ,$$

b)P-value：From sample value to get t test statistic value= $\dfrac{\overline{X} - \mu_0}{S/\sqrt{n}} = t^*$,

$$P\left(t_{n-1} > \left|t^*\right|\right) = \frac{P - value}{2}, P - value < \alpha \Rightarrow reject \quad H_0 .$$

c) Critical value)：

$\left|\overline{X} - \mu_0\right| > t_{\alpha/2,n-1}\, \sigma/\sqrt{n}$ is reject H_0 ,

$$\Rightarrow \overline{X} > \mu_0 + t_{\alpha/2,n-1}\frac{\sigma}{\sqrt{n}} = C_1 或 \overline{X} < \mu_0 - t_{\alpha/2,n-1}\frac{\sigma}{\sqrt{n}} = C_2 \text{ is reject } H_0 .$$

v) type II error and the powerful test：

type II error $= \beta$

$$= P\left(C_2 \leq \overline{X} \leq C_1 \middle| H_1 : \mu = \mu_1 (\neq \mu_0)\right) = P\left(\overline{X} \leq C_1 \middle| \mu = \mu_1\right) - P\left(\overline{X} \leq C_2 \middle| \mu = \mu_1\right)$$

$$= P\left(t_{n-1} = \frac{\overline{X} - \mu_1}{S/\sqrt{n}} \leq \frac{C_1 - \mu_1}{S/\sqrt{n}} \right) - P\left(t_{n-1} \leq \frac{C_2 - \mu_1}{S/\sqrt{n}} \right)$$

powerful test $= 1 - \beta$

$$= 1 - P\left(\overline{X} \leq C_1 \middle| \mu = \mu_1\right) + P\left(\overline{X} \leq C_2 \middle| \mu = \mu_1\right)$$

$$= 1 - P\left(t_{n-1} = \frac{\overline{X} - \mu_1}{S/\sqrt{n}} \leq \frac{C_1 - \mu_1}{S/\sqrt{n}} \right) + P\left(t_{n-1} \leq \frac{C_2 - \mu_1}{S/\sqrt{n}} \right)$$

vi) $(1-\alpha)\times 100\% \quad C.I. \quad for \quad \mu$

$$1 - \alpha = P\left(\left|\overline{X} - \mu\right| \leq Z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \right)$$

$$\overline{X} - t_{\alpha/2,n-1}\frac{\sigma}{\sqrt{n}} \leq \mu \leq \overline{X} + t_{\alpha/2,n-1}\frac{\sigma}{\sqrt{n}}$$

1.5.2.2)*right-tailed test* (*right-sided test*)：

(i) $H_0 : \mu \le \mu_0, H_1 : \mu > \mu_0$, μ_0 is known value.

(ii) $\alpha = P(reject \quad H_0 | H_0 : \mu = \mu_0)$

(iii) $t_{n-1} = \dfrac{\overline{X} - \mu}{S/\sqrt{n}}, \overline{X} = \sum_{i=1}^{n} X_i \Big/ n,$

(iv)test statistic：

a)Z test statistic：

$$t_{n-1} = \frac{\overline{X} - \mu_0}{S/\sqrt{n}}, \quad \frac{\overline{X} - \mu_0}{S/\sqrt{n}} > t_{\alpha,n-1} \Rightarrow reject \quad H_0,$$

b)P-value：From sample value to get t test statistic value= $\dfrac{\overline{X} - \mu_0}{S/\sqrt{n}} = t^*$,

$$P(t_{n-1} > t^*) = P - value, P - value < \alpha \Rightarrow reject \quad H_0 .$$

c) Critical value)：

$$\frac{\overline{X} - \mu_0}{S/\sqrt{n}} > t_{\alpha,n-1} \text{ is reject } H_0 ,$$

$$\Rightarrow \overline{X} > \mu_0 + t_{\alpha,n-1}\frac{\sigma}{\sqrt{n}} = C \text{ is reject } H_0 .$$

v) type II error and the powerful test：

type II error $= \beta$

$$= P(\overline{X} \le C | H_1 : \mu = \mu_1 (> \mu_0)) = P\left(t_{n-1} = \frac{\overline{X} - \mu_1}{S/\sqrt{n}} \le \frac{C - \mu_1}{S/\sqrt{n}} \right)$$

powerful test $= 1 - \beta$

$$= P(\overline{X} > C | \mu = \mu_1) = P\left(t_{n-1} = \frac{\overline{X} - \mu_1}{S/\sqrt{n}} > \frac{C - \mu_1}{S/\sqrt{n}} \right)$$

vi) $(1-\alpha)\times 100\% \quad C.I. \quad for \quad \mu$

$$1 - \alpha = P\left(\overline{X} - \mu \le t_{\alpha,n-1} \times \frac{S}{\sqrt{n}} \right)$$

$$\overline{X} - t_{\alpha,n-1} \times \frac{S}{\sqrt{n}} \le \mu < \infty$$

1.5.2.3)*left-tailed test* (*left-sided test*)：

(i) $H_0 : \mu \geq \mu_0, H_1 : \mu < \mu_0$, μ_0 is known value.

(ii) $\alpha = P(reject \quad H_0 | H_0 : \mu = \mu_0)$

(iii) $t_{n-1} = \dfrac{\overline{X} - \mu}{S/\sqrt{n}}, \overline{X} = \sum_{i=1}^{n} X_i \Big/ n,$

(iv)test statistic：

a)Z test statistic：

$$t_{n-1} = \frac{\overline{X} - \mu_0}{S/\sqrt{n}}, \quad \frac{\overline{X} - \mu_0}{S/\sqrt{n}} < -t_{\alpha,n-1} \Rightarrow reject \quad H_0 ,$$

b)P-value：From sample value to get t test statistic value= $\dfrac{\overline{X} - \mu_0}{S/\sqrt{n}} = t^*$,

$$P(t_{n-1} < t^*) = P - value, P - value < \alpha \Rightarrow reject \quad H_0 .$$

c) Critical value)：

$\dfrac{\overline{X} - \mu_0}{S/\sqrt{n}} < -t_{\alpha,n-1}$ is reject H_0，

$\Rightarrow \overline{X} < \mu_0 - t_{\alpha,n-1} \times S/\sqrt{n} = C$ is reject H_0.

v) type II error and the powerful test：

type II error $= \beta$

$$= P(\overline{X} \geq C | H_1 : \mu = \mu_1 (< \mu_0)) = P\left(t_{n-1} = \frac{\overline{X} - \mu_1}{S/\sqrt{n}} \geq \frac{C - \mu_1}{S/\sqrt{n}} \right)$$

powerful test $= 1 - \beta$

$$= P(\overline{X} < C | \mu = \mu_1) = P\left(t_{n-1} = \frac{\overline{X} - \mu_1}{S/\sqrt{n}} < \frac{C - \mu_1}{S/\sqrt{n}} \right)$$

vi) $(1-\alpha) \times 100\% \quad C.I. \quad for \quad \mu$

$$1 - \alpha = P(-t_{\alpha,n-1} \times S/\sqrt{n} \leq \overline{X} - \mu)$$

$$-\infty < \mu \leq \overline{X} + t_{\alpha,n-1} \times S/\sqrt{n}$$

1.6)example:
The population is Nomral(mu=10,sigma*sigma=25),
There are 10 sample data from the normal population by simulation.

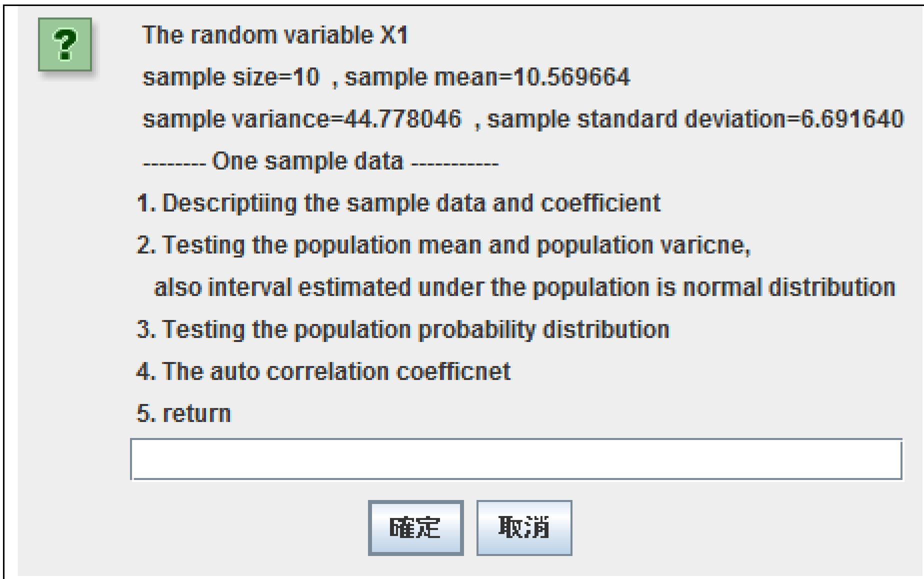
```
          X1
1.8450162861
8.8805820425
25.3510678816
9.3023655721
9.3115949680
7.8615354828
8.4407006524
16.9118047619
4.1705405862
13.6214305395
 X1 is    Normal(mu=10.000000,sigma*sigma=25.000000),


 X1 is    mean=          10.5696638773, s.d.=          6.6916400018, variance=          44.7780459131,
          skewed coefficient=          0.8505579763, kurtosis coefficient=          2.8428374123, MAD=
4.8350623102,
          Q1=          7.8615354828, median=          9.0914738073, Q3=          15.2666176507,
          MIN=          1.8450162861, MAX=          25.3510678816, Range=          23.5060515955,
          Mid-Range=          13.5980420838, C.V.=          0.6330986566, sample size=10


  after storing the sample data is below
                 X1
    1         1.8450162861
    2         4.1705405862
    3         7.8615354828
    4         8.4407006524
    5         8.8805820425
    6         9.3023655721
    7         9.3115949680
    8        13.6214305395
    9        16.9118047619
   10        25.3510678816
=========================================================
 The sample data rank is below
                   X1              rank(X1)
    1         1.8450162861           1.000
    2         8.8805820425           5.000
    3        25.3510678816          10.000
    4         9.3023655721           6.000
    5         9.3115949680           7.000
    6         7.8615354828           3.000
    7         8.4407006524           4.000
    8        16.9118047619           9.000
    9         4.1705405862           2.000
   10        13.6214305395           8.000


    --------------- inference statistiscs ------------------
 * Suppose the population distribution is the normal distribution.
1. one population mean test and mu confidence interval when population sigma is unknown
 H0: mu=0       , mu is population mean
 t(df=9)=4.994921     which formula is t=(X1 sample mean-0)/standard error
 the standard error =sample stand deviation/(n-1)^0.5, n is sample size=10
 left tail test p-value=       0.9997
 right tail test p-value=       0.0003
 two tailes test p-value=       0.0006
  90% confidence interval for mu
  [6.691079 ,    14.448249]
  95% confidence interval for mu
  [5.783340 ,    15.355987]
  99% confidence interval for mu
  [3.690052 ,    17.449275]
```

2. one population sigma confidence interval when population mean is unknown
90% confidence interval for population variance
[23.819406 ,   121.196100]
90% confidence interval for population standard deviation
[4.880513 ,   11.008910]
95% confidence interval for population variance
[21.184474 ,   149.238799]
95% confidence interval for population standard deviation
[4.602659 ,   12.216333]
99% confidence interval for population variance
[17.083562 ,   232.379957]
99% confidence interval for population standard deviation
[4.133227 ,   15.244014]



select 2:

The random variable X1

sample size=10 , sample mean=10.569664

sample variance=44.778046 , sample standard deviation=6.691640

-------- One population mu and sigma test -----------

1. One population mu test , the population sigma is known
   there need input Ho mu value and sigma value
2. One population mu test , the population sigma is unknown
   there need input Ho mu value
3. One population sigma test , the population mu is known
   there need input Ho sigma value and mu value
4. One population sigma test , the population mu is unknown
   there need input Ho sigma value
5. The population disitribution whether normal distribution test
6. return

確定 取消

select 1:

The one population mu test ,H0 mu value and population sigma value

H0: mu= 10

population sigma= 5

one sample population mu test when sigma is known

H0: mu=10.000000 sigma=5.000000, Z test value=6.496418

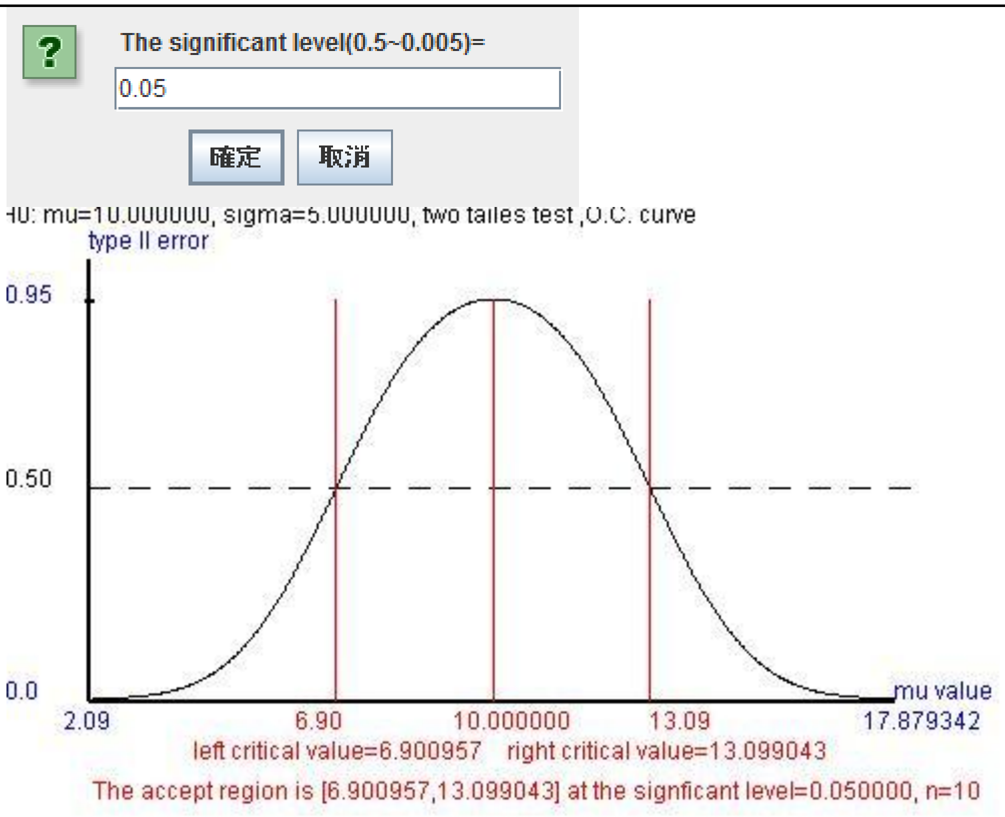
[The sample data X1 O.C. curve, P.F. and test value image]

~~~~~~~~~~~~~~ choose one ~~~~~~~~~~~~~~~~~

1. two tailes Operation characteristic curve
2. two tailes Power function
3. right tail Operation characteristic curve
4. right tail Power function
5. left tail Operation characteristic curve
6. left tail Power function
7. test value image
8. return

確定 取消

selet 1:
~~~~~~~~~~~~~~



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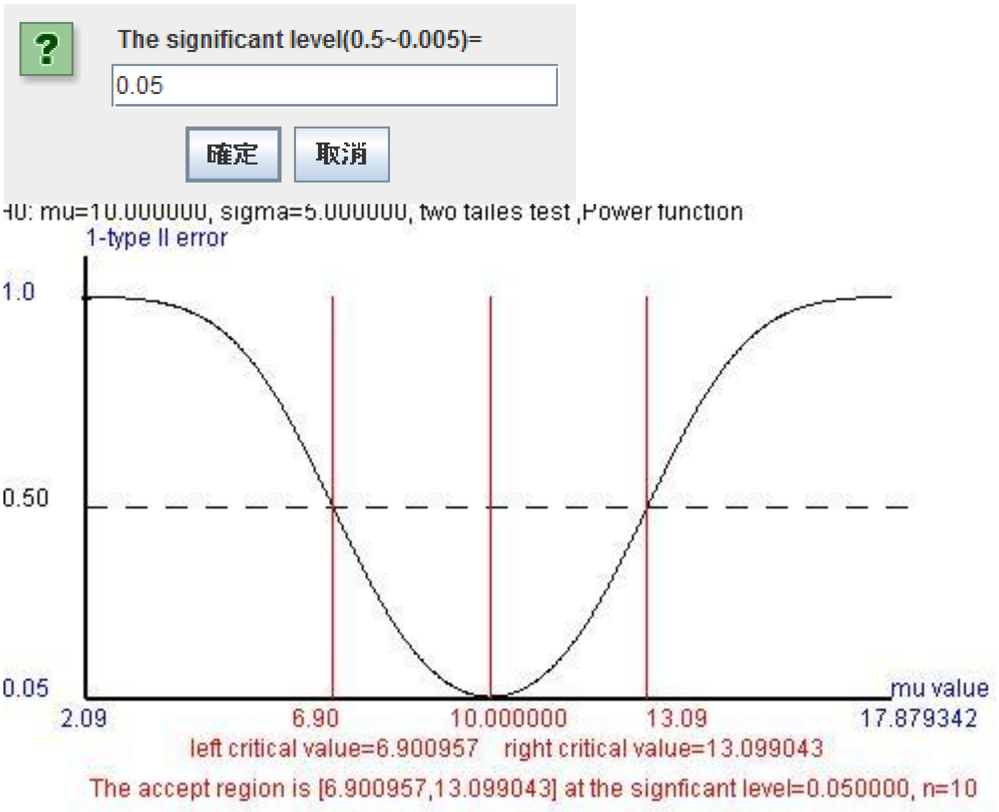

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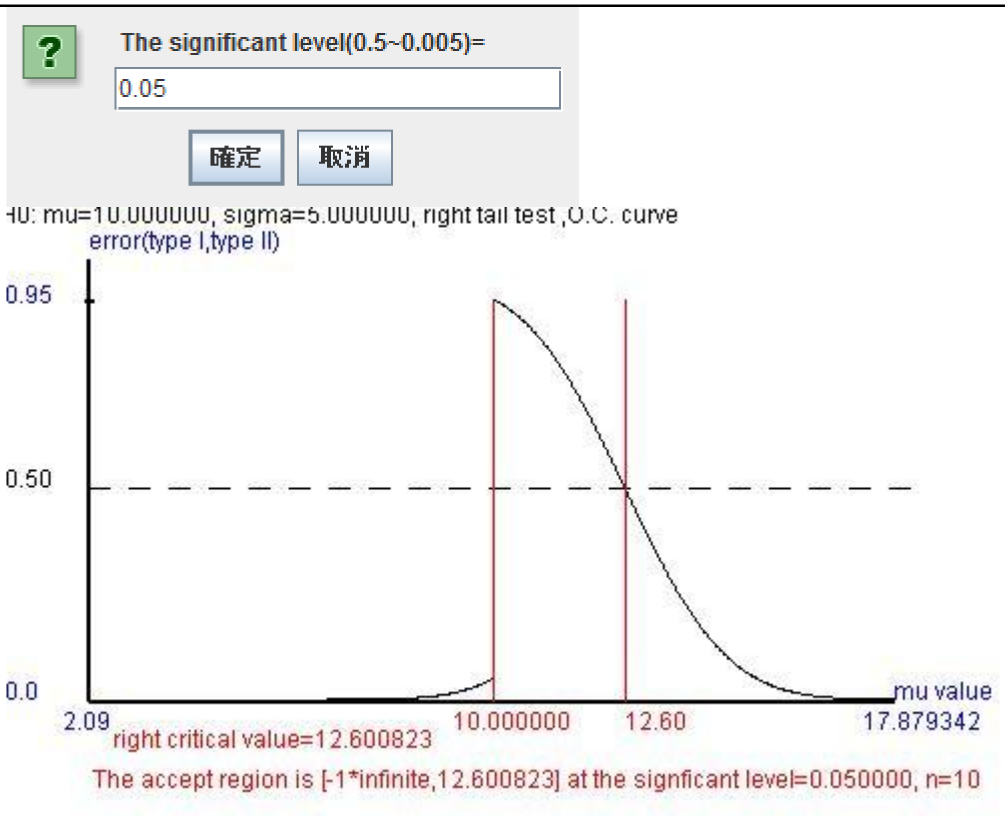

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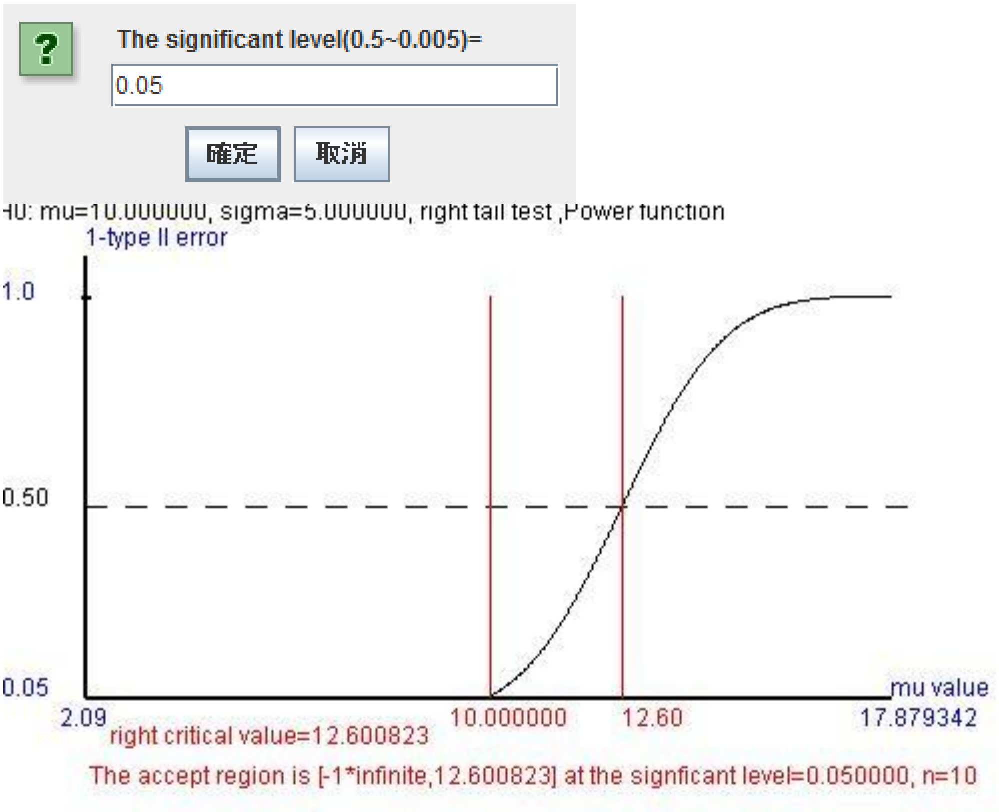

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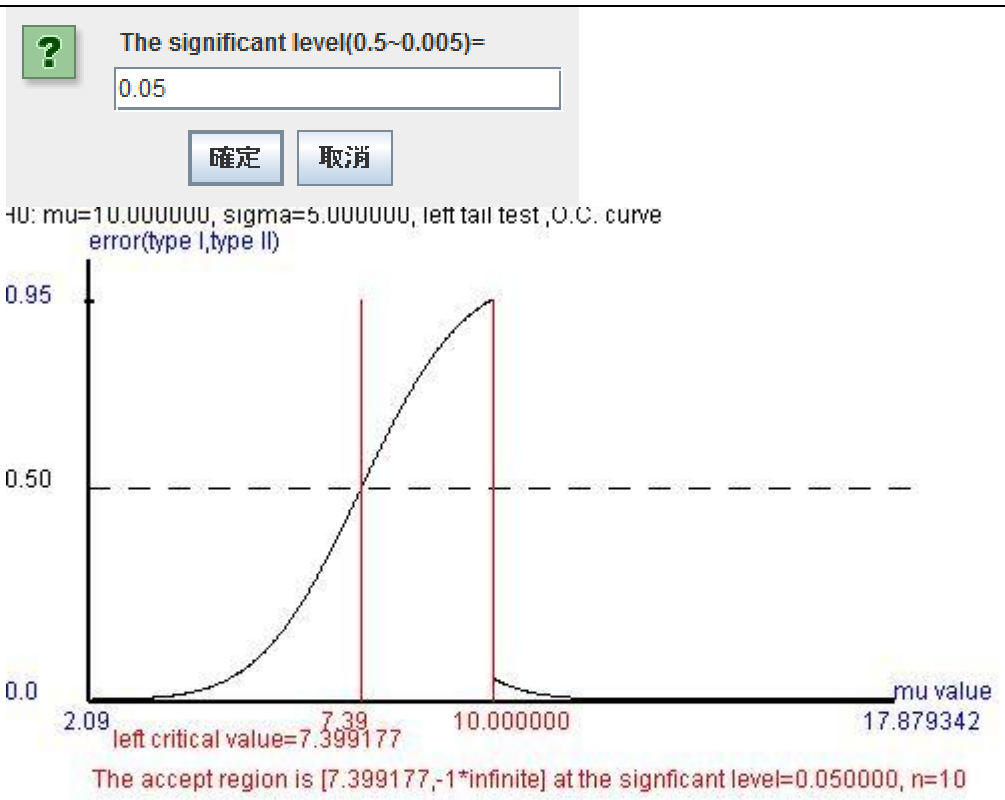

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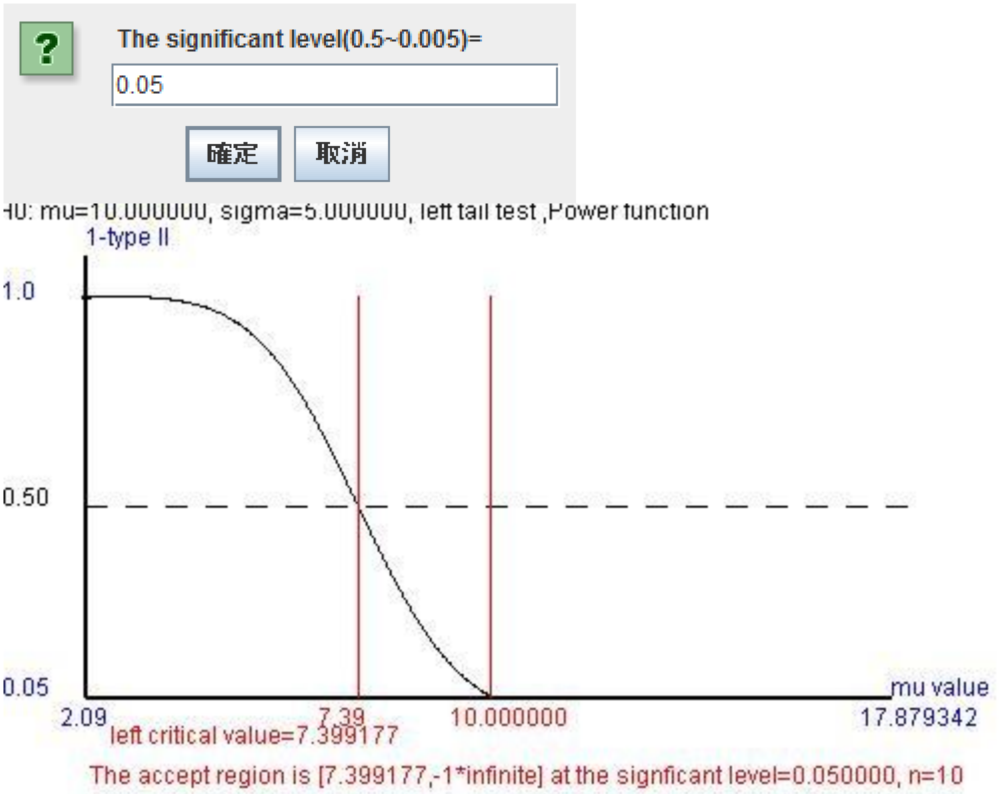

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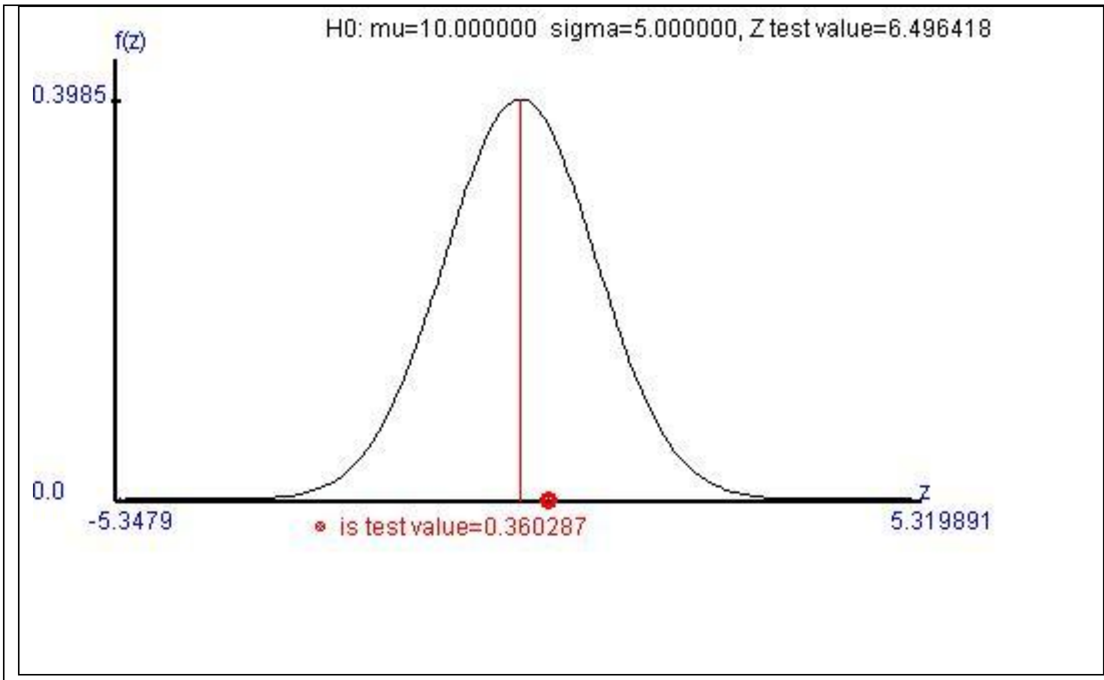
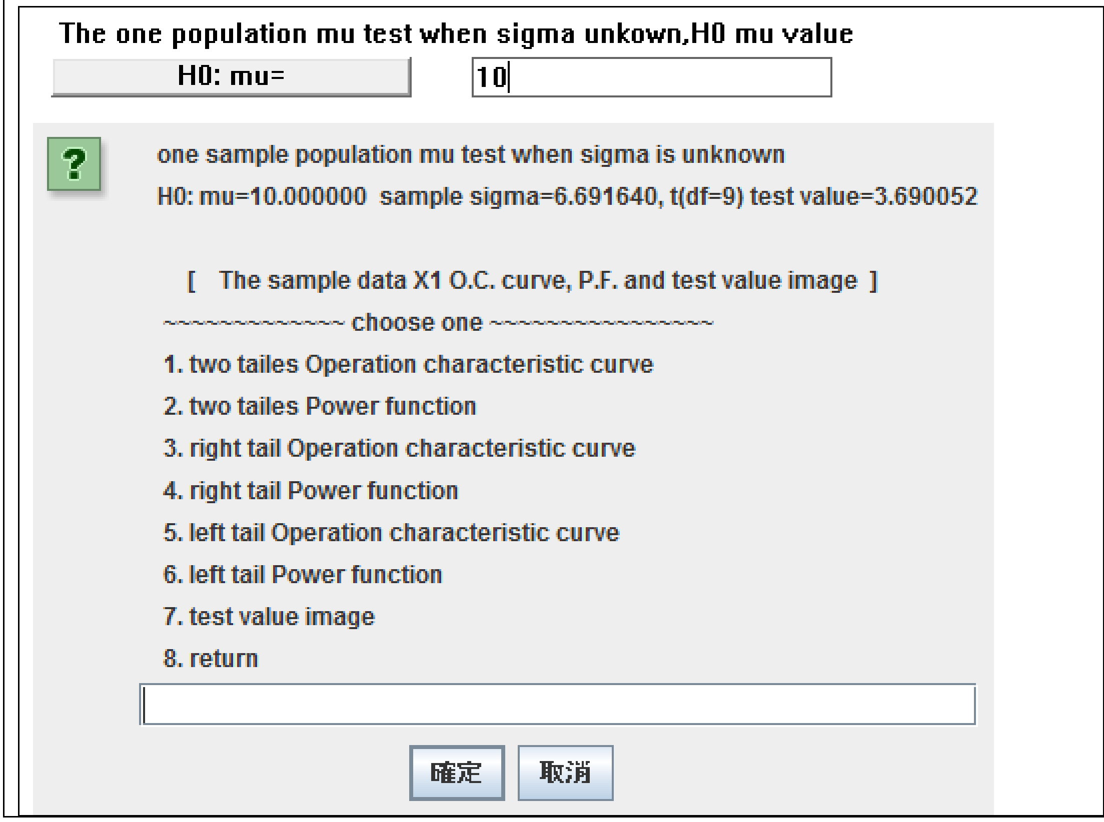


One population mean test , the population standard deviation is knwon
H0: mu=10.000000 , mu is population mean , the population standard deviation=5.000000
The sample mean=10.569664
the test statistic Z=0.360287 ,
which formula is Z=(X1 sample mean-0)/standard error
the standard error =population stand deviation/n^0.5=1.581139, n is sample size=10
left tail test p-value= 0.6407
right tail test p-value= 0.3593
two tailes test p-value= 0.7186
90% confidence interval for mu
[7.968841 , 13.170487]
95% confidence interval for mu
[7.470621 , 13.668707]
99% confidence interval for mu
[6.496418 , 14.642909]

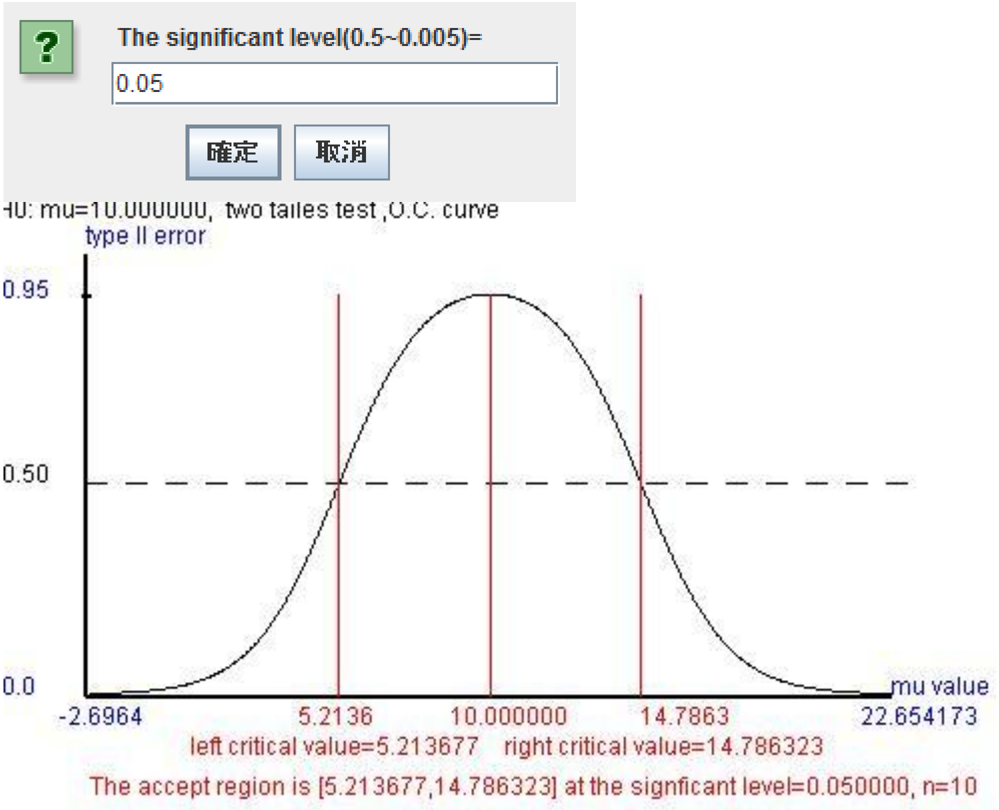
2. One population mu test , the population sigma is unknown

there need input Ho mu value

select 2:



select 1:



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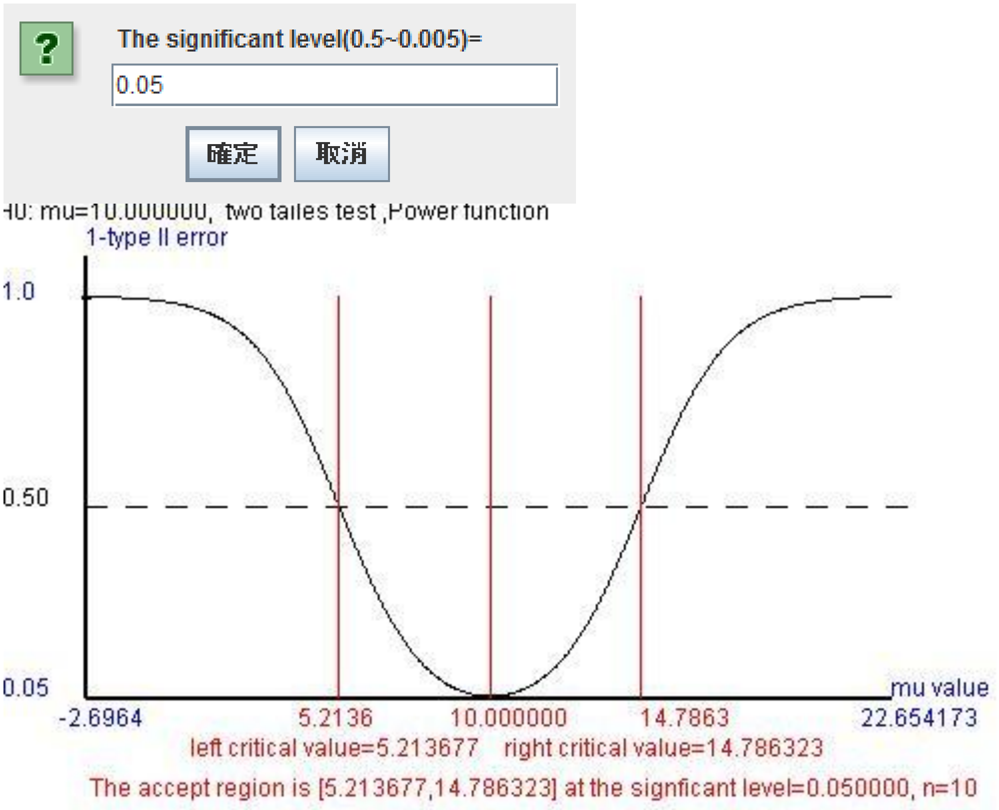

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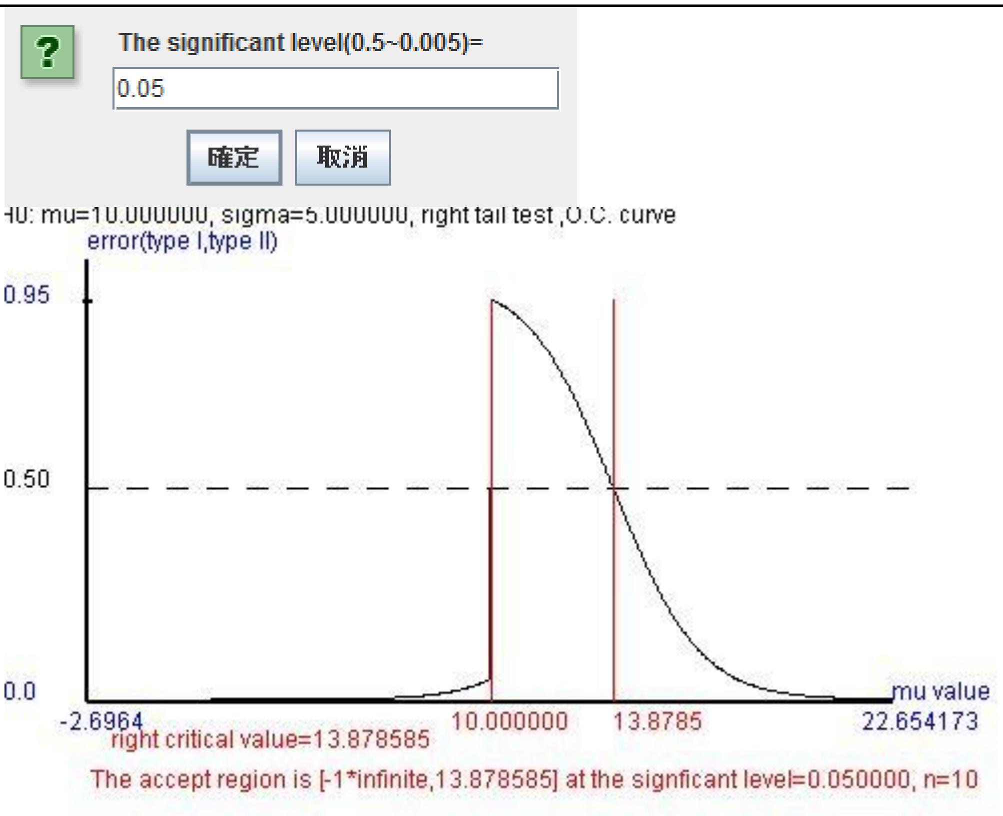

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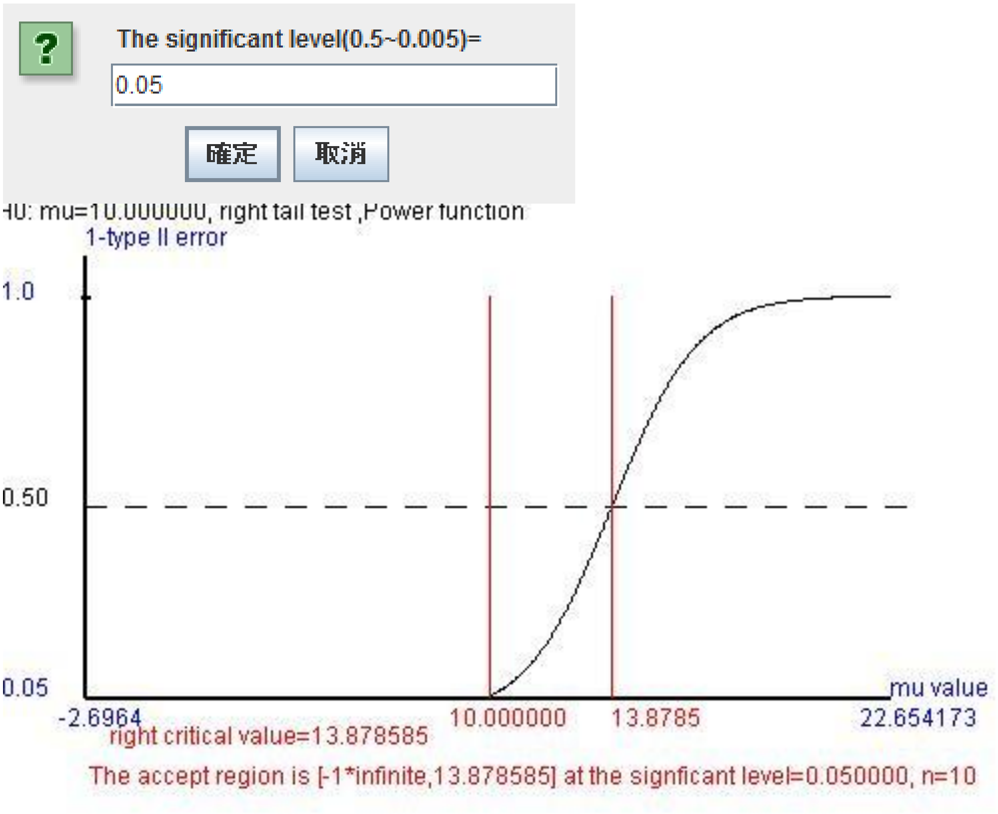

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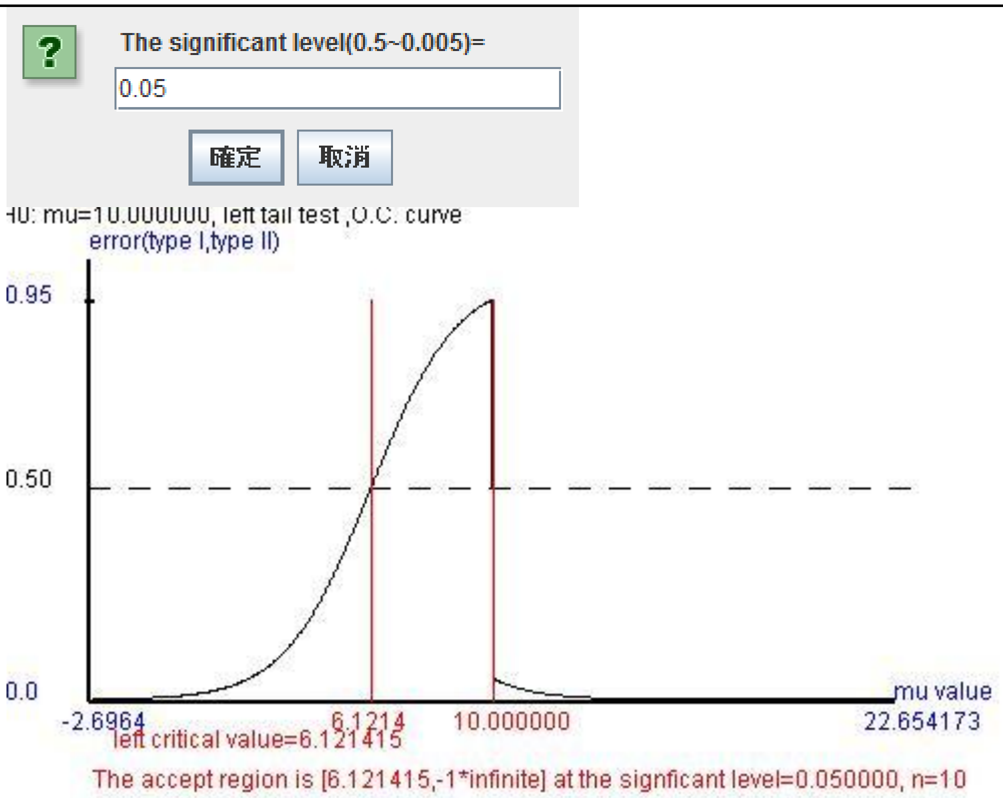
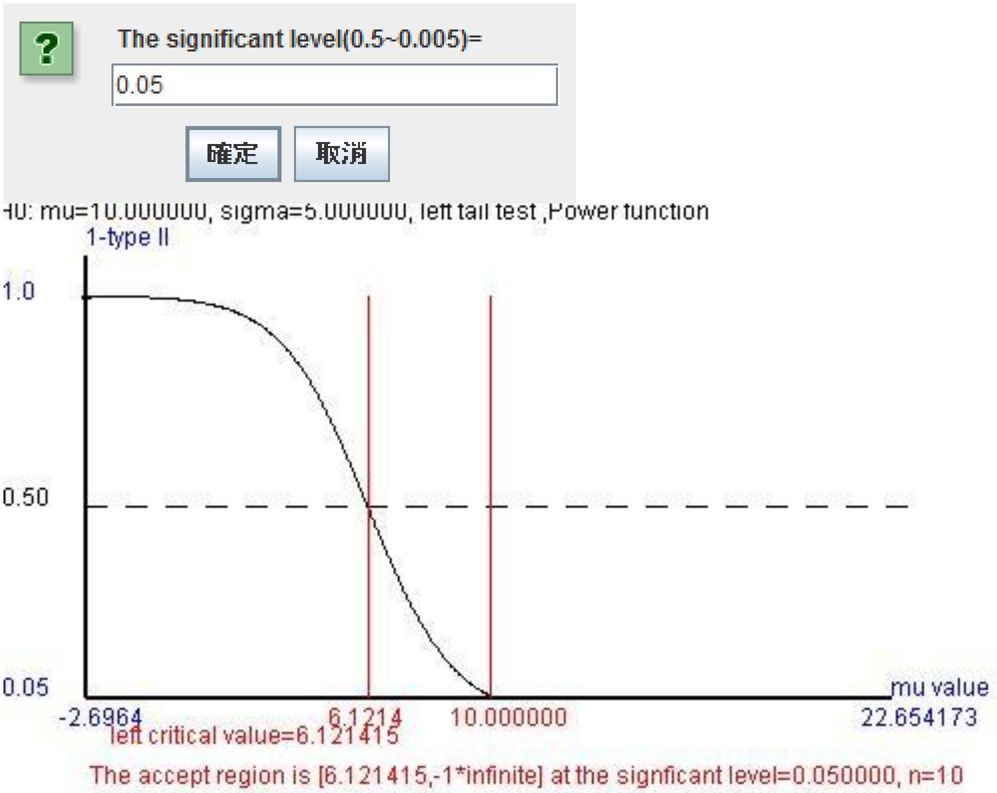
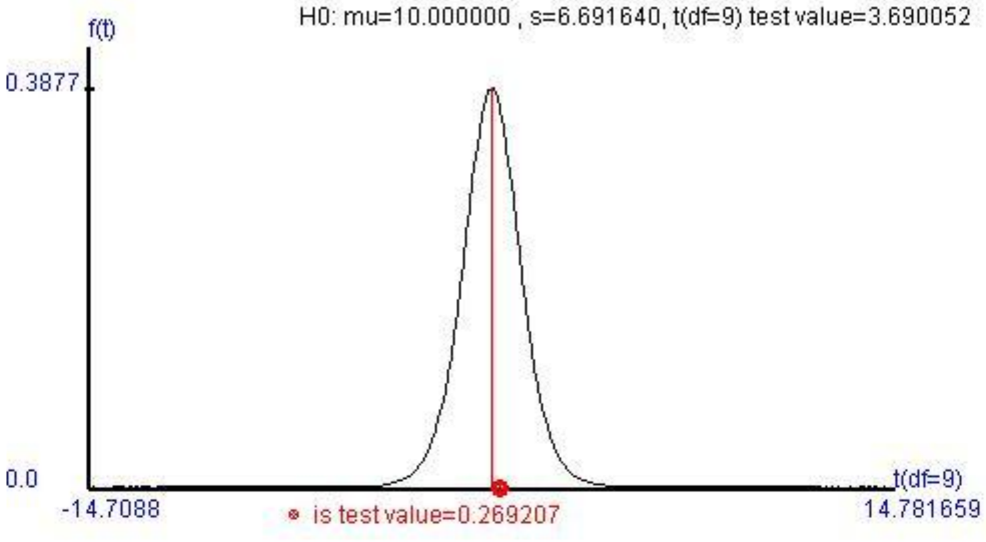


select 6:



select 7:



One population mean test , the population standard deviation is unknown
H0: mu=10.000000 , mu is population mean , the sample standard deviation=6.691640
The sample mean=10.569664
the test statistic t(df=9)=0.269207 ,
which formula is t=(X1 sample mean-0)/standard error
the standard error =sample stand deviation/(n-1)^0.5, n is sample size=10
left tail test p-value= 0.6031
right tail test p-value= 0.3969
two tailes test p-value= 0.7938
90% confidence interval for mu
[6.691079 , 14.448249]
95% confidence interval for mu
[5.783340 , 15.355987]
99% confidence interval for mu
[3.690052 , 17.449275]