Chapter five

The Moment of random variables

1).The central tendency value of random variable is very important to the probability distribution. But the moment means is $E(X^k)$ that means the central tendency value of X^k.

1.1)The software P_S_CCC.exe can calculate the value of $E(X^k)$. The method is the probability distribution transformation and computes the average of new probability distribution. Let $Y = X^k$ and get the new probability distribution of Y and compute the $E(Y)$.
This software cannot offer the moment generating function , factorial moment and characteristic function.

1.2)Of course, the moment of two random variables can be computed by the software P_S_CCC.exe. In first, the new random variable must be gotten by two random variables multiplicative transformation and computed the expected value.
There are two random variables X_1, X_2 and let $Y_1 = X_1 \times X_2$ and computes the $E(Y_1^k) = E((X_1 \times X_2)^k)$.

1.3)The correlation of coefficient also can be gotten in according to above method.
There are two random variables X_1, X_2 and let $Y_1 = X_1 \times X_2$ and computes the
$E(Y_1) = E(X_1 \times X_2), E(X_1), Var(X_1), E(X_2), Var(X_2)$,
$Cov(X_1, X_2) = E(X_1 \times X_2) - E(X_1) \times E(X_2)$,
$$\rho(X_1, X_2) = \frac{Cov(X_1, X_2)}{\sqrt{Var(X_1) \times Var(X_2)}},$$

1.4)The moment $EE(X_2|X_1)$ can use the conditional probability distribution.
For example: $X_1 \sim U(0,1), X_2|X_1 \sim Normal(\mu_2 = X_1, \sigma_2^2 = 1^2)$, getting X_2 marginal probability distribution and computing the $E(X_2)$.
The $E(X_2)$ is $EE(X_2|X_1)$.

1.5) $N \sim f_N(n), X_1, X_2, ... \overset{iid}{\sim} f_X(x)$ and $N, X_1, X_2, ...$ are independent random variables. Let $S_N = X_1 + ... + X_N$ and $E(S_N) = E(N)E(X_1)$,
$E[(S_N)^2] = E(N)E(X_1^2) + E(N(N-1))E(X_1)E(X_1)$,
$$Var(S_N) = E(N)E(X_1^2) + E(N(N-1))E(X_1)E(X_1) - (E(N)E(X_1))^2$$
$$= E(N)Var(X_1) + Var(N)E(X_1)E(X_1).$$