

Chapter two
The marginal probability distribution

1). The marginal probability distribution is just one random variable which value and the probability density function is consist of the probability distribution. The marginal probability distribution can derived to the marginal distribution function and coefficient and marginal conditional probability distribution in a special value region. Suppose $X \sim U(0,1)$,

1.1) the marginal probability density function is $f_x(x) = \begin{cases} 1, & 0 < x < 1, \\ 0, & \text{otherwise,} \end{cases}$

1.2) the marginal distribution function is $F_x(x) = \begin{cases} 0, & x \leq 0, \\ x, & 0 < x < 1, \\ 1, & x \geq 1, \end{cases}$

1.3) $E[H(X)] = \int H(x) \times f_x(x) dx$, $H(X)$ can be $X, X^2, X^3, X \times \cos(X \times \pi), \dots$ etc.

1.4) the marginal conditional probability distribution

is $f_x(x|0.2 < X < 0.7) = \begin{cases} 2, & 0.2 < x < 0.7, \\ 0, & \text{otherwise,} \end{cases}$

This software P_S_CCC.exe can support their functions.

1.5)

One random variable marginal probability distribution will be transferred to the other marginal probability distribution. The transformation need 1) the transformation equation, 2) new random variable range.

The process of transformation in marginal probability distribution.

step 1. The X pdf or df or special distribution name. There are the pdf of X, $f_x(x)$.

step 2. The transferred mathematical equation, $Y = H(X)$,

step 3. The jacobian and $H^{-1}(y)$ is existed, $\frac{dx}{dy} = \frac{dH^{-1}(y)}{dy}$,

step 4. The new random variable pdf $\int f_Y(y) dy = \int f_x(x = H^{-1}(y)) \times \frac{dH^{-1}(y)}{dy} \times dy$.

Sometimes, the transformation is a big problem when the transferred mathematical equation is not matched to the random variable.

For example. $X \sim Normal(\mu, \sigma^2)$, $Y = H(X) = X \times \cos(X \times \pi)$, find the marginal probability distribution of Y.

Because $Y = H(X) = X \times \cos(X \times \pi)$, which inverse function is not existed. In general this transformation cannot be done. But the software P_S_CCC.exe can finish this job.

One random variable will be transferred to two random variables which one is $Y_1 = H_1(X)$ and the other is $Y_2 = H_2(X)$. The Y_1 and Y_2 joint probability density function is $f_{Y_1, Y_2}(y_1, y_2)$ which can be derived by the software P_S_CCC.exe.

The advantage of software P_S_CCC.exe is that can express the each step of transformation in detail.