7)The Durbin Watson test in the multiple regression model.

7.1).The Durbin Watson test of the multiple regression, the model is

$Y_t = \beta_0 + \beta_1 X_{t1} + \beta_2 X_{t2} + ... + + \beta_p X_{p2} + \varepsilon_t, t = 1,2,...,T,\ p >= 2, T - (p+1) >= 1,$

$\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon},$

assumption:

(1) $E(\boldsymbol{\varepsilon}) = \mathbf{0},$

(2) $E(\mathbf{X}^T\boldsymbol{\varepsilon}) = \mathbf{0},$ (3) $\varepsilon_t \sim N(0, \sigma_\varepsilon^2), E(\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}^T) = \sigma_\varepsilon^2 \times \begin{bmatrix} 1 & \rho & \rho^2 & .... & \rho^{T-1} \\ \rho & 1 & \rho & .... & \rho^{T-2} \\ \rho^2 & \rho & 1 & .... & 0 \\ ... & ... & ... & ... & \rho \\ \rho^{T-1} & \rho^{T-2} & .... & \rho & 1 \end{bmatrix}$

$\boldsymbol{\varepsilon} = \mathbf{Y} - \mathbf{X}\boldsymbol{\beta}, \boldsymbol{\varepsilon}\boldsymbol{\varepsilon}^T = (\mathbf{Y} - \mathbf{X}\boldsymbol{\beta})^T (\mathbf{Y} - \mathbf{X}\boldsymbol{\beta}), \frac{\partial(\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}^T)}{\partial\boldsymbol{\beta}} = 0, \hat{\boldsymbol{\beta}} = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{Y},$

$\hat{\mathbf{Y}} = \mathbf{X}\hat{\boldsymbol{\beta}} = \mathbf{X}(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{Y}, \mathbf{e} = \mathbf{Y} - \hat{\mathbf{Y}} = \mathbf{Y} - \mathbf{X}(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{Y},$

The constrains are $\mathbf{e}^\mathbf{T}\mathbf{X}, = \mathbf{0}$ and the degree of freedom is T-p-1.

But the collinearity of independent variables is a condition of $\mathbf{e}^\mathbf{T}\mathbf{X}, = \mathbf{0}$.

$\mathbf{e} = \left(\mathbf{I} - \mathbf{X}(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\right)\mathbf{Y},$ the error variance-covariance matrix is

$E(\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}^T) = E\left[(\mathbf{Y} - \mathbf{X}\boldsymbol{\beta})(\mathbf{Y} - \mathbf{X}\boldsymbol{\beta})^T\right] = E(\mathbf{Y}\mathbf{Y}^\mathbf{T}) - \mathbf{X}\boldsymbol{\beta}E(\mathbf{Y}^\mathbf{T}) - E(\mathbf{Y})\boldsymbol{\beta}^\mathbf{T}\mathbf{X}^\mathbf{T}$

$+ \mathbf{X}\boldsymbol{\beta}\boldsymbol{\beta}^\mathbf{T}\mathbf{X}^\mathbf{T} = E(\mathbf{Y}\mathbf{Y}^\mathbf{T}) - \mathbf{X}\boldsymbol{\beta}\boldsymbol{\beta}^\mathbf{T}\mathbf{X}^\mathbf{T}, E(\mathbf{Y}\mathbf{Y}^\mathbf{T}) = \sigma_\varepsilon^2 \times \begin{bmatrix} 1 & \rho & \rho^2 & .... & \rho^{T-1} \\ \rho & 1 & \rho & .... & \rho^{T-2} \\ \rho^2 & \rho & 1 & .... & 0 \\ ... & ... & ... & ... & \rho \\ \rho^{T-1} & \rho^{T-2} & .... & \rho & 1 \end{bmatrix} + \mathbf{X}\boldsymbol{\beta}\boldsymbol{\beta}^\mathbf{T}\mathbf{X}^\mathbf{T}$

$E(\mathbf{e}\mathbf{e}^T) = \left(\mathbf{I} - \mathbf{X}(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T \setminus\right) E(\mathbf{Y}\mathbf{Y}^\mathbf{T}) \left(\mathbf{I} - \mathbf{X}(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\right)^T$

$= \left(\mathbf{I} - \mathbf{X}(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\right) \left(\sigma_\varepsilon^2 \times \begin{bmatrix} 1 & \rho & \rho^2 & .... & \rho^{T-1} \\ \rho & 1 & \rho & .... & \rho^{T-2} \\ \rho^2 & \rho & 1 & .... & 0 \\ ... & ... & ... & ... & \rho \\ \rho^{T-1} & \rho^{T-2} & .... & \rho & 1 \end{bmatrix} + \mathbf{X}\boldsymbol{\beta}\boldsymbol{\beta}^\mathbf{T}\mathbf{X}^\mathbf{T} \right) \left(\mathbf{I} - \mathbf{X}(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\right)^T$

$= \left(\mathbf{I} - \mathbf{X}(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\right) \left(\sigma_\varepsilon^2 \times \begin{bmatrix} 1 & \rho & \rho^2 & .... & \rho^{T-1} \\ \rho & 1 & \rho & .... & \rho^{T-2} \\ \rho^2 & \rho & 1 & .... & 0 \\ ... & ... & ... & ... & \rho \\ \rho^{T-1} & \rho^{T-2} & .... & \rho & 1 \end{bmatrix} \right) \left(\mathbf{I} - \mathbf{X}(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\right)^T$

$= \sigma_\varepsilon^2 \times \left(\mathbf{I} - \mathbf{X}(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\right) \begin{bmatrix} 1 & \rho & \rho^2 & .... & \rho^{T-1} \\ \rho & 1 & \rho & .... & \rho^{T-2} \\ \rho^2 & \rho & 1 & .... & 0 \\ ... & ... & ... & ... & \rho \\ \rho^{T-1} & \rho^{T-2} & .... & \rho & 1 \end{bmatrix} \left(\mathbf{I} - \mathbf{X}(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\right)^T$

if $\rho = 0, E(\mathbf{e}\mathbf{e}^T) = \sigma_\varepsilon^2 \times \left(\mathbf{I} - \mathbf{X}(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\right)$

$$H_0 : \rho = \rho_0, H_1 : \rho \neq \rho_0, D.W. = \frac{\sum_{t=2}^{T} (e_t - e_{t-1})^2}{\sum_{t=1}^{T} e_t^2}$$

$$New \quad D.W. < W_{1-\alpha/2} \text{ or } New \quad D.W. > W_{\alpha/2} \Rightarrow reject \quad H_0$$

$$H_0 : \rho \leq \rho_0, H_1 : \rho > \rho_0, New \quad D.W. = \frac{\sum_{t=2}^{T} (e_t - e_{t-1})^2}{\sum_{t=1}^{T} e_t^2},$$

$$New \quad D.W. < W_{1-\alpha} \Rightarrow reject \quad H_0$$

$$H_0 : \rho \geq \rho_0, H_1 : \rho < \rho_0, New \quad D.W. = \frac{\sum_{t=2}^{T} (e_t - e_{t-1})^2}{\sum_{t=1}^{T} e_t^2},$$

$$New \quad D.W. > W_{\alpha} \Rightarrow reject \quad H_0$$

7.2).same independent variable value, but auto correlation coefficient is different:

7.2.1)data come from Case 1:

X1 X2 X3 X4 X5 X6

1 : 54.2327801017, -1.0515635183, -2.6129142144, 95.8729376089, 22.6710077518, -0.2330759733,

2 : 13.9250244388, 4.5308629808, -2.8927090373, 89.5992578910, 8.9801011409, -2.0197242513,

3 : 14.6498749197, 1.6952356061, -7.4321385213, 90.7336550626, 32.0067903020, 0.4088939995,

4 : 7.9783225134, 1.7818026806, -6.5196035366, 109.1869631249, 8.9810690255, 0.1523776054,

5 : 8.6762253740, 8.1590653669, -3.0465392653, 102.2317225115, 14.2386734597, 0.1328963108,

6 : 27.7623540858, 1.2328513211, -8.4102613007, 84.5890555624, 9.2839680305, 0.6524649479,

7 : 18.6625587758, 3.2933744224, -4.0086189009, 63.8968287439, 3.2012130715, 0.3515600164,

8 : 39.3319153828, 7.0312539167, -4.8855110925, 124.8773844220, 14.6127561535, 0.8325246655,

9 : 17.9656598313, -0.1333539865, -5.5127281095, 85.5531179580, 15.7406949223, -3.0863445859,

10 : 32.0610021717, 3.4854361980, -1.7108106596, 86.1964533707, 4.1842050838, -1.7323733277,

11 : 13.7528011906, -4.6549833265, -3.9099059591, 91.5492024874, 7.8007722665, -0.4902037820,

12 : 28.2464630681, 7.9737622105, -2.8270247690, 78.6405322222, 14.3222964868, 0.6649429561,

13 : 13.0932431125, 1.1946253885, -8.9544367518, 113.4092231132, 14.0899356091, 1.8632771636,

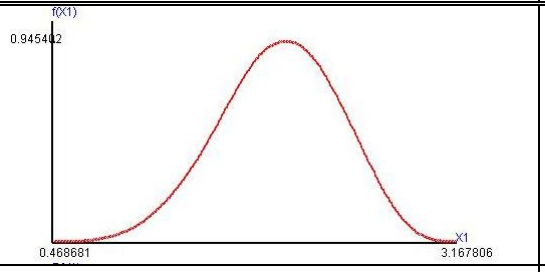
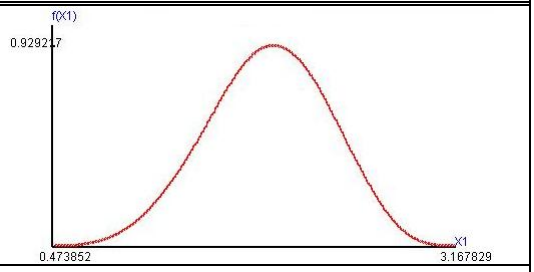
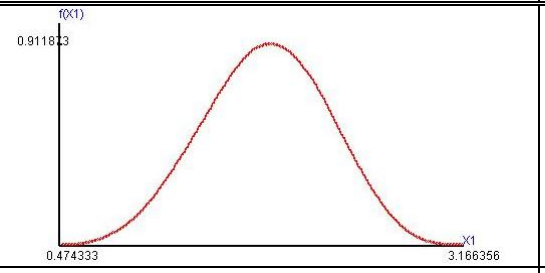
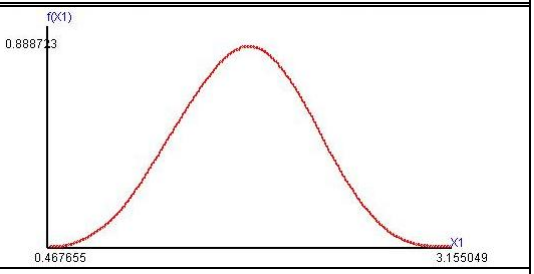
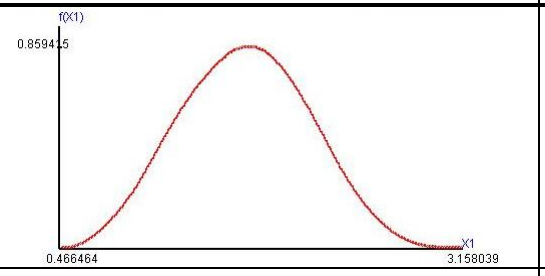
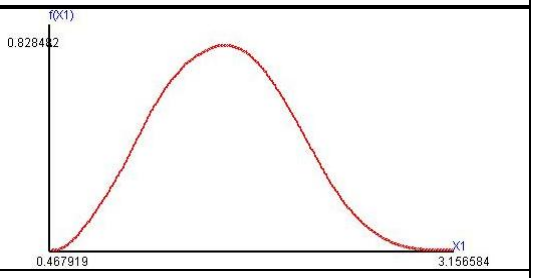
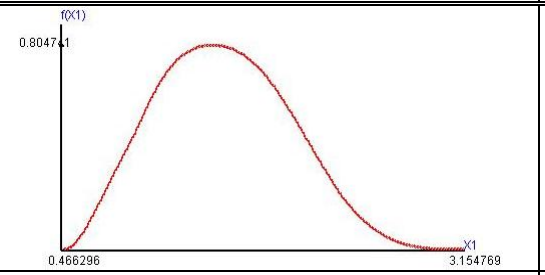
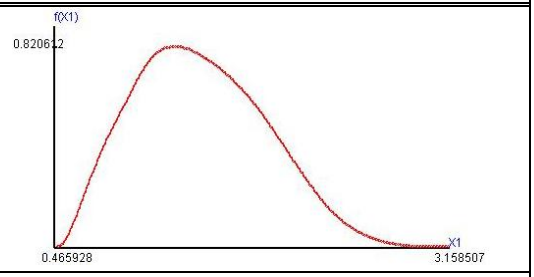
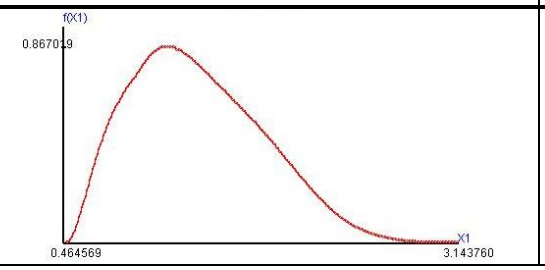
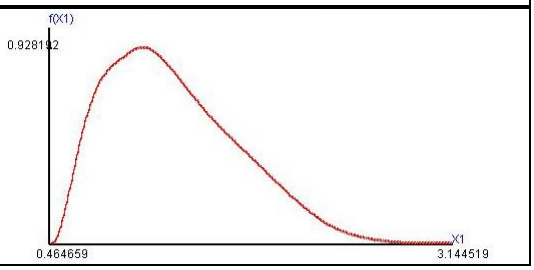
14 : 20.4243954765, -3.9039174396, -5.3385220478, 138.7602214818, 6.0689637971, -2.3724228711,

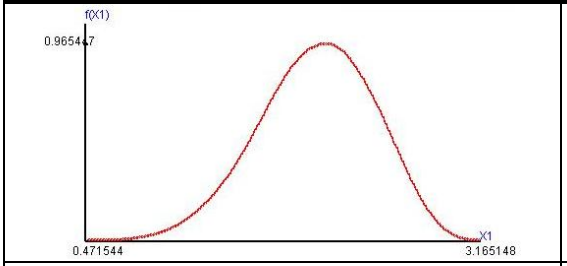
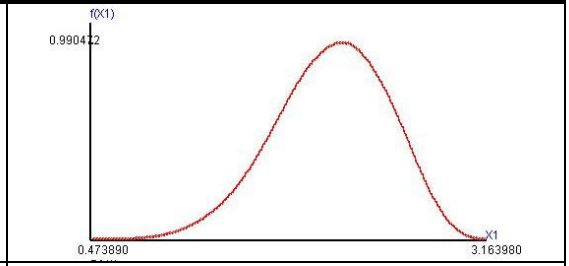
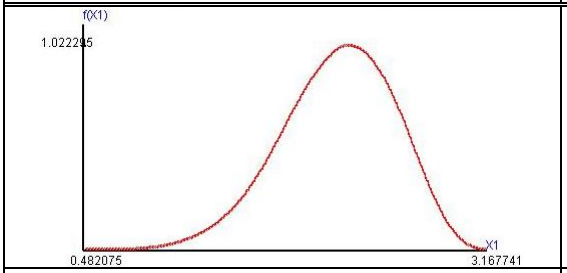
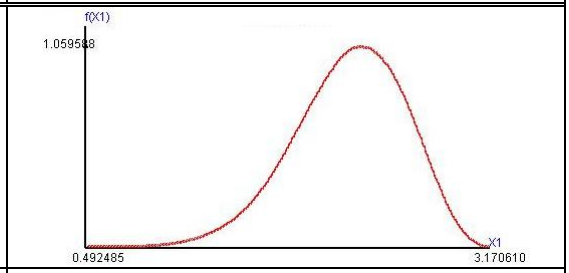
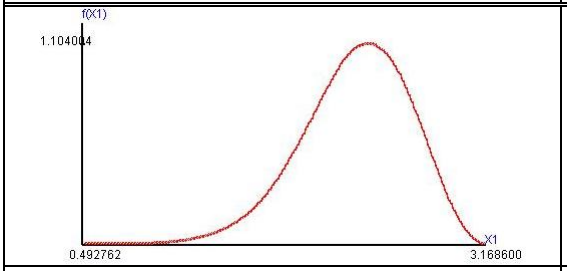
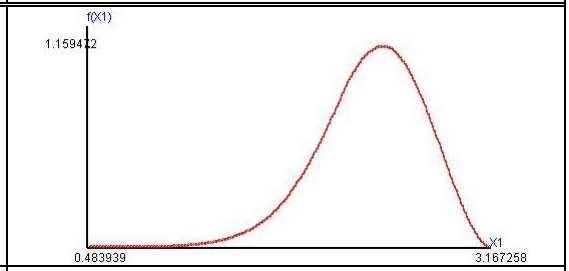
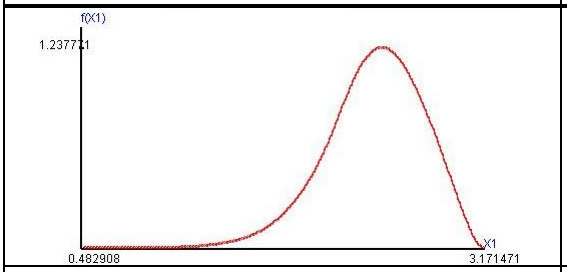
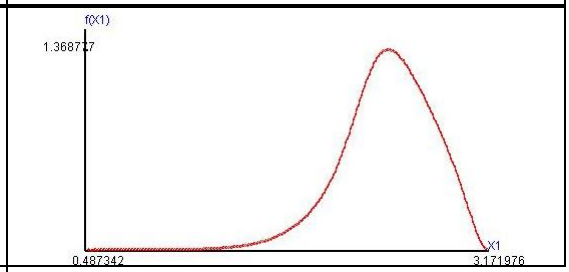
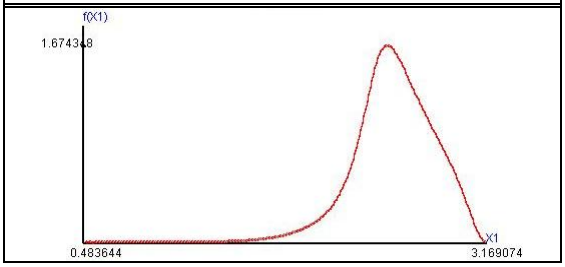
15 : 22.6946305148, 11.6094138960, -1.6969031058, 119.8805947892, 10.5394804672, -0.1477014572,

indepedent sample correlation coefficient----------

	X1	X2	X3	X4	X5
X2	0.024028234	1			
X3	0.314646678	0.405194595	1		
X4	0.005269787	0.001539342	-0.131205559	1	
X5	0.182811853	0.017165054	-0.249082423	0.011680009	1
X6	0.043103752	0.309836262	-0.358269997	-0.005317961	0.246722421

T=15,*df*=15-7=8,

$H_0:\rho=0$	$H_0:\rho=0.1$
	
$H_0:\rho=0.2$	$H_0:\rho=0.3$
	
$H_0:\rho=0.4$	$H_0:\rho=0.5$
	
$H_0:\rho=0.6$	$H_0:\rho=0.7$
	
$H_0:\rho=0.8$	$H_0:\rho=0.9$
	

$H_0 : \rho = -0.1$	$H_0 : \rho = -0.2$
	
$H_0 : \rho = -0.3$	$H_0 : \rho = -0.4$
	
$H_0 : \rho = -0.5$	$H_0 : \rho = -0.6$
	
$H_0 : \rho = -0.7$	$H_0 : \rho = -0.8$
	
$H_0 : \rho = -0.9$	
	

The probability distribution of Durbin Watson test will be modified when the autocorrelation coefficient changed.

7.2.2)Ceofficeint:

Hypothesis	Average	Variance	Ceofficent of Skewed	Coefficient of Kurtosis
$H_0 : \rho = 0$	1.97398	0.16419	-0.20544	2.71414
$H_0 : \rho = 0.1$	1.90876	0.16985	-0.15179	2.66891
$H_0 : \rho = 0.2$	1.84181	0.17566	-0.09755	2.62611
$H_0 : \rho = 0.3$	1.77209	0.18173	-0.04008	2.58306
$H_0 : \rho = 0.4$	1.69855	0.18794	0.02449	2.54071
$H_0 : \rho = 0.5$	1.62031	0.19378	0.10081	2.50595
$H_0 : \rho = 0.6$	1.53716	0.19825	0.19353	2.49312
$H_0 : \rho = 0.7$	1.45024	0.19986	0.30597	2.52335
$H_0 : \rho = 0.8$	1.36281	0.19696	0.43889	2.62080
$H_0 : \rho = 0.9$	1.28080	0.18862	0.58960	2.80419
$H_0 : \rho = -0.1$	2.03839	0.15846	-0.25983	2.76600
$H_0 : \rho = -0.2$	2.10273	0.15238	-0.31513	2.82881
$H_0 : \rho = -0.3$	2.16755	0.14569	-0.37064	2.90596
$H_0 : \rho = -0.4$	2.23319	0.13815	-0.42507	2.99974
$H_0 : \rho = -0.5$	2.29965	0.12951	-0.47685	3.11215
$H_0 : \rho = -0.6$	2.36649	0.11940	-0.52420	3.24742
$H_0 : \rho = -0.7$	2.43253	0.10720	-0.56424	3.41698
$H_0 : \rho = -0.8$	2.49531	0.09175	-0.58986	3.65764
$H_0 : \rho = -0.9$	2.54969	0.07020	-0.57437	4.10326

7.2.3)Crictical value:

Hypothesis	$1-\alpha$			
	0.995	0.99	0.975	0.95
$H_0 : \rho = 0$	0.9108875319	1.0019346112	1.1447008466	1.2754218579
$H_0 : \rho = 0.1$	0.8556318744	0.9409091197	1.0779928744	1.2043884600
$H_0 : \rho = 0.2$	0.8021828240	0.8804755953	1.0108192740	1.1326195224
$H_0 : \rho = 0.3$	0.7508644699	0.8212582121	0.9424158246	1.0594086574
$H_0 : \rho = 0.4$	0.7027624344	0.7645402631	0.8738220607	0.9842328326
$H_0 : \rho = 0.5$	0.6589255368	0.7115369490	0.8074877180	0.9081044914
$H_0 : \rho = 0.6$	0.6205299084	0.6646276872	0.7462573931	0.8347290498
$H_0 : \rho = 0.7$	0.5895489287	0.6256051490	0.6935986315	0.7688821444
$H_0 : \rho = 0.8$	0.5670933001	0.5969086905	0.6530272584	0.7158467386
$H_0 : \rho = 0.9$	0.5543434594	0.5800068612	0.6276954609	0.6802853790
$H_0 : \rho = -0.1$	0.9680763566	1.0635525731	1.2118583025	1.3467333389

$H_0 : \rho = -0.2$	1.0274773234	1.1270225748	1.2810685856	1.4194668499
$H_0 : \rho = -0.3$	1.0900375579	1.1938438960	1.3535389603	1.4948664728
$H_0 : \rho = -0.4$	1.1574375762	1.2659999851	1.4309243123	1.5741027773
$H_0 : \rho = -0.5$	1.2314601670	1.3446633250	1.5144550264	1.6585114126
$H_0 : \rho = -0.6$	1.3137929258	1.4322479776	1.6055966280	1.7493419002
$H_0 : \rho = -0.7$	1.4082978185	1.5307078570	1.7057476483	1.8477247511
$H_0 : \rho = -0.8$	1.5196944095	1.6448720237	1.8188855720	1.9584785373
$H_0 : \rho = -0.9$	1.6613384769	1.7876003331	1.9603242330	2.0964846920

Hypothesis	α			
	0.05	0.025	0.01	0.005
$H_0 : \rho = 0$	2.6146298971	2.7096539487	2.8085852855	2.8686262102
$H_0 : \rho = 0.1$	2.5681626499	2.6686269487	2.7737613709	2.8378628551
$H_0 : \rho = 0.2$	2.5197376564	2.6262360143	2.7377288067	2.8061508343
$H_0 : \rho = 0.3$	2.4687809540	2.5815128198	2.7000260415	2.7731954023
$H_0 : \rho = 0.4$	2.4144725064	2.5335794414	2.6597211288	2.7378492836
$H_0 : \rho = 0.5$	2.3564174333	2.4816528280	2.6161908054	2.6997480897
$H_0 : \rho = 0.6$	2.2941525368	2.4255282349	2.5686560652	2.6582252625
$H_0 : \rho = 0.7$	2.2277390960	2.3647996192	2.5167370571	2.6128497831
$H_0 : \rho = 0.8$	2.1573380309	2.3003506482	2.4606842969	2.5635389008
$H_0 : \rho = 0.9$	2.0842880868	2.2341439888	2.4020501730	2.5116766934
$H_0 : \rho = -0.1$	2.6597172647	2.7496740030	2.8428616426	2.8993378519
$H_0 : \rho = -0.2$	2.7042579673	2.7894200138	2.8770052660	2.9301334890
$H_0 : \rho = -0.3$	2.7487397895	2.8291226840	2.9114023064	2.9608177734
$H_0 : \rho = -0.4$	2.7933149957	2.8690504042	2.9457503134	2.9907666902
$H_0 : \rho = -0.5$	2.8377212186	2.9087297523	2.9791139997	3.0189253742
$H_0 : \rho = -0.6$	2.8809378066	2.9468119775	3.0097957869	3.0443677879
$H_0 : \rho = -0.7$	2.9204566785	2.9803203269	3.0356154268	3.0653876547
$H_0 : \rho = -0.8$	2.9513571335	3.0052448758	3.0540210634	3.0800418830
$H_0 : \rho = -0.9$	2.9634378072	3.0144199424	3.0599774029	3.0842687376

7.3)data come fromCase 2: (The collinearity is very hight)

7.3.1)

X1 X2 X3 X4 X5 X6

1 : 1.0342327801, 0.9938968730, 1.0055483431, 0.9972486251, 2.0174772521, 1.0011128268,

2 : 1.9939250244, 2.0050617260, 2.0044291639, 1.9930661719, 3.9985932430, 1.9892018383,

3 : 2.9946498749, 2.9993904712, 2.9862714459, 2.9938224367, 6.0303541935, 3.0053926267,

4 : 3.9879783225, 3.9995636054, 3.9899215859, 4.0061246421, 7.9985945780, 4.0036825174,

5 : 4.9886762254, 5.0123181307, 5.0038138429, 5.0014878150, 10.0058464462, 5.0035526421,

6 : 6.0077623541, 5.9984657026, 5.9823589548, 5.9897260370, 11.9990123697, 6.0070164330,

7 : 6.9986625588, 7.0025867488, 6.9999655244, 6.9759312192, 13.9906223629, 7.0050104001,

8 : 8.0193319154, 8.0100625078, 7.9964579556, 8.0165849229, 16.0063624223, 8.0082168311,

9 : 8.9979656598, 8.9957332920, 8.9939490876, 8.9903687453, 18.0079181999, 8.9820910361,

10 : 10.0120610022, 10.0029708724, 10.0091567574, 9.9907976356, 19.9919782139, 9.9911175111,

11 : 10.9937528012, 10.9866900333, 11.0003603762, 10.9943661350, 21.9969665824, 10.9993986415,

12 : 12.0082464631, 12.0119475244, 12.0046919009, 11.9857603548, 24.0059617883, 12.0070996197,

13 : 12.9930932431, 12.9983892508, 12.9801822530, 13.0089394821, 26.0056412905, 13.0150885144,

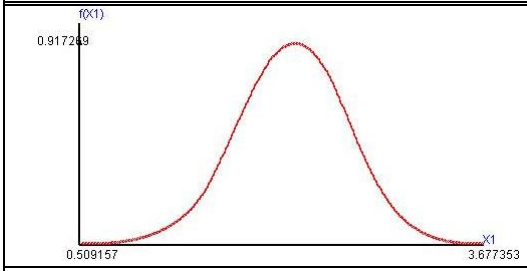
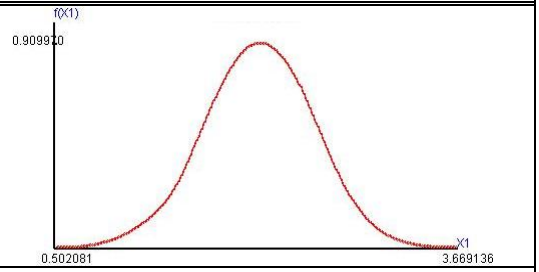
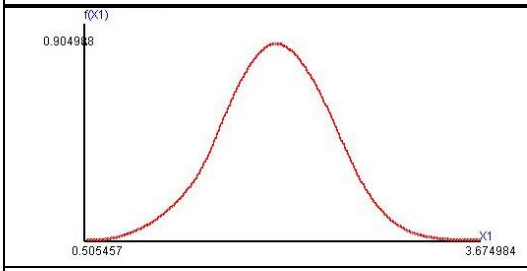
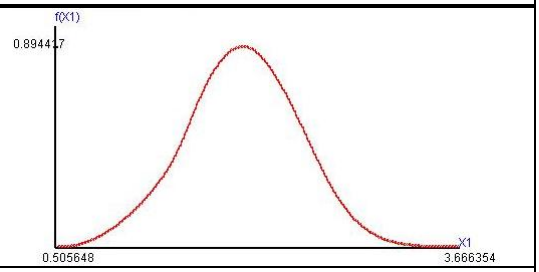
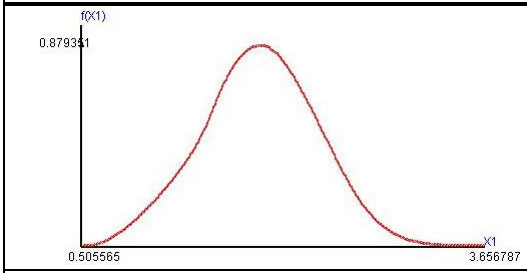
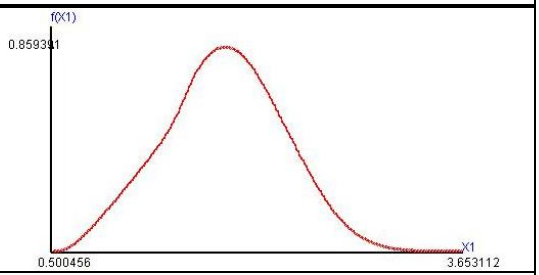
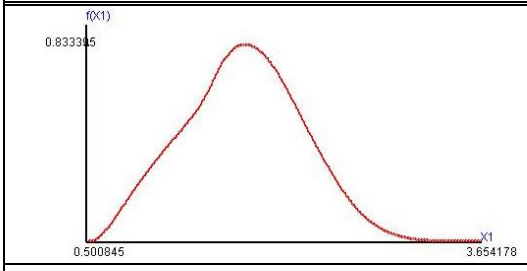
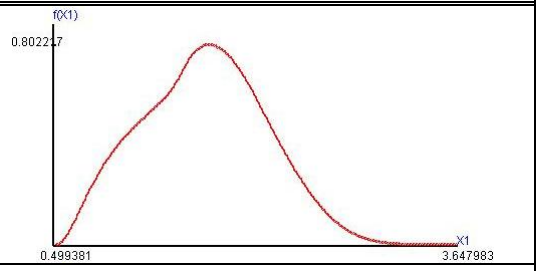
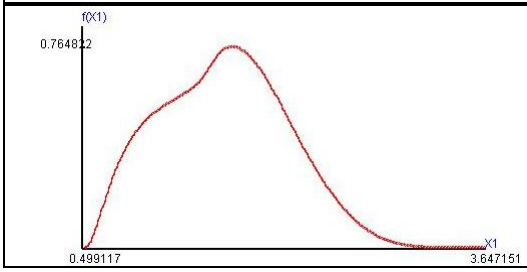
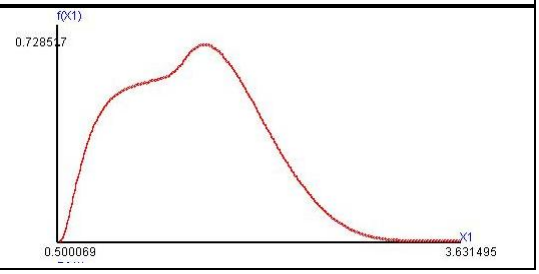
14 : 14.0004243955, 13.9881921651, 13.9946459118, 14.0258401477, 27.9945778811, 13.9868505142,

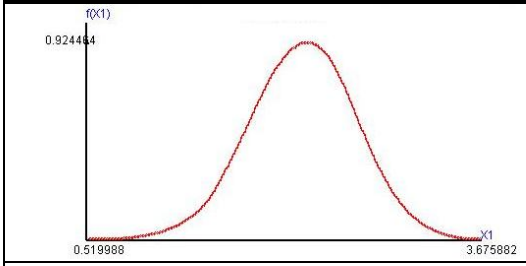
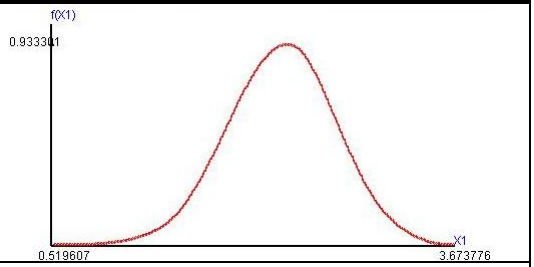
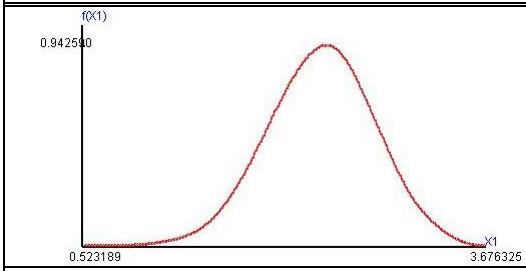
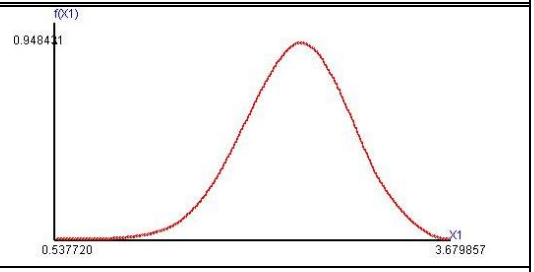
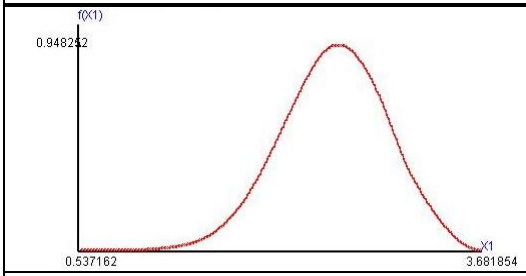
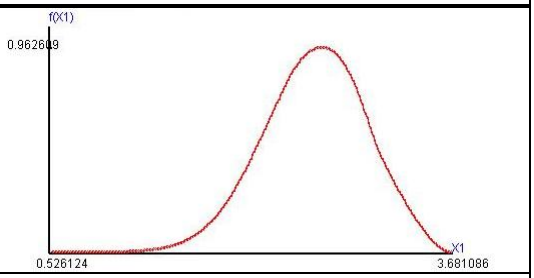
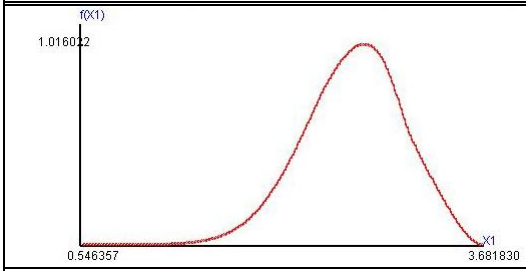
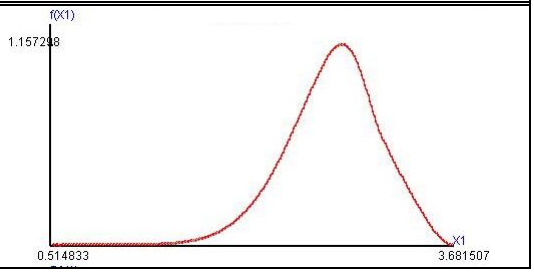
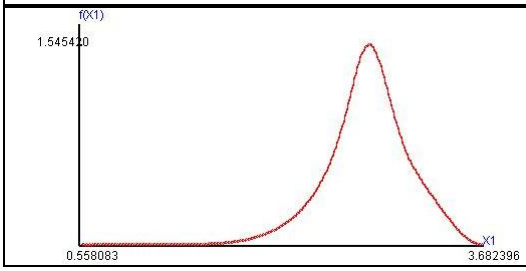
15 : 15.0026946305, 15.0192188278, 15.0092123876, 15.0132537299, 30.0007441110, 15.0016819903,

indepedent sample correlation coefficient----------

	X1	X2	X3	X4	X5
X2	0.999994303	1			
X3	0.999995838	0.999997484	1		
X4	0.999992784	0.999994077	0.999993160	1	
X5	0.999996043	0.999997543	0.999996733	0.999996260	1
X6	0.999994294	0.999997177	0.999994214	0.999994216	0.999997925

T=15,*df*=15-7=8,

$H_0:\rho=0$	$H_0:\rho=0.1$
	
$H_0:\rho=0.2$	$H_0:\rho=0.3$
	
$H_0:\rho=0.4$	$H_0:\rho=0.5$
	
$H_0:\rho=0.6$	$H_0:\rho=0.7$
	
$H_0:\rho=0.8$	$H_0:\rho=0.9$
	

$H_0 : \rho = -0.1$	$H_0 : \rho = -0.2$
	
$H_0 : \rho = -0.3$	$H_0 : \rho = -0.4$
	
$H_0 : \rho = -0.5$	$H_0 : \rho = -0.6$
	
$H_0 : \rho = -0.7$	$H_0 : \rho = -0.8$
	
$H_0 : \rho = -0.9$	
	

The probability distribution of Durbin Watson test will be modified when the autocorrelation coefficient changed.

7.3.2)Ceofficeint:

Hypothesis	Average	Variance	Ceofficent of Skewed	Coefficient of Kurtosis
$H_0 : \rho = 0$	2.16857	0.18509	-0.09951	2.95231
$H_0 : \rho = 0.1$	2.09902	0.18814	-0.08571	2.93599
$H_0 : \rho = 0.2$	2.02930	0.19248	-0.07271	2.90561
$H_0 : \rho = 0.3$	1.95880	0.19838	-0.05773	2.85885
$H_0 : \rho = 0.4$	1.88694	0.20595	-0.03728	2.79484
$H_0 : \rho = 0.5$	1.81338	0.21511	-0.00761	2.71569
$H_0 : \rho = 0.6$	1.73842	0.22551	0.03433	2.62797
$H_0 : \rho = 0.7$	1.66367	0.23639	0.08935	2.54273
$H_0 : \rho = 0.8$	1.59309	0.24666	0.15410	2.47315
$H_0 : \rho = 0.9$	1.53479	0.25497	0.21735	2.42938
$H_0 : \rho = -0.1$	2.23841	0.18300	-0.11606	2.95755
$H_0 : \rho = -0.2$	2.30884	0.18142	-0.13654	2.95500
$H_0 : \rho = -0.3$	2.37991	0.17983	-0.16161	2.94818
$H_0 : \rho = -0.4$	2.45142	0.17756	-0.19169	2.94157
$H_0 : \rho = -0.5$	2.52282	0.17371	-0.22726	2.94196
$H_0 : \rho = -0.6$	2.59323	0.16704	-0.26929	2.96235
$H_0 : \rho = -0.7$	2.66142	0.15566	-0.31953	3.03238
$H_0 : \rho = -0.8$	2.72570	0.13626	-0.37963	3.23183
$H_0 : \rho = -0.9$	2.78201	0.10246	-0.43597	3.84154

7.3.3)Crictical value:

Hypothesis	$1-\alpha$			
	0.995	0.99	0.975	0.95
$H_0:\rho=0$	1.0166190968	1.1243298189	1.2967612373	1.4493849316
$H_0:\rho=0.1$	0.9524502156	1.0527420897	1.2175359018	1.3694607805
$H_0:\rho=0.2$	0.8906199374	0.9823671392	1.1371553732	1.2854867955
$H_0:\rho=0.3$	0.8315437482	0.9137454781	1.0565739517	1.1984857805
$H_0:\rho=0.4$	0.7763522746	0.8484485223	0.9773084428	1.1100210854
$H_0:\rho=0.5$	0.7259254013	0.7878941869	0.9014724670	1.0223337665
$H_0:\rho=0.6$	0.6814432946	0.7335144579	0.8311999974	0.9386149335
$H_0:\rho=0.7$	0.6441193937	0.6871529014	0.7694430340	0.8627336871
$H_0:\rho=0.8$	0.6147375662	0.6501883080	0.7191039161	0.7988299464
$H_0:\rho=0.9$	0.5943397039	0.6243069878	0.6830238393	0.7521199061
$H_0:\rho=-0.1$	1.0827970661	1.1968971324	1.3741387052	1.5244713032
$H_0:\rho=-0.2$	1.1509787492	1.2700218328	1.4495612873	1.5958094483
$H_0:\rho=-0.3$	1.2211096726	1.3441360401	1.5224249167	1.6662608749
$H_0:\rho=-0.4$	1.2941376828	1.4192825882	1.5945491667	1.7382499632
$H_0:\rho=-0.5$	1.3709469413	1.4959716734	1.6766686019	1.8132652910
$H_0:\rho=-0.6$	1.4528067457	1.5757742172	1.7482076231	1.8928980128
$H_0:\rho=-0.7$	1.5407877674	1.6627142330	1.8355797566	1.9792876830
$H_0:\rho=-0.8$	1.6413091042	1.7634000274	1.9360550161	2.0792456767
$H_0:\rho=-0.9$	1.7707930702	1.8944404905	2.0702189446	2.2172579559

Hypothesis	α			
	0.05	0.025	0.01	0.005
$H_0:\rho=0$	2.8649637626	2.9939146398	3.1408450034	3.2342046600
$H_0:\rho=0.1$	2.8011986288	2.9313678968	3.0812014045	3.1795462625
$H_0:\rho=0.2$	2.7399313811	2.8711073424	3.0222796651	3.1239083230
$H_0:\rho=0.3$	2.6815141626	2.8135037516	2.9656735466	3.0686570653
$H_0:\rho=0.4$	2.6255733078	2.7588235042	2.9122898343	3.0154774900
$H_0:\rho=0.5$	2.5716044783	2.7071981358	2.8621369399	2.9661148635
$H_0:\rho=0.6$	2.5196139808	2.6584907334	2.8156710137	2.9206102172
$H_0:\rho=0.7$	2.4702569539	2.6130869099	2.7736328438	2.8797003354
$H_0:\rho=0.8$	2.4255517417	2.5731440049	2.7372837162	2.8446357152
$H_0:\rho=0.9$	2.3901990088	2.5418431282	2.7095396981	2.8188561536
$H_0:\rho=-0.1$	2.9306357075	3.0580488362	3.1997984163	3.2871354368

$H_0 : \rho = -0.2$	2.9975953808	3.1226750813	3.2567797953	3.3368777787
$H_0 : \rho = -0.3$	3.0646516672	3.1855953223	3.3103373116	3.3832558152
$H_0 : \rho = -0.4$	3.1302181177	3.2448599259	3.3593829620	3.4250420851
$H_0 : \rho = -0.5$	3.1912358315	3.2982214575	3.4025764917	3.4611520358
$H_0 : \rho = -0.6$	3.2443779041	3.3433586029	3.4381762510	3.4906282671
$H_0 : \rho = -0.7$	3.2855236515	3.3771078709	3.4637328380	3.5112495901
$H_0 : \rho = -0.8$	3.3086563418	3.3947591042	3.4757843154	3.5203132910
$H_0 : \rho = -0.9$	3.3002513108	3.3853607400	3.4662273008	3.5113346022