



5.迴歸分析(含複迴歸分析)

<p>樣本資料的隨機變數名稱: X1, X2, X3, X4,</p> <p>[樣本資料的計算與圖形]</p> <p>~~~~~ 選擇 ~~~~~</p> <p>Regression analysis and linear model (迴歸分析與線性模式)</p> <p>[Parametric statistics(母數統計學)]</p> <ol style="list-style-type: none"> two sample data , the simple linear model three sample data, the general linear model four or more sample data, the general linear model <p>[Non-parametric statistics(無數統計學)]</p> <ol style="list-style-type: none"> two sample data , the simple linear model storing the simulationg sample data to a special file return <p><input type="text" value="3"/></p>	<p>four or more sample data computation and images. Please select random variables. There are random variables sample data: X1, X2, X3, X4, 1st random variable code (dependnent variable)</p> <p><input type="text" value="4"/></p>
<p>four or more sample data computation and images. Please select random variables. There are random variables sample data: X1, X2, X3, X4, 1st random variable is [X4], the other r.v.'s is different from 1st r.v. other random variables are independent variables, please input the code using ',' separately</p> <p><input type="text" value="1,2,3"/></p>	<p>[The sample data from the below random variables] dependent variable:X4, independent variable:X1,X2,X3 ~~~~~ choose one ~~~~~</p> <ol style="list-style-type: none"> The multiple linear analysis $X4=b0+b1*X1+...+b3*X3$, The multiple linear analysis $X4=b0+b1*X1+...+b3*X3$, [stage method] the criterion is coefficient of determination. The multiple linear analysis $X4=b0+b1*X1+...+b3*X3$, [stage method] the criterion is MSE. The multiple linear analysis $X4=b0+b1*X1+...+b3*X3$, stepwise analysis. The multiple linear analysis ,$X4=b0+H(X0)$, $H(X0)$ is the best estimated equaiton by the maximized coefficient of determination and $X0=k0+k1*X1+...+k3*X3$ The linear model $X4=b0+b1*X1+...+b3*X3+e$, the coefficient of slopes setting the special requirement The no intercept multiple linear analysis $X4=b1*X1+...+b3*X3$, [stage method] the criterion is coefficient of determination. return

Input data(採用模擬器所得樣本)

X1	X2	X3	X4
12.6602747950	8.9175958666	14.8483403924	35.0529883927
12.2198580428	14.0753056367	18.7749373366	47.3771389645
10.6236121600	-4.0871104772	-23.5813896697	-22.4768276312
11.0326863450	0.2423117116	15.1648356427	23.6648832701
9.1182169429	8.0915898932	18.0887649623	34.2872616785
11.5414006831	4.5338884985	-5.4417077181	13.6444626122
7.4139132069	11.0792860544	18.4118353864	36.3376161734
8.3951793930	19.8623912102	31.1508728123	60.3433414930
11.4826777338	18.7802136069	9.3503011745	37.3749919062
8.2597907683	13.4863413999	24.2464594776	46.4053064009
11.7206094576	10.0930212350	2.3141687614	24.5317760507
10.4164393781	5.4984292682	19.2910682797	39.4379988025
8.4384483845	11.7799108538	3.8864638909	27.6396135056
11.3232704150	4.0335653261	6.4133052887	19.5120317551
8.9572692269	12.0868179131	27.7728565354	45.6134492373
7.9614037901	8.6715229751	1.0115873578	17.2797074658
5.8859818622	-8.1010042114	-3.2671992458	-8.8121404241
9.9686504287	4.6050251802	11.0627826111	29.9633358932
9.3630160610	13.5908604671	2.0101061755	23.0803024694
10.6523736866	12.6041319794	32.5823034761	57.2614940030
8.8767874110	2.4657713194	-1.0055518613	8.7588213466
7.5516261368	11.1024566065	29.0674218941	49.5006227295
10.8332817497	14.0863239035	21.1194033476	43.6351239300
9.5135557493	1.3033912267	-5.4370879984	5.8024971430
12.7357814985	13.3897629078	30.5959623186	57.6480874359
11.2333401673	11.0495454322	19.4261255732	40.8679103123
8.2257487485	8.8950230939	-5.3648314154	9.9477037288
9.1041468634	8.1262943496	-7.6260323394	15.2182232921



13.3353524469	16.8072430144	24.1747604558	54.4617357539
9.4574088654	15.8697513618	40.9281326660	67.0309927135
8.2329298254	6.7878004585	3.3414659955	15.2341587517
9.8494029186	6.8439758560	7.8724491555	29.5221972735
8.6238918160	11.5823905790	20.8104395590	42.5472138430
11.2514300220	13.6282862203	18.0004317916	44.7831640923
10.8344016443	11.2221328895	1.2384651605	18.0165636885
10.7237430870	15.0701229545	18.6838459892	44.0051226358
12.0685400965	21.7024437966	21.8854474745	50.6740873327
9.5946618091	6.0093434247	-11.8155429053	5.8886963052
8.9463013541	8.5659316817	1.7874692155	17.2145420029
9.2806872255	13.4075928425	-4.8023112857	22.1982201216
10.6537047418	10.5490037872	12.7295755667	35.0797754433
11.3604071656	-2.8471609521	6.6589908817	20.4186140508
8.9137308074	3.6491702386	3.6702087671	10.9956887287
10.4351392680	16.6904292754	23.1678553482	53.7829351474
11.4936694143	7.6890416970	6.4843255545	26.6783294700
8.3349168320	7.1718316624	-0.5009895166	14.7916663833
9.9020811137	8.1508522583	15.3370635769	37.6425605950
9.1173291812	5.0341663814	22.4521683566	35.9417134467
9.2742942778	5.7169507488	2.2082807218	19.0742572096
10.4371160348	-2.1718638981	6.0805730806	17.1394902164
11.7277182379	13.5636133527	3.1829864742	30.0946681353
14.5229849002	9.4246926849	25.5400518193	49.9191582034
9.4441586506	10.3979855936	6.9014011560	25.5651138901
12.1963531953	3.1549873451	8.0168502854	21.9139739182
9.0482762084	4.9721098098	11.7775982995	27.2030172219
11.5266278297	15.6444807002	11.3710567884	36.2401922961
11.3993372439	14.7943552352	3.4363254404	30.3397450821
12.1432124650	4.9680983529	17.0456901188	28.8651121772
8.8199623819	4.6698289321	10.5428444377	22.9107494564
7.8347183716	13.2516619361	10.1022011367	30.9941195868
11.3320891034	4.9545280783	10.8447124655	30.9310451455
6.5372267798	8.1565957314	8.3129122377	19.9270388341
12.1075679641	9.8044888124	7.3345220622	35.5778999748
7.2310397884	8.7026896022	15.1698623415	25.8519161379
8.3323241340	19.1802266714	28.5690897884	57.1211878701
9.0280288356	16.4800365479	21.6442184787	46.5997841222
11.5885790463	8.8097564881	18.2829591343	37.7381415414
11.7942894169	12.0967820136	21.5527030321	48.2060735559
11.1081479001	9.1730444586	6.1380052003	27.8129274017
8.5385849508	7.8971635930	3.1085042025	24.4598779094
9.0054450950	5.4489548663	-3.3187064237	8.7958202840
11.1489009452	14.0664471162	14.8285897588	35.5609583486
10.8629534241	15.5709460237	28.9084040949	59.4076213026
8.0302492246	10.9190912503	22.1768773872	40.0512865271
5.6598275926	4.3797804646	18.3481562740	31.0169645994
10.3546296638	0.5847680933	3.4542981697	20.4907689729
10.0016734175	8.1589576436	12.3673977693	34.1138442210
9.1862330774	10.5050606924	5.3219040777	24.1728960663
9.1959332514	6.4534820356	11.2470315222	25.6705133943
11.2700948635	22.7686156646	1.9834477893	42.6365647541
8.6669808492	4.7977594113	-2.4601004346	15.2548687195
8.2709400203	12.2953150658	21.3582245359	40.8745544419
5.1886803954	11.9707791643	4.3875309394	20.5870293382
11.5923061899	5.9950019652	-0.6925206356	21.6190979209
8.3310808405	-1.2194221080	-21.3067801559	-20.7713827364
12.0399197493	8.5835709722	15.0632375562	35.8217450974
7.7232696786	10.7882602197	18.2463359219	37.1449865028
11.3379261348	10.3286403187	3.0480022369	26.0290161585
13.7329660326	10.3130659521	-6.7347604581	14.8058455771
11.5852260279	19.7258297565	7.3610235148	42.1564485341
10.7130624736	14.2329553920	14.5607959543	39.7325237662
7.6209475915	10.3614266725	-1.6380220443	16.7777336437
10.0622885554	9.7132617628	5.0484562596	23.8447979371
11.4174134467	11.5524826489	22.3405666911	40.7798127119
9.3407663092	6.8405859423	13.5702868287	31.2403945916
11.1833694606	10.3050286098	9.7247457442	28.9079315841
10.4423171239	15.3325032913	6.8911178606	36.7976322595
5.8168861542	1.7757132067	-0.3525248364	6.5832975557
8.5174742929	17.4469813817	31.9039169511	60.6749721767
9.6804419082	9.8158257429	1.0263120390	23.7284254094
6.7786290257	-0.1334991769	-7.3872613305	-1.0028731102
11.5083395794	15.2554859073	33.7213458626	57.6895935712
10.4281262273	-0.6925707245	3.2598910174	15.8793162142



10.5102287192	17.8475949146	28.3042609496	56.5076309410
11.9693987026	7.7578531356	-4.1740973618	16.1868701961
8.3159111425	11.6324381870	-10.4107208308	9.7731797177
9.1172812971	13.5056294966	9.2205429911	25.1236318334
10.4691884289	14.2972856617	7.0715160309	33.5114523773
9.5464083750	11.2618029546	27.8544371758	52.2835722885
8.6309800723	17.2083259460	24.9202383980	52.3413382744
11.2492615851	11.1362131345	8.6391453657	34.7072537134
12.0160618443	9.8364987675	9.3294650761	31.6131775353
7.3074778812	2.9967817214	2.6288549114	12.2559779012
7.8251921854	8.3208567084	17.4270371314	33.1228459036
11.7119823497	14.7449972160	21.3723269375	52.4628254430
11.6513295314	15.1463732564	15.9899218548	45.9556587678
11.1132469897	5.9072718741	1.0369805280	23.3011075088
9.6007920790	8.7792462028	36.4597746678	57.0378792955
10.1324819097	19.6660714943	24.2597166560	55.5542221404
9.9641544966	7.8594885749	14.5488666190	31.4538217451
11.9751086817	16.5795832061	20.9676808091	51.7479521363
9.5276785234	9.2245347158	14.4200813385	36.4371052889
8.8942400735	17.2588284092	18.3027374984	45.7211340940
8.7830683681	9.3457083304	-6.1430359078	16.5717624586
10.2272982048	8.6736007486	-2.4776774485	18.5538585401
13.1982671477	23.4082188210	31.4640591387	68.6848046183
11.2311970728	9.2885128990	9.6294004891	35.5121210167
9.1820373942	11.6978831702	21.3187335399	45.8794966181
7.0793434226	4.1271718559	-9.5596335528	1.8925216898
10.3485980283	8.0679514939	0.6456377999	22.7971669325
9.8769430344	11.3674225994	28.4845207274	52.4434000669
11.9825399107	19.7813798120	11.4358143837	42.8946198568
10.0227494422	10.1529622088	3.3630991846	25.7818530929
6.6787476329	5.6645130578	27.7200086976	38.5811156886
7.1462155896	9.0264739769	-12.1093039564	6.0503478301
6.4190375591	12.5332936767	-12.9310924355	2.8228759581
8.9159982879	8.2481364891	16.5396342469	27.7469267491
10.2722861546	6.7799410685	7.1523031356	27.5159589063
10.0194458441	3.2712509762	0.1740533117	15.3490509703
9.7720600485	0.2662511850	-12.9092608187	-1.9294291382
10.7893877221	16.1233407694	35.2495256451	59.4491511432
7.9873699138	14.8034874533	16.8193293358	44.5271511166
11.6241341437	4.3804218892	3.0174588242	22.0295278019
9.7846829167	8.2458513704	1.6928998748	19.3402199752
10.7065531178	19.4064243759	16.3312693134	51.9616157723
11.8038901645	10.2529752497	30.1819721496	52.3332383000
12.6578004998	11.0441920244	10.8083491025	35.7942061965
7.1814573299	3.8848352608	2.4131497128	19.0051013710
12.5930073254	24.1454275031	28.2627054391	64.6993846233
8.0644551582	-1.5720363232	-8.5067412589	-4.2527957007
11.1200031814	10.5611028165	0.9849607934	15.4210296494
9.4114779020	10.3749950459	-6.3071132144	19.1131206775
10.0703703941	2.2607481891	1.7525839182	11.6650732513
8.5185161576	6.5861753409	-6.6114695227	9.6253472962
11.2242114762	3.9790218729	4.8301901172	20.1696159903
10.3065227094	13.7907511975	5.9009326373	35.5162483257
7.7279112175	4.6466976220	-3.2721305093	5.9186386112
9.9018702967	7.7649081086	-2.0303977597	20.2688889258
7.2407502021	6.3245633613	-9.4141432968	2.9056361815
8.9774808161	3.6009680002	2.4022312870	19.7147242582
4.9413389072	-3.4587028292	0.9990336926	2.3428904917
12.7741988337	15.0815547469	10.2298727881	37.9590915217
9.2676320087	20.3261677853	18.3499502662	46.8652310904
11.0275294766	5.4776007039	3.2216555702	23.5751079429
11.3068071831	16.7226366248	1.8478182994	33.1727846578
9.9370630558	7.4941773619	13.3648082762	35.6206408826
10.0680032406	8.0090796402	-1.7421415734	11.7807510707
11.8799778134	11.1077862482	3.7995753943	28.1628856800
8.7858346842	8.0628024902	14.0339003004	31.9802156306
10.3294130761	13.8688135582	10.2318188543	37.7775756623
12.6721315844	16.1566305731	32.2963019467	62.9221140308
9.5126904423	15.6046692444	33.1037029691	64.3972809688
9.7405856058	9.6199575916	11.1590072937	30.2550508082
8.4586526536	-2.1062972529	-13.5218788262	-9.3130780269
7.9852288370	11.2106697559	17.9860905719	40.6821043560
9.6578908842	16.3755651866	10.4544575873	39.8129536758
8.6561513325	8.9417884951	-3.4193727619	15.2479991917
10.9956079155	18.8168843166	8.5380626297	38.1188912976



10.5198007815	9.6001483148	-3.9794863595	18.5837172387	
12.2803980219	14.8527961574	24.9825806028	47.0584275429	
7.4026615167	12.4559359975	3.8699508498	21.9529123025	
9.8371885530	6.3509601815	12.0931640655	30.4208630697	
11.2642302401	11.6162667720	22.7767805912	47.6422197468	
11.8877315345	17.9306975136	25.3948591354	57.4676397056	
7.4016193140	9.6570807273	0.4958224608	15.9417005102	
4.9771146003	5.0289415643	-5.0459886290	10.9536425313	
9.0539519387	13.0654335475	13.5981893364	38.5654802839	
12.6171474137	9.7663427614	22.5963903695	47.4555694776	
10.6135711739	9.6608094312	4.6145374221	28.8172804131	
11.8671083134	12.4919474662	9.8336814141	27.9110441472	
10.1669259481	9.6464686694	15.6377994642	38.5329979887	
4.5017998566	3.0755773089	11.3037560432	11.2272359882	
9.8661673424	4.6111654010	3.2275457498	23.6668504843	
11.6057214535	15.7022009077	21.4015801047	51.2467009387	
13.9003423122	13.8777809949	17.7904130641	46.1144505983	
12.4969336395	19.8619768505	10.3004377868	42.7556469233	
8.2597799203	14.8390961596	12.3014399660	34.7977314497	
10.0610733948	15.4154072756	0.7837669468	25.2212233747	
7.7333334611	17.2917500510	14.6721790527	36.2361018275	
5.7705786505	2.6246258437	11.3426461002	16.6864468759	
X1 is	Normal(mu=10.000000,sigma*sigma=4.000000),			
X2 is	Normal(mu=H1,sigma*sigma=25.000000),			
	H1(X1)= X1.			
X3 is	Normal(mu=H1,sigma*sigma=100.000000),			
	H1(X2)= X2.			
X4 is	Normal(mu=H1,sigma*sigma=11.560000),			
	H1(X1,X2,X3)= 1.000000+1.000000*X1+1.000000*X2+1.000000*X3.			
X1 is	mean=	9.8652250732, s.d.=	1.8561882885, variance=	3.4454349625,
	skewed coefficient=-	0.3621357071, kurtosis coefficient=	2.9647520321, MAD=	1.4923180730,
	Q1=	8.6435657024, median=	9.9664024626, Q3=	11.3068071831,
	MIN=	4.5017998566, MAX=	14.5229849002, Range=	10.0211850436,
	Mid-Range=	9.5123923784, C.V.=	0.1881546822, sample size=	200
X2 is	mean=	9.9586935194, s.d.=	5.6567604340, variance=	31.9989386075,
	skewed coefficient=-	0.1776578936, kurtosis coefficient=	3.1026878733, MAD=	4.4123483302,
	Q1=	6.3377617714, median=	9.8261622552, Q3=	13.8777809949,
	MIN=	-8.1010042114, MAX=	24.1454275031, Range=	32.2464317145,
	Mid-Range=	8.0222116458, C.V.=	0.5680223438, sample size=	200
X3 is	mean=	10.0789361104, s.d.=	12.0398018630, variance=	144.9568289006,
	skewed coefficient=	0.0549122715, kurtosis coefficient=	2.6325252185, MAD=	9.7988829914,
	Q1=	1.8176437575, median=	9.7792135791, Q3=	18.3499502662,
	MIN=	-23.5813896697, MAX=	40.9281326660, Range=	64.5095223357,
	Mid-Range=	8.6733714981, C.V.=	1.1945508664, sample size=	200
X4 is	mean=	30.5871639151, s.d.=	16.9634123978, variance=	287.7573601780,
	skewed coefficient=-	0.1812442772, kurtosis coefficient=	2.9748969402, MAD=	13.6390718412,
	Q1=	19.0936889436, median=	30.3803040759, Q3=	42.6365647541,
	MIN=	-22.4768276312, MAX=	68.6848046183, Range=	91.1616322495,
	Mid-Range=	23.1039884935, C.V.=	0.5545925227, sample size=	200
	random variables sample correlation coefficient-----			
r(X1,X2)=	0.3419420085			
r(X1,X3)=	0.2489800180			
r(X1,X4)=	0.4228777986			
r(X2,X3)=	0.5352106262			
r(X2,X4)=	0.7643985744			
r(X3,X4)=	0.9219938824			



Output data

Dependent variable is X4,
 Independent variables are X1,X2,X3
 The correlation matrix is below
 $r(X4,X1)=0.422878,$
 $r(X4,X2)=0.764399,$
 $r(X4,X3)=0.921994,$
 $r(X1,X2)=0.341942,$
 $r(X1,X3)=0.248980,$
 $r(X2,X3)=0.535211,$
 The estimated line is $X4=-1.380403+1.188822*X1+1.024563*X2+0.995765*X3$

ANOVA

Source	df	SS	MS
Regression	3	55419.1946378448	18473.0648792816
error	196	1844.5200375687	9.4108165182
total	199	57263.7146754136	

The F test p value=0.000100

Individual test

variable	coefficient	standard error	t test	p value
intercept	-1.3804025628	1.1819404644	-1.16791	0.24420
X1	1.1888221413	0.1251039606	9.50267	0.00000
X2	1.0245633366	0.0470670211	21.76818	0.00000
X3	0.9957654372	0.0214565862	46.40838	0.00000

MSE= 9.4108165182 , R2=0.967789 , R2(adj)=0.967296
 dependent variable:X4 , sample mean= 30.5871639151 , sample variance=287.757360
 independent variable:X1 , sample mean= 9.8652250732 , sample variance=3.445435
 independent variable:X2 , sample mean= 9.9586935194 , sample variance=31.998939
 independent variable:X3 , sample mean= 10.0789361104 , sample variance=144.956829

----- Regression Coefficient Variance and Covariance Matrix -----
 Var(b0)= 1.3969832614, Cov(b0,b1)= -0.1371944854, Cov(b0,b2)= -0.0022106351,
 Cov(b0,b3)= 0.0025340309,
 Cov(b1,b0)= -0.1371944854, Var(b1)= 0.0156510010, Cov(b1,b2)= -0.0015019794,
 Cov(b1,b3)= -0.0002230800,
 Cov(b2,b0)= -0.0022106351, Cov(b2,b1)= -0.0015019794, Var(b2)= 0.0022153045,
 Cov(b2,b3)= -0.0004994117,
 Cov(b3,b0)= 0.0025340309, Cov(b3,b1)= -0.0002230800, Cov(b3,b2)= -0.0004994117, Var(b3)= 0.0004603851,

~~~~~ individual test ~~~~~

| variable  | coefficient   | standard error | t value | F value   |
|-----------|---------------|----------------|---------|-----------|
| intercept | -1.3804025628 | 1.1819404644   | -1.1679 | 1.3640    |
| X1 slope  | 1.1888221413  | 0.1251039606   | 9.5027  | 90.3008   |
| X2 slope  | 1.0245633366  | 0.0470670211   | 21.7682 | 473.8536  |
| X3 slope  | 0.9957654372  | 0.0214565862   | 46.4084 | 2153.7379 |



```

~~~~~ The goodness of fit for the residual~~~~~
class [1] [2] [3] [4] [5] [6]
[7] [8]
lower limit -3.52896 -2.06906 -0.97736 0.00008 0.97734
2.06894 3.52866
upper limit -3.52896 -2.06906 -0.97736 0.00008 0.97734 2.06894
3.52866
observed no 21.00000 22.00000 30.00000 23.00000 25.00000 22.00000
29.00000 28.00000
probability 0.12500 0.12500 0.12500 0.12500 0.12500 0.12500
0.12500 0.12500
expected no 25.00000 25.00000 25.00000 25.00000 25.00000 25.00000
25.00000 25.00000
chi square 0.64000 0.36000 1.00000 0.16000 0.00000 0.36000
0.64000 0.36000

degree of freedom=6
H0: residual~Normal(0,sigma(error)*sigma(error)), sigma(error) are unknown
pearson chi-square test statistic =3.520000
p-value=0.741300

~~~~~ The run test of residual~~~~~
number of the negative of residual=96
number of the positive ofresidual=104
H0: residualis random , H1: Increasing line or decreasing line
Z=1.300779, p-value=0.903400
H0: residual is random , H1: Oscillation
Z=1.300779, p-value=0.096600
H0: residual is random , H1: Increasing line or decreasing line or Oscillation
Z=1.300779, p-value=0.193200

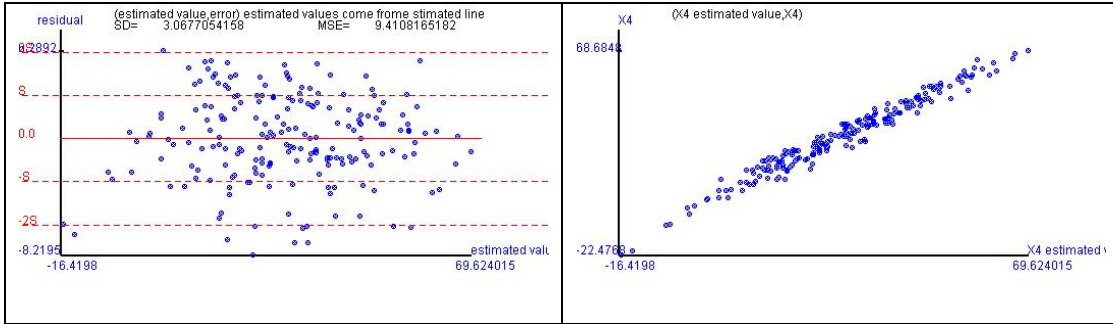
~~~~~ Durbin Watson test ~~~~~
The first order auto regressive error model
Model: e(t)=auto correlation coefficient * e(t-1) + new error (t)
t=2,3,...,200
H0: auto correlation coefficient=0 , H1:auto correlation coefficient > 0
D.W. test=2.303033

H0: auto correlation coefficient=0 , H1:auto correlation coefficient < 0
D.W. test=1.696967

The D.W. table depends on the independent value,
getting the p value must be spending a lot of time.
The Durbin Watson table is wrong in present.

[Please run the Durbin Watson critical value table software
to check the test value is rejected H0 or failed to reject H0.]

```



[ The X4 multi-linear model and regression analysis ]

dependent variable is X4,  
independent variables are  
X1, X2, X3

The line model is  $X4=b_0+ b_1*X1+ b_2*X2+ b_3*X3$

The estimated line is stored in c:\book\_plot\ANOVA\_Mul\_tep.txt

~~~~~ choice one ~~~~~

1. The computation data output
2. The (X4 estimated,error) residual plot error=X4-X4 estimaed value
3. The (X4 estimaed value ,X4) scatter diagram
4. The residual analysis
5. The correlation analysis of dependnent variable and estimated value
6. return

The two populations correlation ceofficient,H0 rho[X4,X4 estimated value]=c

c=

~~ 模擬樣本相關係數的抽樣分配 ~~

Output data

two populations correlation coefficient test  
H0: rho(X4,X4 estimate value)=0.000000  
r(X4,X4 estimate value)=0.983763 ,n=200  
left tail test p-value= 1.0000  
right tail test p-value= 0.0000  
two tailtes test p-value= 0.0000

90% confidence interval for r(X4,X4 estimate value) under rho(X4,X4 estimate value)=0.000000  
[-0.116654, 0.116660]

95% confidence interval for r(X4,X4 estimate value) under rho(X4,X4 estimate value)=0.000000  
[-0.138803, 0.138811]

99% confidence interval for r(X4,X4 estimate value) under rho(X4,X4 estimate value)=0.000000  
[-0.181998, 0.181772]

The X1 value from 0 to 9, but expected number >=5 in each cell  
The frequency distribution is modified as follow

pearson goodness of fit

| X1 value    | [ 2 ]   | [ 3 ]   | [ 4 ]   | [ 5 ]    | [ 6 ]   | [ 7 ]   | [ 8 ]   |
|-------------|---------|---------|---------|----------|---------|---------|---------|
| observed no | 4.00000 | 5.00000 | 9.00000 | 14.00000 | 6.00000 | 3.00000 | 9.00000 |
| probability | 0.10879 | 0.12928 | 0.16806 | 0.17479  | 0.15148 | 0.11253 | 0.15508 |
| expected no | 5.43933 | 6.46394 | 8.40313 | 8.73925  | 7.57402 | 5.62641 | 7.75392 |
| chi square  | 0.38087 | 0.33155 | 0.04240 | 3.16680  | 0.32711 | 1.22601 | 0.20025 |

degree of freedom=5

pearson chi-square test statistic =5.674989  
p-value=0.339100