2).The basic probability distribution name and probability density function.

This program has 60 basic probability distribution,

1)Uniform distribution, $X \sim U(\alpha,\beta)$

$$f_X(x) = \frac{1}{\beta-\alpha}, \alpha \le x \le \beta, -\infty < \alpha < \beta < \infty,$$

2) Normal distribution, $X \sim N(\mu,\sigma^2)$

$$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{(x-\mu)^2}{2\times\sigma^2}\right), -\infty < x < \infty$$

$$-\infty < \mu < \infty, \sigma > 0,$$

3)Shifted exponential distribution, $X \sim Shifted\_\exp onential(\lambda,c)$

$$f_X(x) = \lambda\exp(-\lambda(x-c)), c < x < \infty$$

$$-\infty < c < \infty, \lambda > 0,$$

4)Pareto1 distribution, $X \sim Pareto1(\lambda,c)$

$$f_X(x) = \lambda\times\frac{x^{\lambda-1}}{c^{\lambda}}, 0 < x < c, \lambda > 0, c > 0,$$

5)Pareto2 distribution, $X \sim Pareto2(\lambda,c)$

$$f_X(x) = \lambda\frac{c^{\lambda}}{x^{\lambda+1}}, c < x < \infty, \lambda > 0, c > 0,$$

6)Rayleigh distribution, $X \sim Rayleigh(\lambda,c)$

$$f_X(x) = 2\lambda\times(x-c)\times\exp\left(-\lambda(x-c)^2\right), c < x < \infty$$

$$\lambda > 0, c > 0,$$

7)Double exponential distribution, $X \sim DE(\lambda,\mu)$

$$f_X(x) = \frac{\lambda}{2}\exp\left(-\lambda|x-\mu|\right), -\infty < x < \infty$$

$$-\infty < \mu < \infty, \lambda > 0,$$

8)Lognormal distribution $X \sim Log\_normal(\mu,\sigma^2)$

$$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma x}\exp\left(-\frac{(\ln(x)-\mu)^2}{2\sigma^2}\right), 0 < x < \infty,$$

$$-\infty < \mu < \infty, \sigma > 0,$$

9)Gamma distribution $X \sim Gamma(\alpha,\beta)$

$$f_X(x) = \frac{x^{\alpha-1}}{\Gamma(\alpha)\beta^{\alpha}}\exp\left(-\frac{x}{\beta}\right), 0 < x < \infty, \alpha,\beta > 0,$$

$\Gamma(\ )$: *gamma function*,

10)Beta distribution $X \sim Beta(\alpha,\beta)$

$$f_X(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}x^{\alpha-1}(1-x)^{\beta-1}, 0 < x < 1$$

$\alpha,\beta > 0, \Gamma(\ )$: *gamma function*,

11)Cauchy distribution $X \sim Cauchy(\mu,\sigma)$

$$f_X(x) = \frac{1}{\pi}\times\frac{\sigma}{(x-\mu)^2+\sigma^2}, -\infty < x < \infty,$$

$$\sigma > 0, -\infty < \mu < \infty,$$

12)Arcsin distribution $X \sim Arc\sin(\mu,c)$

$$f(x) = \frac{1}{\pi}\frac{1}{\sqrt{1-\frac{(x-\mu)^2}{c^2}}}, |x-\mu| < c,$$

$$-\infty < \mu < \infty, c > 0,$$

13)Gumble distribution

$X \sim Gumbel(\mu,\sigma)$

$$f_X(x)=\frac{1}{\sigma}e^{-\frac{x-\mu}{\sigma}}e^{-\left(e^{-\frac{x-\mu}{\sigma}}\right)},-\infty<x<\infty,$$

$-\infty<\mu<\infty,\sigma>0,$

14) Triangular 1 distribution

$X \sim Triangular1(\mu,c)$

$$f(x)=\begin{cases}\left|\left(\frac{x-\mu}{c}\right)\times\frac{1}{c}\right|,-c+\mu<x<\mu+c\\ 0,otherwise\end{cases},$$

$-\infty<\mu<\infty,c>0,$

15)Trapezoid distribution

$X \sim Trapezoid(\mu,c)$

$$f_X(x)=$$

$$\begin{cases}\frac{1.5c+x-\mu}{2c^2},\mu-1.5c<x<\mu-0.5c\\ \frac{1}{2c},\mu-0.5c<x<\mu+0.5c\\ \frac{1.5c-x+\mu}{2c^2},\mu+0.5c<x<\mu+1.5c\\ 0,otherwie\end{cases},$$

$-\infty<\mu<\infty,c>0,$

16)U-quadratic distribution

$X \sim U\_quadratic(a,b)$

$$f_X(x)=\alpha(x-\beta)^2,a\le x\le b,-\infty<a<b<\infty,$$

$$\beta=\frac{a+b}{2},\alpha=\frac{12}{(b-a)^3},$$

17) Wingner semicircle distribution

$X \sim Semi\_circle(\mu,R)$

$$f_X(x)=\frac{2}{\pi R^2}\sqrt{R^2-(x-\mu)^2},|x-\mu|\le R,$$

$-\infty<\mu<\infty,R>0,$

18) Logisitic distribution

$X \sim Logistic(\mu,\sigma)$

$$f_X(x)=\frac{e^{-\frac{(x-\mu)}{\sigma}}}{\left(1+e^{-\frac{(x-\mu)}{\sigma}}\right)^2}\times\frac{1}{\sigma},-\infty<x<\infty,$$

$-\infty<\mu<\infty,\sigma>0,$

19)Weibull distribution

$X \sim Weibull(\alpha,\beta,\gamma)$

$$f_X(x)=\gamma\times\left(\frac{x-\alpha}{\beta}\right)^{\gamma-1}\times\frac{1}{\beta}\times\exp\left(-\left(\frac{x-\alpha}{\beta}\right)^{\gamma}\right)$$

$,x>\alpha,\alpha>0,\beta>0,\gamma>0,$

20) Frechet distribution

$X \sim Frechet(\mu,\sigma,\alpha)$

$$f_X(x)=\alpha\left(\frac{x-\mu}{\sigma}\right)^{-\alpha-1}\times\frac{1}{\sigma}\times\exp\left(-\left(\frac{x-\mu}{\sigma}\right)^{-\alpha}\right),$$

$x>\mu,$

$-\infty<\mu<\infty,\sigma>0,\alpha>0$

21) Generalized extreme value distribution(GEV)
$X \sim GEV(\mu, \sigma, c = \xi)$

$$f_X(x) = \frac{1}{\sigma} \times \left(1 + \xi \times \left(\frac{x-\mu}{\sigma}\right)\right)^{(-1/\xi)-1}$$
$$\times \exp\left(-\left[1 + \xi \times \left(\frac{x-\mu}{\sigma}\right)\right]^{-1/\xi}\right), -\infty < x < \infty$$
$$-\infty < \mu < \infty, \sigma > 0, -\infty < \xi = c < \infty$$

22)Pareto3 distribution
$X \sim Pareto3(\lambda, c)$

$$f_X(x) = \lambda\left(1 - \frac{x}{c}\right)^{\lambda-1} \times \frac{1}{c}, 0 < x < c$$
$$\lambda > 0, c > 0$$

23)Triangular2 distribution
$X \sim Triangular2(a, b, c)$

$$f_X(x) = \begin{cases} \frac{2}{b-a} \times \frac{1}{c-a} \times (x-a), a \le x < c \\ \frac{2}{b-a} \times \frac{1}{b-c} \times (b-x), c \le x < b \\ 0, otherwise \end{cases}$$
$$a, b, c \in R, a < c < b$$

24)Triangular3 distribution
$X \sim Triangular3(a, b, c)$

$$f_X(x) = \begin{cases} \frac{2}{b-a} \times \frac{1}{c-a} \times (c-x), a \le x < c \\ \frac{2}{b-a} \times \frac{1}{b-c} \times (x-c), c \le x < b \\ 0, otherwise \end{cases}$$
$$a, b, c \in R, a < c < b$$

25)Log-logistic distribution
$X \sim Log\_Logistic(\alpha, \beta)$

$$f_X(x) = \frac{(\beta/\alpha)(x/\alpha)^{\beta-1}}{\left[1 + (x/\alpha)^{\beta}\right]^2}, x > 0,$$
$$\alpha > 0, \beta > 0,$$

26) Hyperbolic secant distribution
$X \sim Hyper\_secant(\mu, \sigma)$

$$f_X(x) = \frac{1}{2} \operatorname{sech}\left(\frac{\pi}{2} \times \left(\frac{x-\mu}{\sigma}\right)\right) \times \frac{1}{\sigma},$$
$$-\infty < x < \infty, -\infty < \mu < \infty, \sigma > 0$$

27)Kumaraswamy distribution
$X \sim Kumaraswamy(a, b)$

$$f_X(x) = abx^{a-1}(1 - x^a)^{b-1}, 0 < x < 1,$$
$$a > 0, b > 0$$

28)Raised cosine distribution
$X \sim Raised\_cosine(\mu, s)$

$$f_X(x) = \frac{1}{2s}\left[1 + \cos\left(\frac{x-\mu}{s} \times \pi\right)\right],$$
$$\mu - s \le x \le \mu + s, -\infty < \mu < \infty, s > 0$$

29)Gumbel distribution(Type 1)

$X \sim Gumbel(type \quad 1)(a,b)$

$$f_X(x) = ab\exp\left(-\left(be^{-ax} + ax\right)\right),$$
$$-\infty < x < \infty, a > 0, b > 0,$$

30)Gumbel distribution(Type 2)

$X \sim Gumbel(type \quad 2)(a,b)$

$$f_X(x) = abx^{-a-1}\exp\left(-bx^{-a}\right), 0 \le x < \infty,$$
$$a > 0, b > 0,$$

31) Generalized logistic distribution type I

$X \sim G\_Logistic(type \quad I)(\mu,\sigma,\alpha)$

$$f_X(x) = \frac{\alpha}{\sigma} \times \frac{\exp\left(-\left(\frac{x-\mu}{\sigma}\right)\right)}{\left(1+\exp\left(-\left(\frac{x-\mu}{\sigma}\right)\right)\right)^{\alpha+1}},$$
$$-\infty < x < \infty, -\infty < \mu < \infty, \sigma > 0, \alpha > 0,$$

32) Generalized logistic distribution type II

X

$\sim G\_Logistic(type \quad II)(\mu,\sigma,\alpha)$

$$f_X(x) = \frac{\alpha}{\sigma} \times \left(\frac{\exp\left(-\left(\frac{x-\mu}{\sigma}\right)\right)}{1+\exp\left(-\left(\frac{x-\mu}{\sigma}\right)\right)}\right)^{\alpha}$$
$$\times \left(\frac{1}{1+\exp\left(-\left(\frac{x-\mu}{\sigma}\right)\right)}\right), -\infty < x < \infty,$$
$$-\infty < \mu < \infty, \sigma > 0, \alpha > 0,$$

33) Generalized logistic distribution type III

X

$\sim G\_Logistic(type \quad III)(\mu,\sigma,\alpha)$

$$f_X(x) = \frac{\Gamma(2\alpha)}{\Gamma(\alpha)\Gamma(\alpha)} \times \frac{\exp\left(-\alpha \times \left(\frac{x-\mu}{\sigma}\right)\right)}{\left(1+\exp\left(-\left(\frac{x-\mu}{\sigma}\right)\right)\right)^{2\alpha}} \times \frac{1}{\sigma}$$
$$, -\infty < x < \infty, -\infty < \mu < \infty, \sigma > 0, \alpha > 0,$$

34) Generalized logistic distribution type IV

X

$\sim G\_Logistic(type \quad IV)(\mu,\sigma,\alpha,\beta)$

$$f_X(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \times \frac{\exp\left(-\beta\left(\frac{x-\mu}{\sigma}\right)\right)}{\left(1+\exp\left(-\left(\frac{x-\mu}{\sigma}\right)\right)\right)^{\alpha+\beta}}$$
$$\times \frac{1}{\sigma}, -\infty < x < \infty, -\infty < \mu < \infty, \sigma > 0, \alpha > 0, \beta > 0,$$

35)Truncated normal distribution

$X \sim Trucated\_Normal(\mu,\sigma,a,b)$

$$f(x)=\frac{\frac{1}{\sigma}\phi\left(\frac{x-\mu}{\sigma}\right)}{\Phi\left(\frac{b-\mu}{\sigma}\right)-\Phi\left(\frac{a-\mu}{\sigma}\right)}$$

$$=\frac{\frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)}{P(X\le b)-P(X\le a)},$$

$a\le x<b,-\infty<\mu<\infty,\sigma>0,b,a\in R,b>a$

36)Skew normal distribution

$X \sim Skew\_Normal(\mu,\sigma,\alpha)$

$$\phi(x)=\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{x^2}{2}\right),$$

$$\Phi(x)=\int_{-\infty}^{x}\phi(t)dt=\frac{1}{2}\left[1+erf\left(\frac{x}{\sqrt{2}}\right)\right]$$

$$f_X(x)=\frac{2}{\sigma}\phi\left(\frac{x-\mu}{\sigma}\right)\Phi\left(\alpha\times\left(\frac{x-\mu}{\sigma}\right)\right),$$

$-\infty<x<\infty,\alpha>0,-\infty<\mu<\infty,\sigma>0$

37)Folded normal distribution

$X \sim Folded\_Normal(\mu,\sigma)$

$$f_X(x)=\frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{(-x-\mu)^2}{2\sigma^2}\right)$$

$$+\frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right),$$

$0\le x<\infty,-\infty<\mu<\infty,\sigma>0$

38) Logit-normal distribution

$X \sim Logit\_Normal(\mu,\sigma)$

$$f_X(x)=\frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{(\log it(x)-\mu)^2}{2\sigma^2}\right)\frac{1}{x(1-x)},$$

$0\le x\le 1,-\infty<\mu<\infty,\sigma>0$

Logistic function

$$P(t)=\frac{1}{1+e^{-t}},\frac{d}{dt}P(t)=P(t)\times(1-P(t)),$$

$$1-P(t)=P(-t)$$

$$\log it(p)=\log\left(\frac{p}{1-p}\right)=\log(p)-\log(1-p),$$

39)Inverse Gaussian distribution

$X \sim Inverse\_Gussian(\mu,\lambda)$

$$f_X(x)=\left[\frac{\lambda}{2\pi x^3}\right]^{\frac{1}{2}}\exp\left(\frac{-\lambda(x-\mu)^2}{2\mu^2 x}\right),0\le x<\infty,$$

$\mu>0,\lambda>0$

40) Irwin–Hall distribution

$X \sim Irwin\_Hall(n)$

$$U_1,\ldots\ldots,U_n \overset{i.i.d.}{\sim} U(0,1), X = \sum_{k=1}^{n} U_k, n \in N,$$

41)Inverse Gamma distribution

$X \sim Inverse\_Gamma(\alpha, \beta)$

$$f_X(x) = \frac{\beta^{\alpha}}{\Gamma(\alpha)} x^{-\alpha-1} \exp\left(-\frac{\beta}{x}\right), x > 0,$$

$$\alpha > 0, \beta > 0,$$

42)t distribution

$X \sim t(\upsilon = df)$

$$f_X(x) = \frac{\Gamma\left(\frac{\upsilon+1}{2}\right)}{\sqrt{\upsilon\pi}\,\Gamma\left(\frac{\upsilon}{2}\right)} \left(1 + \frac{x^2}{\upsilon}\right)^{-\left(\frac{\upsilon+1}{2}\right)}, -\infty < x < \infty,$$

$$\upsilon = df \in N, \Gamma(\ ): gamma \quad function,$$

43)Chi-square distribution

$X \sim \chi^2(\upsilon = df)$

$$f_X(x) = \frac{x^{\upsilon/2-1}}{\Gamma\left(\frac{\upsilon}{2}\right) 2^{\frac{\upsilon}{2}}} \exp\left(-\frac{x}{2}\right), x > 0, \upsilon = df \in N,$$

44)Inverse χ_{υ}^2 distribution(the first kind)

$X \sim Inverse\_\chi^2(I)(\upsilon = df)$

$$f_X(x) = \frac{2^{-\upsilon/2}}{\Gamma\left(\frac{\upsilon}{2}\right)} x^{-\frac{\upsilon}{2}-1} \exp\left(-\frac{1}{2x}\right), x > 0,$$

$$\upsilon = df \in N,$$

45)Inverse χ_{υ}^2 distribution(the second kind)

$X \sim Inverse\_\chi^2(II)(\upsilon = df)$

$$f_X(x) = \frac{\left(\frac{\upsilon}{2}\right)^{\upsilon/2}}{\Gamma\left(\frac{\upsilon}{2}\right)} x^{-\frac{\upsilon}{2}-1} \exp\left(-\frac{\upsilon}{2x}\right), x > 0,$$

$$\upsilon = df \in N,$$

46)Scaled Inverse χ_{υ}^2 distribution

$X \sim Scaled\_Inverse\_\chi^2(\upsilon = df)$

$$f_X(x) = \frac{\left(\frac{\sigma^2\upsilon}{2}\right)^{\upsilon/2}}{\Gamma\left(\frac{\upsilon}{2}\right)} x^{-\frac{\upsilon}{2}-1} \exp\left(-\frac{\upsilon\sigma^2}{2x}\right), x > 0$$

$$\upsilon = df \in N, \sigma > 0,$$

47) F distribution

$X \sim F(df1 = \upsilon_1, df2 = \upsilon_2)$

$$f(x) \frac{\Gamma\left(\frac{df_1 + df_2}{2}\right)}{\Gamma\left(\frac{df_1}{2}\right)\Gamma\left(\frac{df_2}{2}\right)} \times \left(\frac{df_1}{df_2}\right)^{\frac{df_1}{2}}$$

$$\times x^{\frac{df_1}{2}-1} \times \left(1 + \frac{df_1}{df_2} x\right)^{-\frac{df_1+df_2}{2}}, x > 0,$$

$$\upsilon_1 = df_1 \in N, \upsilon_2 = df_2 \in N,$$

$$\Gamma(\ ): gamma \quad function,$$

48)Fisher's z-distribution
$X \sim Fisher's \quad \mathrm{Z}(df1, df2)$

$$Z = \frac{1}{2}\log F, F(df1, df2) = \frac{\chi^2_{df1}/df1}{\chi^2_{df2}/df2},$$

$$\chi^2_{df1} \sim Gamma\left(\frac{df1}{2}, 2\right), \chi^2_{df2} \sim Gamma\left(\frac{df2}{2}, 2\right),$$

$\chi^2_{df1}, \chi^2_{df2}$ *are independent r.v.'s*,

$df1, df2 \in N,$

49)Non-central t distribution
$X \sim Noncentral \quad t(\mu, \upsilon)$

$Z \sim N(0,1), V \sim \chi^2_{\upsilon},$

$$f_Z(z) = \frac{1}{\sqrt{2\pi}}\exp\left(-\frac{z^2}{2}\right), -\infty < z < \infty$$

$$f_V(v) = \frac{v^{df/2-1}}{\Gamma\left(\frac{df}{2}\right)2^{\frac{df}{2}}}\exp\left(-\frac{v}{2}\right), v > 0$$

$$X \sim T = \frac{Z+\mu}{\sqrt{V/\upsilon}}, -\infty < x < \infty, -\infty < \mu < \infty, \upsilon \in N,$$

50)Non-central χ^2_{υ} distribution
$X \sim Noncentral \quad \chi^2(\upsilon, \lambda)$

$X_i \sim N(\mu_i, \sigma_i^2), i = 1,2,\ldots,k$

$X_1, \ldots\ldots, X_k$ *are independent r.v.'s*

$$X = \sum_{i=1}^{k}\left(\frac{X_i}{\sigma_i}\right)^2, \lambda = \sum_{i=1}^{k}\left(\frac{\mu_i}{\sigma_i}\right)^2$$

$$X \sim \chi^2\left(\upsilon = k, \lambda = \sum_{i=1}^{k}\left(\frac{\mu_i}{\sigma_i}\right)^2\right)$$

51)Non-central F distribution
$X \sim Noncentral \quad F(\upsilon_1, \upsilon_2, \lambda)$

$X_1 \sim Non-central \quad \chi^2(\upsilon_1, \lambda)$

$X_2 \sim \chi^2(\upsilon_2), X_1, X_2$ *are independent*

$$X = \frac{X_1/\upsilon_1}{X_2/\upsilon_2} \sim Non-central \quad F(\upsilon_1, \upsilon_2, \lambda)$$

52)Generalized normal distribution
$X \sim Generalized\_Normal$
(μ, α, β)

$$f_X(x) = \frac{\beta}{2\alpha\Gamma\left(\frac{1}{\beta}\right)} \times \exp\left(-\left(\frac{|x-\mu|}{\alpha}\right)^{\beta}\right),$$

$-\infty < x < \infty, \alpha > 0, \beta > 0, -\infty < \mu < \infty$

53)Generalized half normal distribution
$X \sim Generalized\_half\_Normal$
(μ, α, β)

$$f_X(x) = \frac{\beta}{\alpha\Gamma\left(\frac{1}{\beta}\right)} \times \exp\left(-\left(\frac{|x-\mu|}{\alpha}\right)^{\beta}\right),$$

$\mu < x < \infty, \alpha > 0, \beta > 0, 0 < \mu < \infty$

54)Maxwell distribution

$X \sim Maxwell(\sigma)$

$$Z_1, Z_2, Z_3 \overset{i.i.d.}{\sim} N(0,1),$$
$$X = \sigma \times \sqrt{Z_1^2 + Z_2^2 + Z_3^2},$$
$$f_X(x) = \frac{x^2}{\Gamma\left(\frac{3}{2}\right) 2^{\frac{1}{2}} \sigma^3} \exp\left(-\frac{x^2}{2\sigma^2}\right),$$
$$x > 0, \sigma > 0$$

55)n independent Z square plus and root distribution

$X \sim n\_Z\_root(\sigma)$

$$Z_1, Z_2, \ldots\ldots, Z_n \overset{i.i.d.}{\sim} N(0,1),$$
$$X = \sigma \times \sqrt{Z_1^2 + Z_2^2 + \ldots. + Z_n^2},$$
$$f_X(x) = \frac{x^{n-1}}{\Gamma\left(\frac{n}{2}\right) 2^{\frac{n-2}{2}} \sigma^n} \exp\left(-\frac{x^2}{2\sigma^2}\right),$$
$$x > 0, \sigma > 0, \Gamma(\ ): gamma \quad function,$$

56) Hotelling's T-square distribution

$X \sim T_{p,m}^2$

$$\frac{m-p+1}{pm} X \sim F(p, m-p+1)$$
$$X \sim \frac{p \times m}{m-p+1} \times Y, Y \sim F(p, m-p+1)$$

57)Beta prime distribution

$X \sim Beta\_prime(\alpha, \beta)$

$$f_X(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1} (1+x)^{-\alpha-\beta}, x > 0$$
$$\alpha, \beta > 0, \Gamma(\ ): gamma \quad function,$$

58) Lévy distribution

$X \sim Levy(\mu, c)$

$$f_X(x) = \sqrt{\frac{c}{2\pi}} \times \frac{\exp(-c/2(x-\mu))}{(x-\mu)^{\frac{3}{2}}},$$
$$\mu \le x < \infty, -\infty < \mu < \infty, c > 0,$$
$$F(x) = erfc\left(\sqrt{c/2(x-\mu)}\right)$$
$$-\infty < \mu < \infty, c > 0$$

59)Bernoulli distribution

$X \sim B(1, p)$

$$f_X(x) = p^x (1-p)^{1-x}, x = 0,1,$$
$$0 < p < 1$$

60)Binomial distribution

$X \sim B(n, p)$

$$f_X(x) = \binom{n}{x} p^x (1-p)^{n-x}, x = 0,1,\ldots,n$$
$$0 < p < 1, n \in N,$$

The above probability distribution can be derived from the software program in directly.