Chapter seven The order statistic

1)The introduction.

There are random samples which size is n, the random sample values is sorting from the smallest value to the largest value.

2). $X_1,...,X_n \overset{iid}{\sim} f_X(x)$

Y_j = The jth order statistic of $X_1,...,X_n$,

$f_{Y_1,...,Y_n}(y_1,...,y_n) = n! \times f_{X_1,...,X_n}(x_1 = y_1,..,x_n = y_n)$

$= n! \times f_X(x = y_1) \times .... \times f_X(x = y_n), -\infty < y_1 < ... < y_n < \infty,$

$$J = \frac{\partial(y_1,...,y_n)}{\partial(x_1,...,x_n)} = n!,$$

$$f_{Y_j}(y_j) = \frac{n!}{(j-1)! \times (n-j)!} \times (F_X(y_j))^{j-1} \times f_X(x = y_j) \times (1 - F_X(y_j))^{n-j},$$

$-\infty < y_j < \infty,$

$$f_{Y_j,Y_k}(y_j, y_k) = \frac{n!}{(j-1)! \times (k-j-1) \times (n-k)!} \times (F_X(y_j))^{j-1} \times f_X(x = y_j)$$

$$\times (F_X(y_k) - F_X(y_j))^{k-j-1} \times f_X(x = y_k) \times (1 - F_X(y_k))^{n-k}$$

$-\infty < y_j < y_{j=k} < \infty, j < k$

let $F_X(y_j) = U, j = 1,2,....,n$

$$f_U(u) = \frac{n!}{(j-1)! \times (n-j)!} \times (u)^{j-1} \times (1-u)^{n-j}, 0 < u < 1, U \sim Beta(j, n-j+1)$$

let $F_X(y_j) = U_1, F_X(y_k) = U_2, j,k = 1,2,....,n, j < k$

$$f_{U_1,U_2}(u_1,u_2) = \frac{n!}{(j-1)! \times (k-j-1) \times (n-k)!} \times (u_1)^{j-1} \times (u_2 - u_1)^{k-j-1} \times (1-u_2)^{n-k}$$

$0 < u_1 < u_2 < 1,$