Chapter four

The three or more random variables joint probability distribution

1).The joint probability distribution is three or more random variables which values and the joint probability density function is consist of the probability distribution. The joint probability distribution can derived to the joint distribution function, coefficient, joint conditional probability distribution in a special value region, the conditional probability distribution and the marginal probability distribution.

This chapter is about the two random variables joint probability distribution.

Suppose $X_1 \sim U(0,1), X_2 \sim U(-1,1), X_3 \sim U(-2,2), X_1, X_2$ and X_3 are independent random variables.

1.1)the joint probability density function is

$$f_{X_1,X_2,X_3}(x_1,x_2,x_3)=\begin{cases}\frac{1}{8},0<x_1<1,-1<x_2<1,,-2<x_2<2,\\ 0,otherw,ise\end{cases},$$

1.2)the joint distribution function is

$$F_{X_1,X_2,X_3}(x_1,x_2,x_3)=\begin{cases}0, x_1\le 0 \quad or \quad x_2\le -1 \quad or \quad x_3 \le -2\\ \frac{x_1\times(x_2+1)\times(x_2+2)}{8},0<x_1<1,-1<x_2<1,-2<x_2<2,\\ \frac{(x_2+1)\times(x_2+2)}{8}, x_1\ge 1,-1<x_2<1,-2<x_2<2,\\ \frac{x_1\times(x_2+2)}{8},0<x_1<1, x_2\ge 1,-2<x_2<2,\\ \frac{x_1\times(x_2+1)}{8},0<x_1<1,-1<x_2<1, x_2\ge 2,\\ x_1,0<x_1<1, x_2\ge 1\\ 1, x_1\ge 1, x_2\ge 1, x_3\ge 2\end{cases}$$

1.3)The marginal moment or joint monet,

$$E[H(X_1,X_2,X_3)]=\int\int H(x_1,x_2,x_3)\times f_{X_1,X_2,X_3}(x_1,x_2,x_3)\times dx_1\times dx_2\times dx_3,$$

$H(X_1,X_2,X_3)$ can be $X_1, X_1+X_2, X_1+X_2+X_3, (X_1+X_2+X_3)^3,$

$(X_1+X_2+X_3)\times\cos(X_2\times\pi)\times\exp(-X_3),....$ etc.

1.4) The joint conditional probability distribution in a special value region,

$$f_{X_1,X_2,X_3}(x_1,x_2,x_3|-1\le x_1+x_2+x_3\le 1), F_{X_1,X_2,X_3}(x_1,x_2,x_3|-1\le x_1+x_2+x_3\le 1),.$$

1.5) The conditional probability distribution,

$$f_{X_1|X_2=x_2}(x_1|X_2=x_2)=\frac{f_{X_1,X_2}(x_1,x_2)}{f_{X_2}(x_2)},\ f_{X_2|X_1=x_1}(x_2|X_1=x_1)=\frac{f_{X_1,X_2}(x_1,x_2)}{f_{X_1}(x_1)},$$

$$F_{X_1|X_2=x_2}(x_1|X_2=x_2)=\int_0^{x_1} f_{X_1|X_2=x_2}(x_1|X_2=x_2)dx_1,$$

$$F_{X_2|X_1=x_1}(x_2|X_1=x_1)=\int_0^{x_2} f_{X_2|X_1=x_1}(x_2|X_1=x_1)dx_2,$$

$$E[H(X_1)|X_2=x_2]=\int H(x_1)\times f_{X_1|X_2=x_2}(x_1|X_2=x_2)dx_1,$$

$$E[H(X_2)|X_1=x_1]=\int H(x_2)\times f_{X_2|X_1=x_1}(x_2|X_1=x_1)dx_2,$$

$$f_{X_1,X_2|X_3=x_3}\left(x_1,x_2|X_3=x_3\right)=\frac{f_{X_1,X_2,X_3}\left(x_1,x_2,x_3\right)}{f_{X_3}\left(x_3\right)},$$
$$E\left[H\left(X_1,X_2\right)|X_3=x_3\right]=\int\int H\left(x_1,x_2\right)\times f_{X_1,X_2|X_3=x_3}\left(x_1,x_2|X_3=x_3\right)\times dx_1\times dx_2,,$$

1.6) The marginal probability distribution,

$$f_{X_1}\left(x_1\right)=\int\int f\,f_{X_1,X_2,X_3}\left(x_1,x_2,x_3\right)\times dx_2\times dx_3,$$
$$f_{X_2}\left(x_2\right)=\int\int f\,f_{X_1,X_2,X_3}\left(x_1,x_2,x_3\right)\times dx_1\times dx_3,$$
$$f_{X_3}\left(x_3\right)=\int\int f\,f_{X_1,X_2,X_3}\left(x_1,x_2,x_3\right)\times dx_1\times dx_2,$$

This software P_S_CCC.exe can support their functions, but the can show the one variable, two random variables and three random variables probability distribution only.

1.7)Three or more random variables joint probability distribution will be transferred to the other joint probability distribution or marginal probability distribution. The transformation need 1)the transformation equation, 2) new random variable range. The process of transformation in marginal probability distribution.

step 1. The X_1,X_2,X_3 joint pdf or df or their special distribution name. There are the joint pdf of $X_1,X_2,X_3,,$ $f_{X_1,X_2,X_3}\left(x_1,x_2,x_3\right)$.

step 2. The transferred mathematical equation,

$$Y_1=H_1\left(X_1,X_2,X_3\right),Y_2=H_2\left(X_1,X_2,X_3\right),Y_3=H_2\left(X_1,X_2,X_3\right),$$

step 3. The jacobian and $X_1=G_1\left(Y_1,Y_2,Y_3\right),\ X_2=G_2\left(Y_1,Y_2,Y_3\right),X_3=G_3\left(Y_1,Y_2,Y_3\right),$

are existed, $J=\frac{\partial\left(x_1,x_2,x_3\right)}{\partial\left(y_1,y_2,y_3\right)}\begin{vmatrix}\frac{\partial G_1\left(y_1,y_2,y_3\right)}{\partial y_1} & \frac{\partial G_1\left(y_1,y_2,y_3\right)}{\partial y_2} & \frac{\partial G_1\left(y_1,y_2,y_3\right)}{\partial y_3}\\ \frac{\partial G_2\left(y_1,y_2,y_3\right)}{\partial y_1} & \frac{\partial G_2\left(y_1,y_2,y_3\right)}{\partial y_2} & \frac{\partial G_2\left(y_1,y_2,y_3\right)}{\partial y_3}\\ \frac{\partial G_3\left(y_1,y_2,y_3\right)}{\partial y_1} & \frac{\partial G_3\left(y_1,y_2,y_3\right)}{\partial y_2} & \frac{\partial G_3\left(y_1,y_2,y_3\right)}{\partial y_3}\end{vmatrix},$

step 4. The new random variables joint pdf

$$\int\int\int f_{Y_1,Y_2,Y_3}\left(y_1,y_2,y_3\right)dy_3dy_2dy_1=\int\int\int f_{X_1,X_2,X_3}\left(x_1,x_2,x_3\right)\times J\times dx_3\times dx_2\times dx_1.$$
$$f_{Y_1}\left(y_1\right)=\int\int f_{Y_1,Y_2,Y_3}\left(y_1,y_2,y_3\right)dy_2dy_3,\ f_{Y_2}\left(y_2\right)=\int\int f_{Y_1,Y_2,Y_3}\left(y_1,y_2,y_3\right)dy_1dy_3,$$
$$f_{Y_3}\left(y_3\right)=\int\int f_{Y_1,Y_2,Y_3}\left(y_1,y_2,y_3\right)dy_1dy_2,$$

Sometimes, the transformation is a big problem when the transferred mathematical equation is not matched to the random variables.

1.8)How to find the joint probability distribution in the software P_S_CCC.exe.

1.8.1) X_1, X_2, X_3 marginal probability distribution are known and X_1, X_2 and X_3 are independent random variables.

$$f_{X_1,X_2,X_3}(x_1,x_2,x_3) = f_{X_1}(x_1) \times f_{X_2}(x_2) \times f_{X_3}(x_3).$$

1.8.2) X_1 marginal probability distribution are known and X_2 conditional probability distribution are known, but the parameters are function of X_1 and the X_3 conditional probability distribution of X_1 and X_2.

$$f_{X_1,X_2,X_3}(x_1,x_2,x_3)$$
$$= f_{X_1}(x_1) \times f_{X_2|X_1=x_1}(x_2|X_1 = x_1) \times f_{X_3|X_1=x_1,X_2=x_2}(x_3|X_1 = x_1, X_2 = x_2).$$

1.8.3) X_1, X_2, X_3 marginal probability distribution are known and X_1, X_2 and X_3 are independent random variables. But the values range of X_1, X_2 and X_3 are the special range constrains.

$$f_{X_1,X_2,X_3}(x_1,x_2,x_3) = f_{X_1}(x_1) \times f_{X_2}(x_2) \times f_{X_3}(x_3).$$
$$f_{X_1,X_2,X_3}(x_1,x_2,x_3|a \le X_1 + X_2 + X_3 \le b) = \frac{f_{X_1,X_2,X_3}(x_1,x_2,x_3)}{P(a \le X_1 + X_2 + X_3 \le b)},$$

1.8.4) X_1 marginal probability distribution are known and X_2 conditional probability distribution are known, but the parameters are function of X_1 and the X_3 conditional probability distribution of X_1 and X_2.

The values range of X_1, X_2 and X_3 are the special constrains.

$$f_{X_1,X_2,X_3}(x_1,x_2,x_3)$$
$$= f_{X_1}(x_1) \times f_{X_2|X_1=x_1}(x_2|X_1 = x_1) \times f_{X_3|X_1=x_1,X_2=x_2}(x_3|X_1 = x_1, X_2 = x_2),$$
$$f_{X_1,X_2,X_3}(x_1,x_2,x_3|a \le X_1 + X_2 + X_3 \le b) = \frac{f_{X_1,X_2,X_3}(x_1,x_2,x_3)}{P(a \le X_1 + X_2 + X_3 \le b)},$$

Note: i.i.d. is independently and identically distributed.